
# PHYSICS

## Class – VIII

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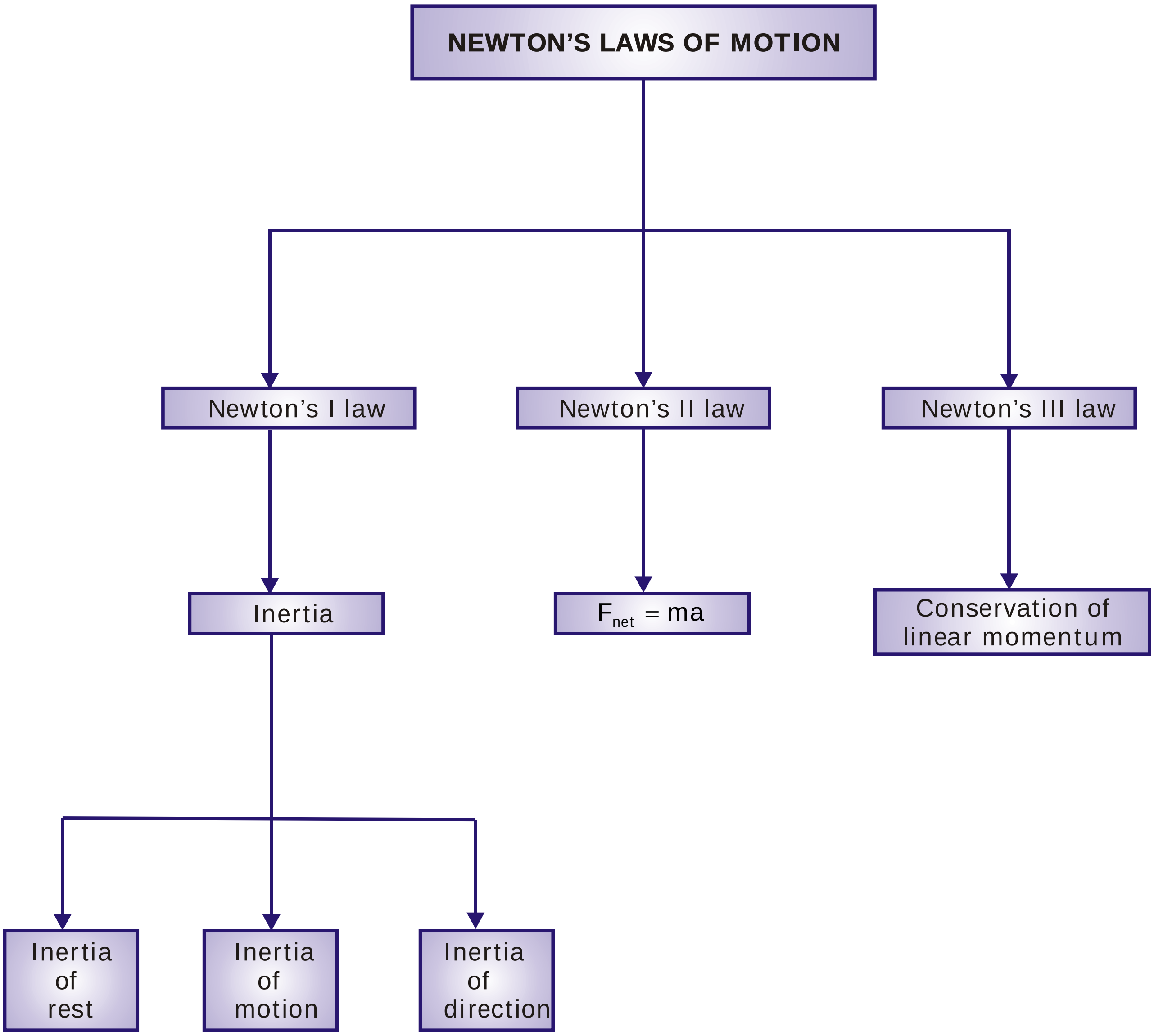
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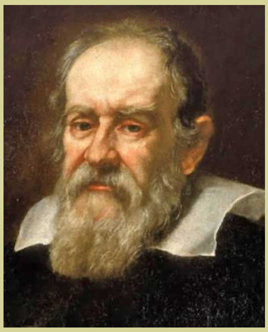
## KNOW YOUR SCIENTIST

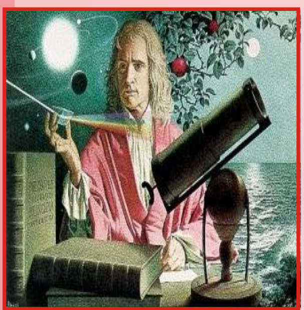


**Galileo Galilei (1564 - 1642)**

### Galileo Galilei (1564 - 1642)

Galileo Galilei was a key figure in the scientific revolution in Europe about four centuries ago. Galileo proposed the concept of acceleration. From experiments on motion of bodies on inclined planes or falling, freely, he contradicted the Aristotelian notion that a force was required to keep a body in motion and that heavier bodies fall faster than lighter bodies under gravity. He thus arrived at the law of inertia that was the starting point of the subsequent epochal work of Isaac Newton. Newton brought out another masterpiece optics that summarized his work on light and colour.

## Issac Netwon (1643 - 1727)



In 1684, encouraged by his friend Edmund Halley, Netwon embarked on wring what was to be The principia Mathematica was one of the greatest scientific works ever published. He enunciated all three laws of motion and the universal law of gravitation, which explained all the three Kepler's laws of planetary motion. The book was packed with a host of path - breaking achievements: basic principles of fluid mechanics, mathematics of wave motion, calculation of masses of the earth, the sun and other planets, explanation of the precession of equinoxes, the tides, etc. Newton brought out another masterpiece Opticks that summarized his work on light and colour.

# FORCE

## SYNOPSIS-1

1. **Newton's first law of motion :**
A body at rest will remain at rest and a body in motion will remain in uniform motion, unless it is compelled by an external force to change its state of rest or of uniform motion.
Newton's first law of motion is also called **Galileo's law of inertia.**
2. **Inertia :**Inertia of a body may be defined as **the tendency of a body to oppose any change in its state of rest or uniform motion.**
3. **Mass and inertia :** Inertia is an inherent property of each body by virtue of which it has a tendency to resist the change in its state of rest or state of uniform motion. The property of inertia is because of the mass of body. The greater the mass, the greater is the inertia of body.
4. **Inertia of rest :** The tendency of a body by virtue of which it cannot change its state of rest by itself is called inertia of rest.
5. **Inertia of motion :** The tendency of a body by virtue of which it cannot change its state of motion by itself is called inertia of motion.
6. **Inertia of direction :** The tendency of a body to oppose any change in its direction of motion by itself is known as inertia of direction.
7. **Linear momentum :-**Quantity of motion possessed by a moving body is known as momentum of the body. ( or ) The total quantity of motion contained in a body is called momentum.
8. **Mathematical expression :** Momentum of a body is equal to the product of the mass (m) of the body and the velocity (V) of the body. It is denoted by **P = m×v** .
9. **Units of momentum :** S.I. unit of momentum = S.I unit of mass × S.I unit of velocity = kg × m/s = kg m/s. Similarly C. G. S. unit of momentum is **g cms$^{-1}$.**
The direction of momentum of a body is same as that of the direction of the velocity of the body.
10. **Change of momentum :** If 'u' and 'v' are the initial and final velocity of a body then its, initial momentum = mu final momentum = mv
Now change of momentum = final momentum – initial momentum = mv – mu
11. **Newton's second Law of motion :**Newton's second law gives the quantitative definition of force in other words it measures force.According to the Newton when force is applied upon the body then acceleration is produced.
12. **Newton's 2$^{nd}$ Law in terms of momentum :**The rate of change of momentum of an object is proportional to the net force applied on the object. The direction of the change of momentum is the same as the direction of the net force.
13. **Absolute Units of Force :** S.I unit of force = kg m/s$^2$ = newton
C.G.S unit of force = g cm/s$^2$ = dyne.
14. **Definition of newton (N) :** 1 newton is that much force which produces an acceleration of 1 m/s$^2$ in a body of mass 1 kg.
15. **Definition of dyne :** The force is said to be 1 dyne if it produces 1 cm/s$^2$ acceleration in a body of 1g mass.

**16. Relation between newton and dyne :**

1 newton (N) = 1 kg $m/s^2$= 1000 g × 100 $cm/s^2$

= 100000 g $cm/s^2$ = $10^5$ $gcm/s^2$ = $10^5$ dyne

**17. Gravitational Units of force :** Gravitational units of force is the force which produces an acceleration equal to the acceleration due to gravity in a unit mass.

**18.** a) Gravitational unit of force in S.I. system is kilogram weight or kilogram force.

1 kg wt = 1 kg × 9.8 $m/s^2$ (on earth)

1 kg wt (kgf) = 9.8 kg $m/s^2$ = 9.8 N(on earth)

b) Gravitational unit of force in C.G.S. system is 1 gram weight or gram force.

1 g wt = 1g × 980 $cm/s^2$ (on earth)

1 g wt = 980 g cm $/s^2$ = 980 dyne(on earth)

**19. Newton's Third Law :**'To every action, there is an equal and opposite reaction' Action and reaction force are equal in magnitude but opposite in direction.

i.e. Action = – Reaction

**20. Linear Momentum :** If a body of mass m is moving with a velocity v then its momentum is given as $\vec{p} = m\vec{v}$. Momentum is a vector quantity. It is mesurment of motion of a body.Its units are kgm/s in SI system and gcm/s in CGS system.

# WORKSHEET-1

**CUQ** 1. Newton's first law of motion is also called _______law of inertia.

1) Newton's 2) Charles 3) Galileo's 4) Coulombs

2. Newton's second law gives the quantitative definition of ________

1) Mass 2) Velocity 3) Speed 4) Force

3. Force = Mass (Final velocity - initial velocity) ÷ _________

1) Velocity 2) Time 3) Speed 4) Distance

4. Mathematically force is equal to ______.

1) Mass ÷ Acceleration 2) Mass ÷ Time

3) Mass × Acceleartion 4) Mass × Time

5. The quantity of motion of a body is best represented by

1) its mass 2) its speed

3) its velocity 4) its linear momentum

6. The behavior of a body under zero resultant force is given by

1) Newton's third law of motion 2) Newton's second law of motion

3) Newton's first law of motion 4) Newton's law of gravitation

7. Which law of Newton defines an 'inertial frame of reference'?

1) First law of motion 2) Second law of motion

3) Third law of motion 4) Law of gravitation

8. The statement "acceleration is *zero* if and only if the net force is *zero*" is valid in

1) non-inertial frames

2) inertial frames

3) both in an inertial frames and non-inertial frames

4) neither inertial frames nor non-inertial frames

9. You lunge forward when your car suddenly comes to a halt and you are thrown backward when your car rapidly accelerates. Which law of Newton in involved in these?
   1) third law 2) second law 3) first law 4) law of gravitation
10. Which of the following is the most significant law of motion given by Newton?
   1) First law of motion 2) Second law of motion
   3) Third law of motion 4) Zeroth law of motion

## JEE MAINS

**Single Correct Choice Type:**

1. Force generally denotes
   1) a push 2) a pull 3) neither (1) nor (2) 4) Both (1)&(2)
2. A force may be defined as
   1) An external cause which changes the state of rest
   2) An external cause which tends to change the state of rest
   3) Both (1) & (2) 4) neither (1) nor (2)
3. A rider falls forward, when a galloping horse stops suddenly is an example of
   1) Inertia of rest 2) Inertia of direction
   3) Inertia of motion 4) Both(1) & (2)
4. When a fan is switched off, it continues to move due to
   1) Inertia of rest 2) Inertia of motion
   3) Inertia of direction 4) Both (1) &(2)
5. 1 gram weight = ________gm. cm / $s^2$
   1). 9.8 2) 98 3) 980 4) 9800
6. 1kg.wt =______ N
   1). 9.8 2) 98 3) 980 4) 9800
7. 1kg.wt =______ kg.m.$s^{-2}$.
   1). 9.8 2) 98 3) 980 4) 9800
8. If action force acting on a body is gravitational in nature, the reaction force
   1) may be a contact force 2) must be gravitational too
   3) may be a gravitational or contact force 4) may be a force of any origin
9. Action and reaction can never balance out because
   1) they are equal but not opposite always
   2) they are unequal in magnitude even though opposite in direction
   3) though they are equal in magnitude and opposite in direction they act on different bodies
   4) they are unequal in magnitudes
10. Inertia of a body has a direct dependence on
   1) Velocity 2) Mass 3) Area 4) Volume
11. Two bodies having masses 1kg and 4kg. In these two which will have more inertia.
   1) 1kg 2) 4 kg 3) Same 4) Can't say
12. When two bodies made of different material (iron, copper) having same mass. Then their inertia
   1) more for iron 2) more for copper 3)same for both 4)can't say

**Multi Correct Choice Type:**

13. Force effects
    1) A stationary object into motion 2) A moving body to stop
    3) Direction of moving body 4) Dimensions of the body

**Reasoning Type:**

14. *Statement I :* When a carpet suddenly jerked, the dust flies off.
    *Statement II :* The tendency of a body to continue in its state of rest , even when some external force is applied on it is called inertia of rest
    1) Both Statements are true, Statement II is the correct explanation of Statement I.
    2) Both Statements are true, Statement II is not correct explanation of Statement I.
    3) Statement I is true, Statement II is false.
    4) Statement I is false, Statement II is true
15. Pickup the correct statements from the following:
    a) Force is not required to keep a body moving with uniform velocity on surface
    b) During the motion of a body it moves with uniform velocity
    c) In the absence of a force the body moves with uniform velocity
    d) Internal forces cannot impart any motion to the body
    1) a,b are true 2) c,d are true 3)a,c are true 4)b,d are true
16. Identify the correct statements in the following
    a) Inertia is defined with the help of Newtons Ist law of motion
    b) Newton's 2nd law helps us to measure the force
    c) A body in rotation posseses inertia
    d) mass is the measure of inertia
    1) a,b & c are true 2) b,c & d are true
    3) c,d & a are true 4) a,b,c & d are true
17. Pick out the examples of inertia of rest in the following.
    a) Ripen fruits can be made to fall by moving violently the branch
    b) When a moving bus is stopped suddenly the passengers fall forward.
    c) A coin is kept on a cardboard placed on a tumbler. When the cardboard is given an impulse if flies off but the coin falls into the tumbler.
    d) When a bullet is fired on to a glass plane a fine hole is formed.
    1) a,b & c are true 2) a,b are true 3)a,b & d are true 4) a,c & d are true

**Matrix Match Type:**

18.

| Column - I | Column - II |
|---|---|
| a) Rocket motion | 1) Normal reaction force |
| b) Inertia | 2) Force of friction |
| c) Magnitude of force | 3) Newton's first law |
| d) Contact forces | 4) Newton's second law |
| | 5) Newton's third law |

19. S I: If two objects of different masses have same momentum, the lighter body posses greater velocity.
S II: For all bodies momentum always remains same if force applied is same. For a given time
1) Both I & II True
2) Both I & II False
3) Statement I is True & Statement II is False
4) Statement I is False & Statement II is True

**Multi Correct Choice Type:**

20. Choose the correct statements:
1) Both action and reaction are forces
2) Action and rection act simultaneously but act on differnt bodies
3) Action and reaction cannot cancel each other
4) Action and reaction forces occurs in pairs only

21. Choose the correct statements:
1) 1kg-wt = 9.8 kg m $/s^2$
2) 1g-wt = 980 g m $/s^2$
3) 1kg-wt = 980 g cm $/s^2$
4) 1g-wt = 980 dyne

**Comprehension Type:**

Inertia of a body may be defined as the tendency of a body to oppose any change in its state of rest or uniform motion.

22. Two bodies 'A' and 'B' of same masses are moving with velocities V and 2 V respectively. Then the ratio of their inertia is
1) 1 : 1 2) 2 : 1 3) 3 : 1 4) 4 : 1

23. Which of the following has the largest inertia ?
1) A pin
2) A pen
3) Your physics book
4) Your loaded school bags

24. An athlete runs some distance before taking a long jump, because
1) It helps him to gain energy
2) It helps to apply large force
3) It gives himself large amount of inertia
4) None of these

## SYNOPSIS-2

**1. Types of forces:** There are basically three forces which are commonly encountered in mechanics.

**(a) Field forces (non contact forces) :** These are the forces in which contact between two objects is not necessary. Gravitational force between two bodies and electrostatic force between two charges are two examples of field forces. Weight (W = mg) of a body comes in this category.

**Weight:** The gravitational force of attraction of the earth acting on a body is known as its weight. Near the surface of the earth it is the product of mass of the body M and gravitational acceleration g, thus weight W = Mg and acts vertically downward.

(b) **Contact forces:** Two bodies in contact exert equal and opposite forces on each other. If the contact is frictionless the contact force is perpendicular to the common surface and known as normal reaction.

If, however, the objects are in rough contact and move (or have a tendency to move) relative to each other without losing contact then frictional force arise which oppose such motion. Again each object exerts a frictional force on the other and the two forces are equal and opposite. This force is perpendicular to normal reaction. Thus, the contact force (F) between two objects is made up of two forces.

(i) Normal reaction (N) (ii) Force of friction (f)

(c) **Attachment to Another Body (Strings, ropes, springs):**

Tension (T) in a string and spring force (F = kx) come in this group

**Tension :** When a body is connected through a string or rope a force may act on the body by the string or rope due to the tendency of extension. This force is called tension. While pulling a bucket of water from well, you exert upward force on bucket through tension only. Tension acts at, all contact points between body and rope/string along the length of the string to the either side of contact point. If some force is applied on string/rope Tension in the string/rope become equal to applied force at the point.

2. **Spring force:** Consider a light spring tied to a vertical wall, which is being pulled to right and the final elongation of spring is x and at that moment force applied is, say F.

**What is the force applied by spring on stretching agent ?**

Applying IIIrd law of Newton you easily say that spring also applied force F on stretching agent. Ideal spring follows Hooke's law which say that force applied by spring on bodies connected to it is proportional to extension or compression (Change over nature length) and is always opposite to extension or compression.

So $\overline{F} \propto -\overline{x}$, $\overline{F} = -k\overline{x}$ ; Where k is a constant that is characteristic of the spring known as spring constant or force constant.

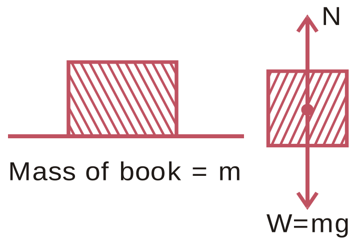
3. **Frictional force:** When surface of a body slides over surface of another body, each body exerts a force known as frictional force

**FREE BODY DIAGRAM**

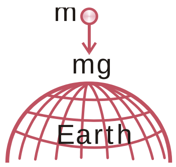
No system, natural or man made, consists of a single body alone or is complete in itself. A single body or a part of the system can, however, be isolated from the rest by appropriately accounting for its effect on the remaining system.

A free body diagram (FBD) consists of a diagrammatic representation of a single body or a subsystem of bodies isolated from its surroundings showing all the forces acting on it.

Consider, for example, a book lying on a horizontal surface.

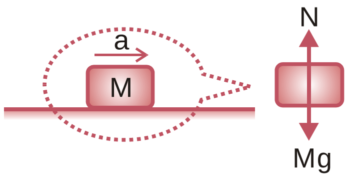



The following examples show the free body diagrams of different bodies



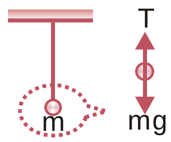

(A)

F.B.D of a particle in gravitational field. The earth pulls the particle of mass m by a force mg



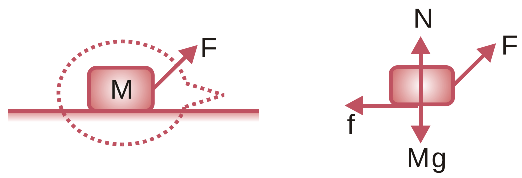

(B)

F.B.D of a block placed on a horizontal surface. Two vertical forces act on the block: The earth pulls the block downward by mg, and the surface pushes the block upward by N




(C)

F.B.D of a ball suspended by a string. Two vertical forces act on the ball: The earth pulls the ball downward by mg and the string pulls the ball upwards by T.




(D)

F.B.D of a block placed on a rough surface being pulled by an external force. There are four forces acting on the block: the gravitational pull Mg; the normal reaction N; the external force F; and the tangential force of friction f.

To draw the free body diagram of any body keep in mind the following points.

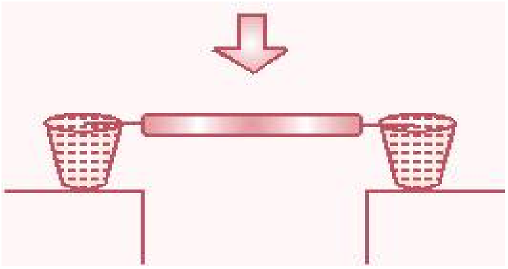
1. We have to represent weight for every body. (If we assume pulleys are mass-less then we will not represent weight)
2. Observe whether the body is in contact with the other body (surface). If it is in contact with any surface then we have to represent normal reaction force and frictional force [ if any ].
3. If there is any string attached with the body then we have to represent tension force

## ACTIVITY :-

Set a five-rupee coin on a stiff card covering an empty galss tumbler standing on a table. Give the card a sharp horizontal flick with a finger. If we do it fast then the card shoots away, allwoing the coin to fall vertically into the glass tumber due to its inertia.

The inertia of the coin tries to maintain its state of rest even when the card flows off.

## ACTIVITY :-



A wooden scale is placed horizontally with its two ends resting on two glass tumblers containing water as shown in figure. Strike suddenly the centre of the ruler with a long rod. Observe that neither the glass tumblers break nor the water spill out. But the scale gets broken into two pieces. Why does this happen?

# WORKSHEET-2

CUQ 1. Weight of an object is always directed ______.

1) vertically downwards 2) vertically upwards

3) parallel to the surface 4) inclined

2. Whenever two surfaces are in contact, they press (or push) each other by a force called _______.

1) non-contact force 2) contact force

3) gravitational force 4) Magnetic Force

3. The component of the contact force perpendicular to the surface is called _____.

1) normal reaction 2) frictional force 3) weight 4) speed

4. Which of the following are catogorised into contact forces

1) Frictional force 2) Tension forces as applied through strings

3) Force exerted during collision 4) All of these

5. The magnitude of non-contact forces

1) Depends on distance of seperation

2) Decreases with increase in separation

3) Increases with increase in separation 4) Both (1) & (2)

6. Which of the following are catogorised into non-contact forces

1) Gravitational force 2) Frictional force

3) Normal force 4) All of these

7. The component of the contact force along the surface (or parallel) in contact is called _____.

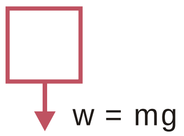
1) normal reaction 2) frictional force 3) weight 4) speed

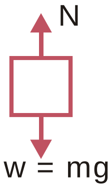
8. The number of normal forces acting on a body depends on

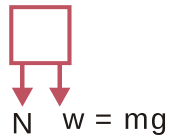
1) number of points or surfaces of contact    2) mass of body

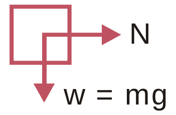
3) colour of the body    4) both (1) and (2)

9. The free body diagram of the freely falling body (neglecting air resistance)

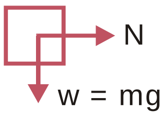
1) 

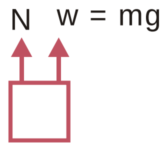

2) 

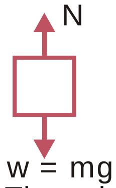

3) 

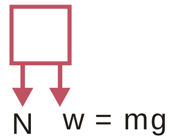

4) 


10. A book lying on a smooth horizontal table. Then the free body diagram of the book.

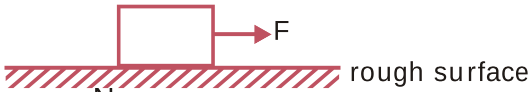

1) 

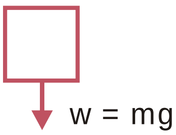

2) 

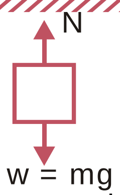

3) 

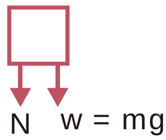

4) 

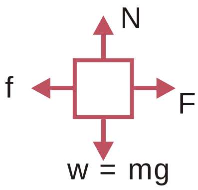

11. A book moving on a rough horizontal table. Then the free body diagram of the book (f is frictional force).




1) 


2) 


3) 

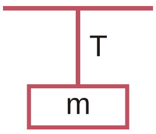

4) 

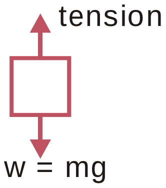

12. Tension force always _____ a body.

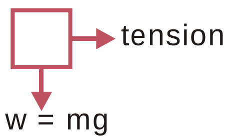
1) pushes    2) pulls

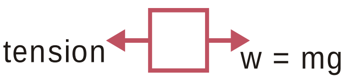
3) sometimes pushes, some times pulls    4) Don't pull or Don't push

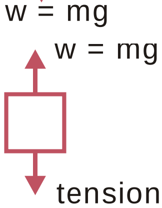
13. The free body diagram of a block 'm' is



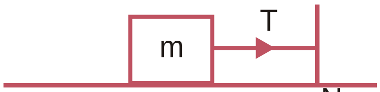

1) 

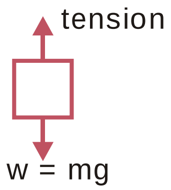

2) 

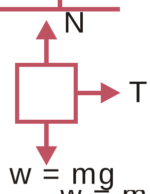

3) 

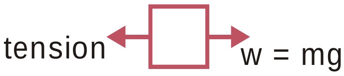

4) 

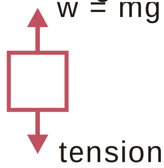
14. The free body diagram of a block ‘m’ is




1) 


2) 


3) 


4) 


## JEE MAINS

**Single Correct Choice Type:**

1. A free diagram is a
   1) Diagrammatic representation of a single body or a sub-system of bodies showing all the forces acting on it.
   2) Diagrammatic representation of a single body or a sub-system of bodies without showing all the forces acting on it.
   3) Both (1)&(2)   4) Neither (1)nor(2)
2. In free body diagram
   1) The object of interest is isolated from its surroundings
   2) Interactions between the object and the surroundings are represented in terms of forces.
   3) Both (1)&(2)   4) Neither (1)nor(2)
3. A book kept on the table, FBD of the book consists of forces indicating
   1) Weight of the object   2) Normal force exerted by the table
   3) Normal force exerted by the book   4) Both (1) & (2)
4. Whenever two surfaces are in contact, they press ( or push) each other by a force
   1) contact force   2) reaction force   3) normal force   4) all of these
5. Tension across massless pulley or frictionless pulley remains
   1) zero   2) constant   3) unequal   4) none of these
6. The weight of an object can be written as
   1)mg   2)ma   3)both(1) and (2)   4)neither (1) nor (2)
7. A book kept on the table, FBD of the book consists of forces indicating
   1) weight of the object   2)Normal force exerted by the table
   3) both (1) and (2)   4) neither (1) nor (2)

8. The tension in the rope when they become slack

1) Maximum 2) minimum 3) Zero 4) Can't say

9. Force applied by spring on bodies connected to it is proportional to extension or compression and is always opposite to extension or compression, this law is called

1) Hooke's law 2) Newton's law

3) Gravitational law 4) None of these

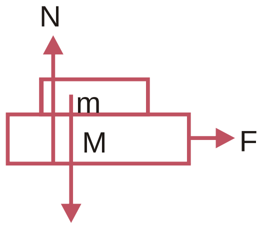
10. When a body is connected through a string or rope a force may act on the body by the string or rope due to he tendency of extension, this force is called

1) Frictional force 2) Magnetic force

3) Electrostatic force 4) Tension force

## Comprehension Type

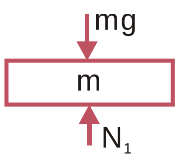
Two blocks of masses M and m that are placed as shown and a force of magnitude F is acting on M.

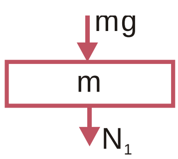


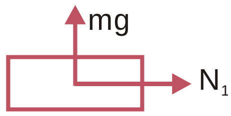

11. The free body diagram of mass m consists of_____ number of forces

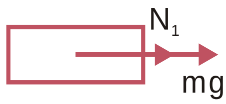
1) Two 2) Three 3) Four 4) Five

12. Then the free body diagram of 'm' is (no friction)

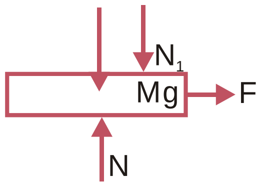
1) 

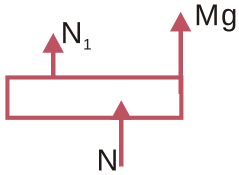
2) 

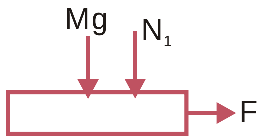
3) 

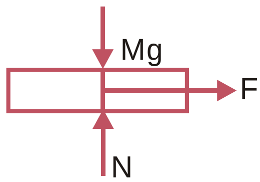
4) 


13. In the above case,the free body diagram of M is

1) 

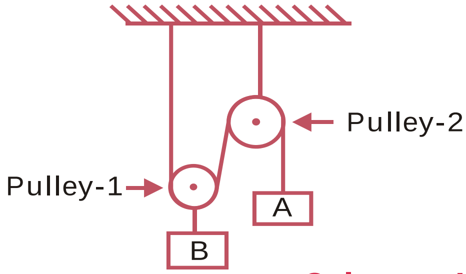
2) 

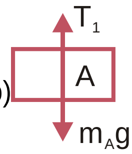
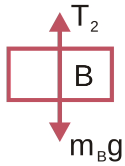
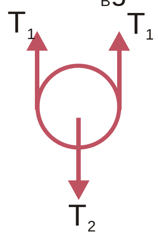
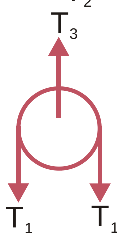
3) 

4) 

### Matrix Match Type:

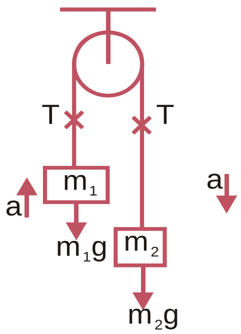
14. Two blocks A and B of masses $m_A$ and $m_B$ are arranged in the diagram as shown.




| Column - I | Column - II |
|---|---|
| a) FBD of $m_A$ | p)   |
| b) FBD of $m_B$ | q)   |
| c) FBD of pulley 1 | r)   |
| d) FBD of pulley 2 | s)   |

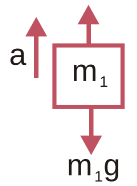
### Comprehension Type:

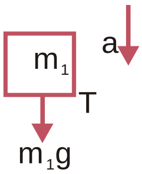
Two masses $m_1$ and $m_2$ are attached to a flexible inextensible massless rope which passes over a frictionless and massless pulley as shown. $(m_2 > m_1)$

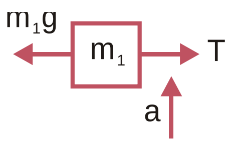


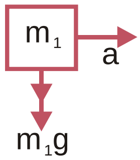
15. The free body diagram of mass $m_2$ consists of____number of forces
1) Two 2) Three 3) Four 4) Five

16. The free body diagram of '$m_1$'

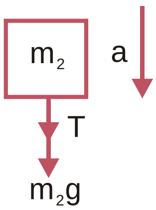
1) 

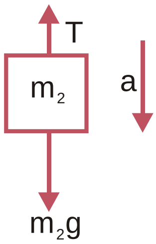
2) 

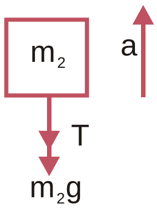
3) 

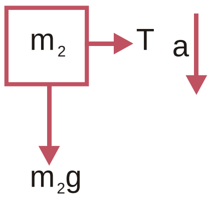
4) 


17. The free body diagram of '$m_2$' is

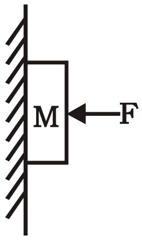
1) 

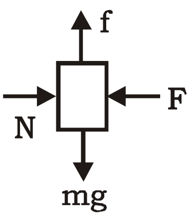
2) 

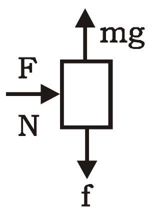
3) 

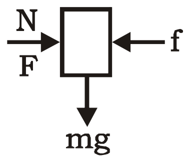
4) 

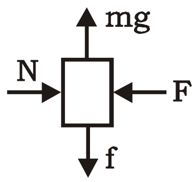

18. Free body diagram of Hock pressing towards rough wall with a force F is



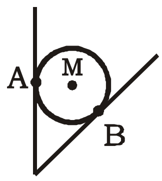

1) 

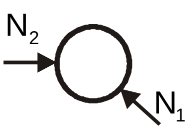
2) 

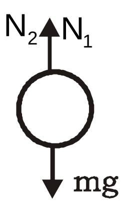
3) 

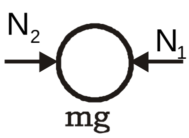
4) 

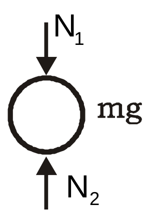

19. Free body diagram of sphere placed in between two planks is ( it is in contact at ( A & B)




1) 

2) 

3) 

4) 

# SYNOPSIS-3

## Law of conservation of momentum :

According to this law, the total momentum of a system remains constant if no net external force acts on the system.

That is, momentum of a system. $\vec{p}$ = constant, if net external force acting on it is zero (i.e. $\vec{F}_{external} = 0$).

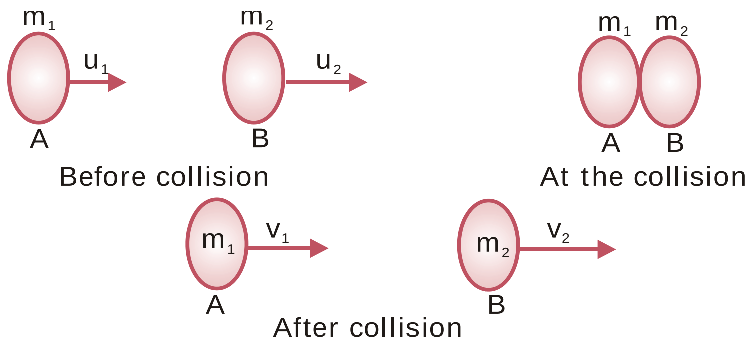
## Conservation of momentum from third law of motion :

If a number of bodies collide with one another the total momentum of the bodies just before collision is equal to the total momentum just after collision.

**Example :** Let a moving ball collides with another stationary ball lying on a ground. Observe, what happens after collision? The moving ball will slow down i.e., its velocity decreases after colliding with the stationary ball. On the other hand, the stationary ball begins to move i.e. its velocity increases after collision. We know, momentum of a body = mass of the body × velocity of the body (i.e. $\vec{P} = m\vec{v}$). Therefore, the momentum of moving ball decreases after collision and the momentum of the stationary ball increases after collision. Thus, we find that when two balls collide with each other, then moving ball loses momentum and the stationary ball gains momentum. The loss of momentum of one ball is equal to the gain of momentum of other ball. However, the total momentum of these colliding balls before and after the collision remains the same. This is the law of conservation of momentum.

## Derivation :

Let us consider two marbles A and B having masses $m_1$ and $m_2$ moving with initial velocities, $u_1$ and $u_2$ ($u_1 > u_2$) in the same direction.




After collision the two bodies move with velocities $v_1$ and $v_2$ in the same direction say as shown in figure.

Momentum of body A before collision = $m_1u_1$

Momentum of body B before collision = $m_2u_2$

$\therefore$ Total momentum of body A and B before collision = $m_1 u_1 + m_2 u_2$

**Similarly :**

momentum of body A after collision = $m_1v_1$

momentum of body B after collision = $m_2v_2$

$\therefore$ Total momentum of body A and B after collision = $m_1 v_1 + m_2 v_2$

Suppose the two marbles collide with each other for 't' seconds.

The momentum of marble A before and after collision is $m_1u_1$ and $m_1v_1$ respectively.

$\Rightarrow$ Change of momentum of body A during the collision = $m_1v_1 - m_1u_1$

$\therefore$ The rate of change of momentum during the collision of body A will be

$$= \frac{\text{change of momentum}}{\text{Time taken}} = \frac{m_1v_1 - m_1u_1}{t} = \frac{m_1(v_1 - u_1)}{t}$$

Similarly, the rate of change of momentum of marble B will be

$$\frac{m_2v_2 - m_2u_2}{t} = \frac{m_2(v_2 - u_2)}{t}$$

If the force exerted by marble A on B is $F_1$ and that by B on A is $F_2$, then according to Newton's 2nd law of motion,

$$F_1 = \frac{m_1(v_1 - u_1)}{t} \quad \text{\_\_\_\_\_\_\_\_ (1)} \qquad F_2 = \frac{m_2(v_2 - u_2)}{t} \quad \text{\_\_\_\_\_\_\_\_ (2)}$$

According to Newton's 3rd law of motion, the force exerted by marble A on B and marble B on marble A are equal and opposite to each other.

$$\therefore F_1 = -F_2$$

$$\Rightarrow \frac{m_1(v_1 - u_1)}{t} = -\frac{m_2(v_2 - u_2)}{t} \Rightarrow m_1v_1 - m_1u_1 = -m_2v_2 + m_2u_2$$

$\Rightarrow m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$ or, $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

i,e. total momentum before collision = total momentum after collision.

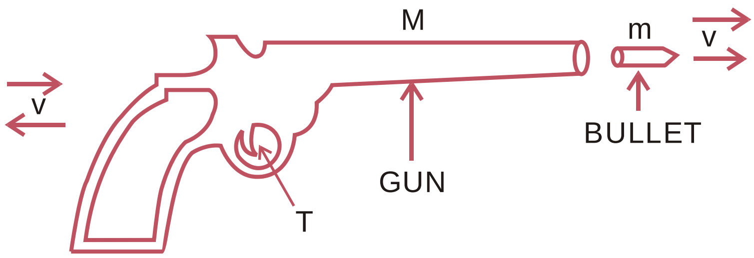
**Note :** If the two bodies stick together after collision, then they move with common velocity v (say) then $m_1u_1 + m_2u_2 = m_1v + m_2v \Rightarrow v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}$

**Examples to illustrate the law of conservation of momentum :**

1. **Recoil of a gun :** We have already explained the recoil of a gun with the help of Newton's third law of motion. However, the recoil of a gun can also be explained with the help of the law of conservation of momentum.

The bullet inside the gun and the gun itself forms a system. Thus, the system is (gun + bullet). Before firing, the gun and the bullet are at rest, therefore, momentum of the system before firing is zero.

When the bullet is fired, it leaves the gun in the forward direction with certain momentum. Since no external force acts on the system, so the momentum of the system (gun + bullet) must be zero after firing. This is possible only if the gun moves backward with a momentum equal to the momentum of the bullet. That is why gun recoils or moves backward.




Gun and bullet

The velocity with which the gun moves backward after firing a bullet is known as recoil velocity.

**Calculation of Recoil Velocity of a gun :**

Let, mass of the bullet = m;

velocity of the bullet after firing = $\vec{v}$;     mass of the gun = M

Recoil velocity of the gun after firing = $\vec{V}$

Since the system is at rest before firing. So momentum of the system (gun + bullet) before firing = 0

Total momentum of the system (gun + bullet) after firing

= momentum of gun + momentum of bullet = $M\vec{V} + m\vec{v}$

According to the law of conservation of momentum,

$M\vec{V} + m\vec{v} = 0 \Rightarrow M\vec{V} = -m\vec{v}$ or     $\vec{V} = -\left(\frac{m}{M}\right)\vec{v}$

Negative sign shows that the direction of the velocity of the gun after firing is opposite to the direction of the velocity of the bullet.

Action and reaction being equal and opposite and acting simultaneously for same duration, have equal and opposite impulses. They produce equal and opposite changes of momentum in the pair of bodies involved. It keeps the total momentum of the two body system constant (conserved)

**Impulsive Force :** A large force which acts for a small interval of time is called impulsive force.

**Impulse :** Impulse of a force is defined as the change in momentum produced by the given force and it is equal to the product of force and the time for which it acts.

According to Newton's 2nd law of motion

$$\vec{F} = m\vec{a} = m\left(\frac{\vec{v}-\vec{u}}{t}\right) = \frac{m\vec{v}-m\vec{u}}{t} \Rightarrow \vec{F}t = m\vec{v}-m\vec{u}$$

Impulsive force = change in momentum.

**Note :** Impulse is a vector quantity, whose direction is same as that of force.

**Unit :** S. I. unit of impulse = N s or kg m/s

C.G.S unit of impulse = dyne second or g cm/s

**Ex :** 1) The force with which a hammer strikes a nail.

2) The force with which a bat hits a cricket ball.

**Elastic & Inelastic Collisions :** During collisions, there will be an exchange of momentum between the bodies. However the kinetic energy of the bodies may remain constant or change. Accordingly, the collisions are classified into two kinds. They are

1) Elastic Collisions and 2) Ineleastic collisions.

**Elastic Collisions :** The collisions in which both mometum and kinetic energy are conserved are known as elastic collisions. The collision between nuclei and fundamental particles are elastic collisions.

**Inelastic Collisions :** The collisions in which kinetic energy is not conserved but law of conservation of momentum holds good are are known as ineleastic collisions. When two bodies stick together after collision, the collision is said to be completly inelastic. The collision bewteen a bullet and its target when the bullet remains embedded in the target is an example of perfectly inelastic collision. During these collisions the loss of kinetic energy appears in the form of heat or as excitation energy as in the case of atomic collisions.

## ACTIVITY :-

Take a big rubber balloon and inflate it fully. Tie its neck using a thread. Also using adhesive tape, fix a straw on the surface of this balloon.

Pass a thread through the straw and hold one end of the thread in your hand or fix it on the wall.

Ask your friend to hold the other end of the thread or fix it on a wall at some distance.

Now remove the thread tied on the neck of balloon. Let the air escape from the mouth of the balloon.

Observe the direction in which the straw moves.

# WORK SHEET-3

CUQ 1. Mutual forces between two bodies are always

1) Unequal and opposite 2) Equal and opposite
3) Equal and in the same direction 4) Unequal

2. The recoil of a gun can be explained with the help of the
1) Newton's First Law 2) Newton's Second Law
3) Newton's Third Law 4) None of these

3. According to Newton's Second Law of motion impulsive force is equal to
1) $m\vec{v}$ 2) $m\vec{u}$ 3) $m\vec{v} - m\vec{u}$ 4) $m\vec{v} + m\vec{u}$

4. The momentum of moving ball __________ after collision
1) Zero 2) Decreases 3) Increases 4) None

5. The momentum of the stationary ball __________ after collision.
1) Zero 2) Decreases 3) Increases 4) None

6. When a body is travelling at constant velocity, the net force on it is ___
1) < 1 2) > 1 3) 0 4) $\infty$

7. The momentum and energy are conserved in this collision.
1) Inelastic 2) Elastic
3) Perfectly elastic 4) In any collision

8. The momentum only conserved but not energy in this collision
1) Inelastic 2) Elastic
3) Perfectly inelastic 4) In any collision

9. After collision they stick together in this collision
1) Perfectly inelastic 2) Perfectly elastic
3) Elastic 4) Inelastic

## JEE MAINS

### Single Correct Choice Type:

1. The momentum of the system is conserved
1) Always 2) Never
3) Only in absence of external force 4) Only when an external force acts

2. Conservation of linear momentum is a direct consequence of

1) Newton's first law of motion 2) Newton's second law of motion

3) Newton's third law of motion 4) None of these

3. When two bodies of masses $m_1$ and $m_2$ moving with velocities $u_1$ and $u_2$ in the same direction collide with each other and $v_1$ and $v_2$ are their velocities after collision in the same direction, then

1) $m_1v_1 + m_2v_2 = m_2u_2 - m_1u_1$ 2) $m_1v_1 + m_2v_2 = m_1u_1 - m_2u_2$

3) $m_2u_2 + m_2u_1 = m_2v_1 + m_1v_2$ 4) $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

4. Which of the following is not a vector quantity?

1) mass 2) Impulse 3) Momentum 4) Force

5. Choose the correct answer

1). S.I Unit of impluse is N-S 2) C.G.S Unit of Impluse in N-S

3). Impluse is a scalar quantity 4) Impluse has no direction

6. A bullet of mass 100g is fired from a gun of mass 20 kg with a velocity of $100ms^{-1}$. Then the velocity of recoil of the gun is

1) –1.5 m/s 2) –0.5 m/s 3) –2.5 m/s 4) 3.5 m/s

7. Statement I: Collision between two particles is not necessarily associated with physical contact betwen them.

Statement II: Only in physical contact momentum transfer takes place.

1) Both I & II True 2) Both I & II False

3) Statement I is True & Statement II is False

4) Statement I is False & Statement II is True

8. The car A of mass 1500 kg travelling at 25 m/s collides with another car B of mass 1000 kg travelling at 15 m/s in the same direction. After collision the velocity of car A becomes 20 m/s. The velocity of car B after the collision is

1) 12.2 m/s 2) 11.5 m/s 3) 22.5 m/s 4) 5.22 m/s

9. A body of mass 2Kg moving with uniform velocity of $40ms^{-1}$ collides with another body at rest. If the two bodies move together with a velocity of $25ms^{-1}$, mass of the other body is

1) 0.6 Kg 2) 0.9 Kg 3) 1.2 Kg 4) 1.5Kg

10. A body of mass 6Kg travelling with a velocity of $10ms^{-1}$ collides elastically with a body of mass 4Kg travelling at a speed of $5ms^{-1}$ in opposite direction and comes to rest. Then velocity of the second body is

1) 0 2) $6ms^{-1}$ 3) $8ms^{-1}$ 4) $10ms^{-1}$

11. A truck weighing 2500 kg and moving with a velocity 28 $ms^{-1}$ collides with a stationary car weighing 300 kg. The truck and the car move together after the impact. Then their common velocity is impact. Then their common velocity is

1) 25 m/s  2) 20 m/s  3) 15 m/s  4) 10 m/s

12. A body of mass 20Kg moving with a velocity of $20ms^{-1}$ collides with another body of mass 40Kg moving in the same direction with a velocity of $10ms^{-1}$. After collision, if they move together, their common velocity is

1) $\frac{10}{3}ms^{-1}$  2) $\frac{20}{3}ms^{-1}$  3) $\frac{40}{3}ms^{-1}$  4) $\frac{50}{3}ms^{-1}$

13. When two bodies of masses $m_1$ and $m_2$ moving with velocities $u_1$ and $u_2$ inthe opposite direction collide with each other and move together after collision in the same direction with a common velocity v, then

1) $v = \frac{m_1u_1 - m_2u_2}{m_1 + m_2}$  2) $m_2u_1 - m_1u_2 = (m_1 + m_2)v$

3) $m_1u_1 - m_2u_2 = (m_1 - m_2)v$  4) $m_1u_1 + m_2u_2 = m_1v - m_2v$

14. A sphere of mass 25Kg moving with a velocity of $40ms^{-1}$ collides with another sphere of mass 15Kg which is at rest. After collision if they move with the same velocity, that velocity is equal to

1) $25ms^{-1}$  2) $40ms^{-1}$  3) $15ms^{-1}$  4) $12ms^{-1}$

15. Two spheres of masses 2Kg and 3Kg travelling in opposite direction with velocities $8ms^{-1}$ and $6ms^{-1}$ collide. If the collision is perfectly inelastic, then final velocity is

1) 0.1m/s  2) 0.2m/s  3) 0.3m/s  4) 0.4m/s

**Reasoning Type:**

16. *Statement I :* A large force which acts for a small interval of time is called impulsive force.

*Statement II :* The total momentum of a system remains constant if no net external force acts on the system.

1) Both Statements are true, Statement II is the correct explanation of Statement I.

2) Both Statements are true, Statement II is not correct explanation of Statement I.

3) Statement I is true, Statement II is false.

4) Statement I is false, Statement II is true.

17. A neutron of mass $1.67 \times 10^{-27}$ Kg moving with a speed of $3 \times 10^{6}$ $ms^{-1}$ collides with a deutron of mass $3.34 \times 10^{-27}$ Kg at rest. After collision they both stick together and form a trition. Velocity of triton is

1) $10^{8}\ m\ s^{-1}$ 2) $10^{10}\ m\ s^{-1}$

3) $10^{6}\ m\ s^{-1}$ 4) $10^{4}\ m\ s^{-1}$

**Matrix Match Type:**

18.

| **Column - I** | **Column - II** |
|---|---|
| a) Impulse | 1) Dimensional formula is $[MLT^{-1}]$ |
| b) Momentum | 2) Area of force time graph |
| c) Body is at rest | 3) Quantity of motion |
| d) Force | 4) Linear momentum is zero |
| | 5) Rate of change of momentum |

19. Statement I : kg m/s is the unit of impulse.

Statement II: kg m/s is the unit of mass.

1) Both I & II True 2) Both I & II False

3) Statement I is True & Statement II is False

4) Statement I is False & Statement II is True

**Multi Correct Choice Type:**

20. A truck of mass 500 kg moving at 4 m/s collides with another truck of mass 1500 kg moving in the same direction at 2 m/s. What is their common velocity just after the collision if they move together ?

1) 2 m/s 2) 2.5 m/s 3) 3 m/s 4) 3.5 m/s

21. A man and a cart approach each other. Mass of the main is 64Kg and his velocity is 5.Kmph. Mass of the cart is 32kg and its velocity is 1.8Kmph. If the main jumps into the cart, the velocity of the cart becomes

1) 3Kmph 2) 4.11 Kmph 3) 1.8 Kmph 4) 5.4 Kmph

22. Two masses $m_A$ and $m_B$ moving with velocities $V_A$ and $V_B$ in opposite directions collide elastically. After th2at the masses $m_A$ & $m_B$ moving with velocities $V_B$ & $V_A$ respectively. The ratio $m_A$ & $m_B$ is

1) 1 2) $V_A / V_B$ 3) $\frac{m_A / m_B}{m_A}$ 4) $\frac{V_A - V_B}{V_A + V_B}$

# KEY & SOLUTIONS

## FORCE

## WORK SHEET-1

**CUQ:** 1) 3  2) 4  3) 2  4) 3  5) 4  6) 3  7) 1  8) 2
9) 3  10) 2

### JEE MAINS

1) 3  2) 3  3) 3  4) 2  5) 3  6) 1  7) 1  8) 2
9) 3  10) 2  11) 2  12) 3
13) 1,2,3,4  14) 1  15) 2,4  16) 1,2,3,4  17) 1,2,3
18) a-5,b-3,c-4, d-1,2  19) 1  20) 1,2,3,4  21) 1,3,4
22) 1  23) 4  24) 3

## WORK SHEET-2

**CUQ:** 1. 1  2. 2  3. 1  4. 4  5. 4
6. 1  7. 2  8. 1  9. 1  10. 3
11. 4  12. 2  13. 1  14. 2

**JEE MAINS :**

1. 1  2. 3  3. 4  4. 4  5. 2
6. 1  7. 3  8. 3  9. 1  10. 4
11. 1  12. 1  13. 1  14. a-p; b-q; c-r; d-s
15. 1  16. 1  17. 2  18. 1  19. 1.

## WORK SHEET-3

**CUQ:** 1. 2  2. 3  3. 3  4. 2  5. 3
6. 3  7. 2  8. 1  9. 1

**JEE MAINS :**

1. 4  2. 3  3. 4  4. 1  5. 1  6. 2
7. 4  8. 3  9. 3  10. 4  11. 1  12. 3
13. 1  14.2  15. 1  16. 1  17. 3
18. a-1,2; b-1,3; c-4; d-5  19. 3  20. 2  21. 2  22. 1

## HINTS & SOLUTIONS

6. $m_1v_1 = m_2v_2$ $m_1 = 100g = 0.1kg$ , $m_2 = 20kg$

$v_1 = 100m / s$ ; $0.1 \times 100 = -20v_2$ ; $v_2 = 0.5m / s$

8. $m_A = 1500kg$ $m_B = 1000kg$

$u_1 = 25m / s$ $u_2 = 15m / s$

$v_1 = 20m / s$ $v_2 = ?$

According to conservation of momentum

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

1500 x 25 + 1000 x 15 = 1500 x 20 + 1000 x $v_2$

$v_2 = 22.5m/s$

9. $m_1u_1 + m_2u_2 = (m_1 + m_2)v$ ; 10. $m_1u_1 - m_2u_2 = m_1v_1 + m_2v_2$

11. $m_1u_1 + m_2u_2 = (m_1 + m_2)v$

# FRICTION

## FRICTION SINOPSIS-1

**FRICTION:** When surface of a body slides over surface of another body, each body exerts a force known as frictional force on the other. Friction is the parallel component of contact force between two bodies in contact.

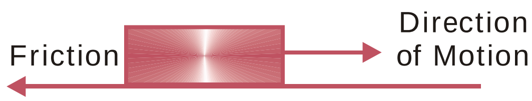
The property by virtue of which an opposing force is generated between two rough surfaces in contact with each other and which opposes the sliding of one surface over the other is known as friction.

Frictional Force: The force which opposes the sliding or relative motion of two bodies in contact with each other, is called frictional force. Frictional force some times may be in the direction of motion of the body

Ex1: If you are walking due east frictional force is due east only

Ex 2: Engine is connected rear wheels of a car. When the car is accelerated, direction of frictional force on the rear wheels will be in the direction of motion and on the front wheels in the opposite direction of motion

**CAUSE OF FRICTION**: Surfaces are not generally perfectly smooth. The origin of friction is due to surface irregularities of molecular scale. Even a highly polished surface would not be free from such irregularities.




When a body is pulled over a surface the relative motion is resisted due to inter-locking of surface irregularities and an opposing force is developed. This force is called force of friction.

Friction should not be misunderstood that it opposes the relative motion. Some times it supports or produces relative motion also. A vehicle accelerates on a road only because the frictional forces on the vehicle due to the road drives it. It is not possible to accelerate a vehicle on a smooth frictionless road.

When a block is resting on another block and lower block is pulled by a force, it is the frictional force which accelerates the upper block.

**Advantage of Friction:**

* Friction plays an important role in our daily life. While walking friction between the ground and shoes prevent us from slipping
* Without friction motion cannot be conveyed by belts from motor to machine
* Vehicles will not come to rest even if the brakes are applied when there is no friction between tyres and the road.
* When there is no friction knots cannot be tied.
* Nails and screws do not hold the boards together without friction.

**Disadvantage of Friction:**

* Friction causes wear and tear of moving parts of the machinery.
* Friction generates heat in machine parts which damages the machinery.

Methods of Reducing Friciton:

Polishing: Friction can be reduced by making the surface in contact polished and smooth. This will remove the irregularities of the surfaces thereby breaking the inter–locking.

**Note:** Polishing the surfaces to higher degree leads to increase of friction.

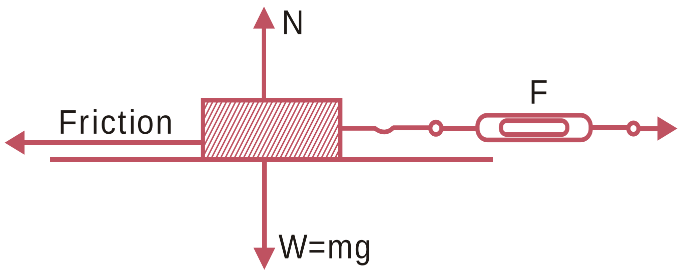
Lubricants: Friction can be reduced by using lubricating materials such as grease, oil or graphite on the surfaces. Friction can be reduced by introducing a layer of gases between sliding surfaces. Lubricants should have low density, high viscosity and they should be non volatile.

Ball Bearings: By using ball and roller bearings, the sliding friction is converted into rolling friction, thereby the friction is reduced.

Streamlining: Automobiles and aeroplanes are streamlined to reduce the friction due to air

**TYPES OF FRICTION :**

i) Static friction ii) Kinetic or dynamic friction iii) Rolling friction



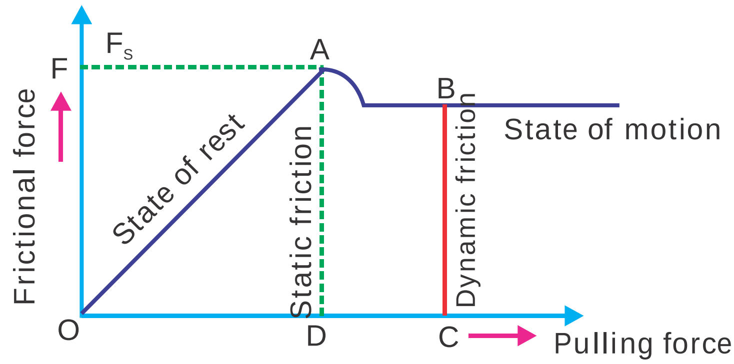

**Static friction:** Suppose a block is placed on a horizontal surface. A spring balance is attached to it to apply and measure the applied force.

When a horizontal force F is applied, the block does not move. The friction force becomes F opposing the motion of the body.

The static friction is self adjusting. So, when the applied force is gradually increased, the force of friction also increases but when the applied force reaches a particular value $F_L$ the body starts moving. Frictional force cannot increase beyond $F_L$. This is called static friction or limiting friction.

(If a force is applied to pull the body and it does not move, the friction acts which is equal in magnitude and opposite in direction to the applied force, i.e., friction is self adjusting force. Further, as the body is at rest, the friction is called static friction.

If the applied force is increased, the force of static friction also increases. If the applied force exceeds a certain (maximum) value, the body starts moving. This maximum force of static friction upto which body does not move is called limiting friction. Thus static friction is self-adjusting force with an upper limit called limiting friction).

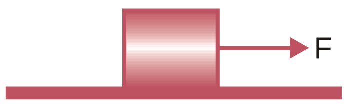



This limiting force of friction ($F_L$) is found experimentally to depend on normal reaction(N). Hence, $f_{max} \propto N \Rightarrow F_L = \mu_s N \Rightarrow \mu_s = \frac{F_L}{N}$

where $\mu_s$ is a dimensionless constant and called coefficient of static friction which depends on the nature of the surfaces in contact.

To understand the above graph, study the following problem:

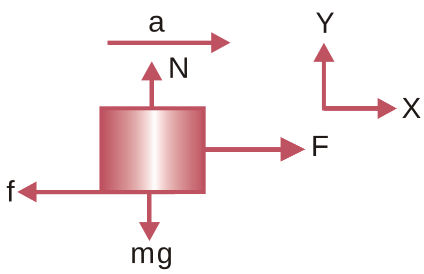
Suppose a block of mass 1 kg is placed over a rough surface and a horizontal force F is applied on the block as shown in figure.




Now, let us see what are the values of force of friction f and acceleration of the block a if the force F is gradually increased.

Given that $\mu_s = 0.5, \mu_k = 0.4$ and g = 10 $m/s^2$.

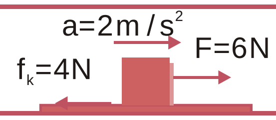
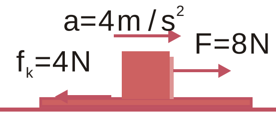
Sol: Free body diagram of block is




$\sum F_y = 0$, $\therefore$ N – mg = 0 or N = mg = (1) (10) = 10 N

$f_L = \mu_s N = (0.5)(10) = 5N$; $f_k = \mu_k N = (0.4)(10) = 4N$

Below is explained in tabular form, how the force of friction f depends on the applied force F. (mass=1 kg)

| F | f | $F_{net}=F-f$ | Acceleration of block $a=\frac{F_{net}}{m}$ | Diagram |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 |  |
| 2N | 2N | 0 | 0 |  |
| 4N | 4N | 0 | 0 |  |
| 5N | 5N | 0 | 0 |  |
| 6N | 4N | 2N | $2m/s^2$ |  |
| 8N | 4N | 4N | $2m/s^2$ |  |

## Kinetic or dynamic friction :

When the body starts moving, the frictional force slightly decreases and this is called dynamic friction.

The dynamic friction is not a self adjusting force. It is independent of surface area of contact and also the relative speed between the surfaces which are in contact. Experimentally it is well established that the kinetic friction is less than the static friction and is found to be, Kinetic friction $\propto$ Normal friction,

$F_k \propto N \Rightarrow F_k = \mu_k N \Rightarrow \mu_k = \frac{F_k}{N}$ where $\mu_k$ is known as coefficient of kinetic friction.

Note: When two bodies slip over each other the force of friction is called kinetic friction, but when they do not slip but have a tendency to do so the force of friction is called static friction.

## Rolling Friction :

When a body like a wheel or sphere or cylinder rolls on a surface the friction developed is known as rolling friction.

This is due to deformation of the surface on which the body rolls and also due to the deformation of the rolling body at the point of contact.

Rolling friction is always minimum. That is why ball bearings are used to reduce friction by converting sliding friction into rolling friction in machine parts.

Rolling friction $\propto$ Normal reaction $F_R \propto N \Rightarrow F_R = \mu_R N \Rightarrow \mu_R = \frac{F_R}{N}$

where $\mu_R$ is known as Rolling friction.

The three types of frictions are in the order $F_s > F_k > F_R$.

So for a given pair of solids $\mu_s > \mu_k > \mu_R$

**Laws of Friction:**

* Friction opposes relative motion between two surfaces in contact and is always tangential to the surface of contact.
* Friction depends on the nature of the two surfaces in contact i.e., nature of materials, surface finish, temperature of the two surfaces etc.
* Friction is independent of the area of contact between the two surfaces
* Friction is directly proportional to the normal reaction acting on the body.
* Coefficient of static friction ($\mu_s$) depends on the nature of the two surfaces in contact and is independent of area of the contact.
* Static friction opposes impending motion. The term impending motion means, motion that would take place (but does not actually take place) under the applied force, if friction were absent.

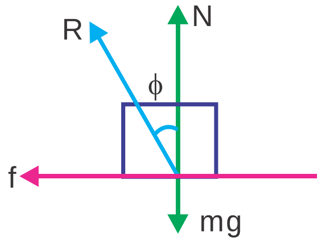
The law of static friction may thus be written as

$f_s \leq \mu_s N$. Where the dimensionless constant $\mu_s$ is called the coefficient of static friction and N is the magnitude of the normal force. $(f_s)_{max} = \mu_s N$

* Coefficient of kinetic friction ($\mu_k$) $= \frac{f_k}{N}$
* Coefficient of kinetic friction ($\mu_k$) is independent of velocity.
* Coefficient of rolling friction $(\mu_R) = \frac{f_R}{N}$
* Rolling friction depends on area of the surfaces,

**Note :** $\mu_s > \mu_k > \mu_R$

**Contact Force and Angle of Friction:** Normal reaction N and frictional force *f* are two components of the total reaction force contact force of the surface on the block. Angle between total reaction R and normal reaction N is called angle of friction. Friction is parallel component of contact force to the surfaces. Normal force is perpendicular component of contact force to the surfaces.




$R = \sqrt{f^2 + N^2}$

When the block is static $\tan\phi = \frac{f}{N}; \phi \le \phi_s$

When the block is in impending state, $\tan\phi_s = \frac{\mu_s N}{N} = \mu_s$

Where $\phi_s \to$ maximum angle of friction.

When block is sliding, $\tan\phi_k = \frac{\mu_k N}{N} = \mu_k$

Since $\mu_s > \mu_k$, is follws that $\phi_s > \phi_k$.

**Normal Reaction (N):**

* When a body rests on another, the force acting on the bottom surface of the body is called the normal reaction. This is always perpendicular to the surface.
* When the body lies on a horizontal surface
  N = W = mg
* When the body lies on an inclined surface
  N = W cos$\theta$ = mg cos$\theta$ .
  Normal force always gives rise to frictional force in the direction opposite to the tendency of motion.
* The angle between the normal reaction N and the resultant of f and N is called the angle of friction
* When a body is moving on a rough inclined plane which makes an angle $\theta$ with the horizontal the frictional force acting on it is $f = \mu_k mg\cos\theta$
  Motion On a Horizontal Rough Surface:
* When a body of mass m on a horizontal rough surface is pulled or pushed with a horizontal force F,
* If the body does not move, the frictional force is equal to the applied force in backward direction.
* If the body is about to move. Then the Friction force, $f = \mu_s N = \mu_s mg$
* If the applied force $F > \mu_k mg$, the body moves forward.
  Then the frictional force = $\mu_k mg$

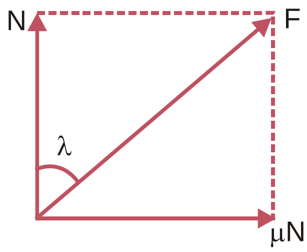
The body moves forward with an acceleration $a = \dfrac{F - \mu_k mg}{m}$

* If a minimum force, required to move the body is applied and it is further continued, the body moves with an acceleration. $a = (\mu_s - \mu_k)g$

**ANGLE OF FRICTION $(\lambda)$ :**

At a point of rough contact, where slipping is about to occur the two forces acting on each object are the normal reaction N and frictional force $\mu N$. The resultant of these two forces is F and it makes an angle $\lambda$ with the normal where

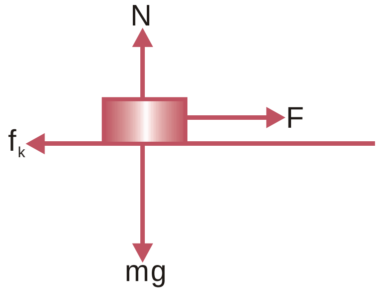
$$\tan\lambda = \frac{\mu N}{N} = \mu \quad \text{or} \quad \lambda = \tan^{-1}(\mu)$$




This angle $\lambda$ is called the angle of friction.

**Motion of a body on the rough horizontal plane**

In the diagram given, 'F' is the horizontal force, pulls the body on a rough horizontal plane of co-efficient of kinetic friction $\mu_k$. If 'F' is greater than limiting friction, the body slides and kinetic friction $f_k$ comes to play and acts to the left, tangentially along surface of contact of the body and the surface.




But from the laws of friction $f_k = \mu_k N = \mu_k mg$

if the acceleration produced be 'a', the resultant force acting on the body

$F - f_k = ma \qquad \therefore a = \dfrac{F - f_k}{m}$

**Note :** When a body of mass m is at rest on a horizontal surface the maximum frictional force acting is $\mu_s mg$, So the minimum force required to move the body is $F = \mu_s mg$.

1) If $F > \mu_s mg$ the body moves with uniform acceleration then, net force acting on the body is $F_R = F - \mu_k mg$, where $\mu_k$ = coefficient of kinetic friction.

So acceleration of the body $a = \frac{F_R}{m} = \frac{F - \mu_k mg}{m}$

2) If the body moves with uniform velocity, external force F = kinetic friction = $\mu_k mg$.

3) If the force applied on the body is equal to that required just to move the body $F = \mu_s mg$, the kinetic friction acting on the body $F_k = \mu_k mg$.

So net force acting on the body = $F - \mu_k = \mu_s mg - \mu_k mg = mg(\mu_s - \mu_k)$

$\therefore$ acceleration of the body = $g(\mu_s - \mu_k)$

4) If a body moving on a horizontal surface is retarded by frictional forces, the retardation produced $a = \mu g$.

# FRICTION WORKSHEET-1

**CUQ**

1. The force which opposes the sliding or relative motion of two bodies in contact with each other is called

   1) Normal force 2)Frictional force
   3) Static friction 4) Rolling friction

2. Static friction is a

   1) It is a borizontal force 2) Sliding force
   3) Self adjusting force 4) kinetic force

3. The maximum force of static friction upto which body does not move is called

   1) Rolling friction 2) Frictional force
   3) Static friction 4) Limiting friction

4. When the body starts moving, the frictional force slightly decreases this is called

   1) Kinetic friction 2) Dynamic friction
   3) Both (1) and (2) 4) Rolling friction

5. When a body like a wheel or sphere or cylinder rolls on a surface. the froction developed is known as

   1) kinetic friction 2) Rolling friction
   3) Static friction 4) Dynamic friction

6. Eqution for the motion of a body moves with uniform velocity, then external force F=

1) $\mu_k mg$ 2) $\mu_s mg$ 3) $mg$ 4) $\mu_k$

7. Friction is independent of

1) Nature of the two surfaces 2) Temperature of the two surfaces

3) Area of contact between two surfaces 4) Velocity

8. If a body moving on a horizontal surface is retarded by frictional forces, the retardation produced a =

1) $\mu_k g$ 2) $\mu g$ 3) $\mu_s mg$ 4) $mg$

9. When a body rests on another, the force acting on the bottom surface of the body is called

1) Rolling friction 2) Frictional force 3) Kinetic friction 4) Normal reaction

10. Sand is dusted on the railway tracks during rainy season to

1) make it always wet 2) increase friction

3) to reduce consumption of fuel 4) none

## JEE MAIN & JEE ADVANCED

**LEVEL-1 Single Correct Choice Type:**

1. If $F_L$ = 10 N and Applied Force is 0 N, then Frictional Force acting on the body is

1) 10 N 2) 5 N 3) 7.5 N 4) 0 N

2. If the Angle of Friction is $45^0$, then the Coefficient of Static Friction ($f_S$) =

1) 1 2) $\frac{1}{\sqrt{2}}$ 3) 0 4) $\infty$

3. A book of weight 20N is pressed between two hands and each hand exerts a force of 40N. If the book just starts to slide down. Coefficient of friction is

1. 0.25 2. 0.2 3.0.5 4. 0.1

4. A block of mass 10kg is on a rough horizontal surface. A horizontal force 4N is applied on it. If coefficient of static friction between those two surfaces is 0.5 frictional force between the two surface in contact is

1. 20 N 2. 49 N 3. 4.9 N 4. 4 N

5. A horizontal force of $4\sqrt{3}$ kg–wt is just sufficient to pull a body of 8 Kg–wt on a horizontal surface. The coefficient of friction between the surfaces is

1) $\frac{\sqrt{3}}{4}$ 2) $\sqrt{\frac{3}{4}}$ 3) $\frac{\sqrt{3}}{6}$ 4) $\sqrt{\frac{3}{2}}$

6. A body of mass 60kg is pushed with just enough force to start it moving on a rough surface with $\mu_s = 0.5$ and $\mu_k = 0.4$ and the force continues to act afterwards. The acceleration of the body is

1) $0.98\ m/sec^2$ 2) $3.92 m/sec^2$ 3) $4.90 m/sec^2$ 4) Zero

**Statement type:**

7. *Statement-I:* When a bicycle is being pedaled, the friction on the front wheel is in a direction opposite to the motion of bicycle

*Statement-II:* The rear wheel while being pedalled, pushes the front wheel on rough road due to which the friction opposes the relative motion

1) Both Statements are true.

2) Both Statements are false.

3) Statement I is true, Statement II is false.

4) Statement I is false, Statement II is true.

**LEVEL-2 & 3 Single Correct Choice Type:**

8. A man of mass 65 kg. is standing stationary with respect to a conveyor belt which is accelerating with $1 m/s^2$. If $\mu_s$ is 0.2, the net force on the man and the maximum acceleration of the belt so that the man is stationary relative to the belt are $\left(g = 10 m/s^2\right)$

1. zero, $2 m/s^2$ 2. $65N, 2m/s^2$ 3. zero, $1 m/s^2$ 4. $65N, 1m/s^2$

9. A block of mass 2 kg is placed on the floor. The coefficient of static friction is 0.4. If a force of 2.8 N is applied on the block parallel to floor. the force of friction between the block and floor (taking $g = 10 m/S^2$) is :

1. 2.8 N 2. 8 N 3. 2N 4. Zero

10. A body of mass 2 kg rests on a rough inclined plane making an angle $30^0$ with the horizontal. The coefficient of static fricition between the block and the plane is 0.7. The fricitional force on the block is:

1. 9.8 N 2. $0.7 \times 9.8 \times \sqrt{3}$N 3. $9.8 \times \sqrt{3}$N 4. $0.7 \times 9.8$

11. Two boxes of same material start moving on surface, one with a velocity of $10ms^{-1}$ and another with a velocity V. If the ratio of stopping times for the two boxes is 1:3, then the value of V is

1) $3.3ms^{-1}$ 2) $10ms^{-1}$ 3) $30ms^{-1}$ 4) $50ms^{-1}$

12. A body is struck to the front part of the truck The coefficient of friction between the body and truck is $\mu$. The minimum acceleration with which the truck should travel so that the body does not fall down is

1. $\mu / g$ 2. $\mu g$ 3. $g / \mu$ 4. $\mu^2 g$

13. A motor of power 0.98 kW drags a mass 100kg through 4m on a surface having coefficient of friction 0.3. Time of that mass is

1) 0.4 sec 2) 0.8 sec 3) 1.2 sec 4) 1.6 sec

14. A 10kg mass is resting on a horizontal surface and horizontal force of 80N is applied. If $\mu$ = 0.2, the ratio of acceleration with out and with friction is ($g = 10ms^{-2}$)

1. 3/4 2. 4/3 3. 1/2 4. 2

**Comprehension type :**

An object of mass 10kg moving with a velocity of 5.6 $ms^{-1}$ on a horizontal rough surface comes to rest after traveling a distance of 16m then.

15. The coefficient of friction is

1) 0.1 2) 0.2 3) 0.5 4) 0.7

16. The work done against friction is

1) 128J 2) 156.8J 3) 56.2J 4) 25.32J

17. The body comes to rest in the time is

1) 2.6 s 2) 3.2 s 3) 4.2 s 4) 5.7 s

**LEVEL-4 & 5 Single Correct Choice Type:**

18. An eraser weighing 2N is pressed against the black board with a froce of 5N. If the coefficient of friction is 0.4, the force parallel to the black board is required to slide the eraser upwards with uniform velocity is

1. 2N 2. 2.8N 3. 4N 4. 4.8N

**Integer Answer Type:**

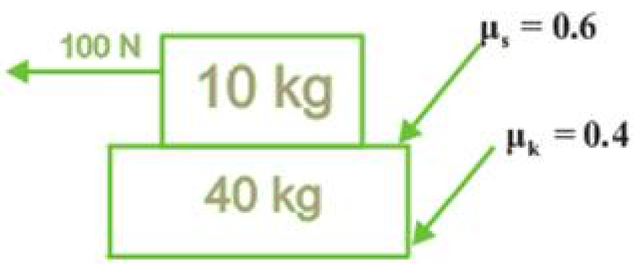
19. A block of weight 100N is pushed by a force F on a horizontal rough plane moves with an acceleration 1 $m/s^2$, when force is doubled its acceleration becomes 10$m/s^2$. The coefficient of friction is_________$10^{-1}$ ($g=10ms^{-2}$)

**Matrix Match Type:**

20. **Column - I** **Column - II**

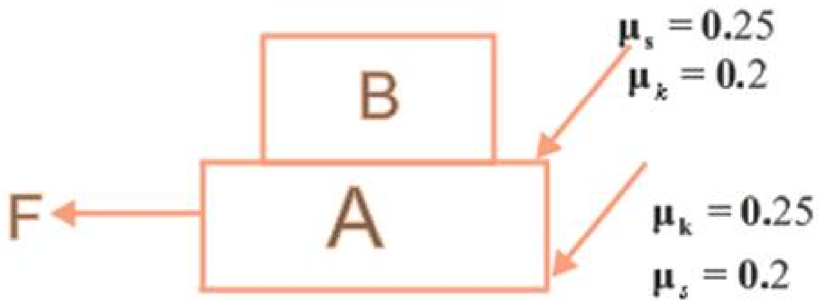
| Column - I | Column - II |
|---|---|
| a) static friction | 1) constant for given pair of surfaces |
| b) Limiting friction | 2) dimension less quantity |
| c) Kinetic friction | 3) self adjusting |
| d) Rolling friction | 4) has the least magnitude for a given normal reaction |

21. A 40 kg slab rests on a frictionless floor. A 10 kg block rests on top of the slab. The static coefficent of friction bewtween the block and the slab is 0.60 while the kinetic coefficient of friction is 0.40. The 10 kg block is acted upon by a horixontal force of 100 N. The resulting acceleration of the slab will be




1) $1.5 ms^{-2}$ 2) $2.0 ms^{-2}$ 3) $10 ms^{-2}$ 4) $1.0 ms^{-2}$

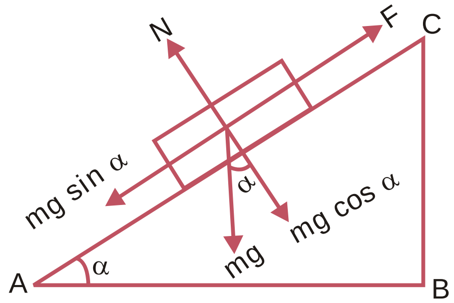
22. Block B of mass 2 kg rests on block A of mass 10 kg. All surfaces are rough with the value of coefficient of friction as shown in the figure. Find the minimum force F that should be applied on block A to cause relative motion between A and B. $(g = 10 m/s^2)$




1) 24 N 2) 30 N 3) 48 N 4) 60 N

# FRICTION SYNOPSIS-2

**ANGLE OF REPOSE :** The angle $\alpha$ with which the inclined plane makes an angle with the horizontal when a body placed on it just begins to slide down is called the angle of repose.

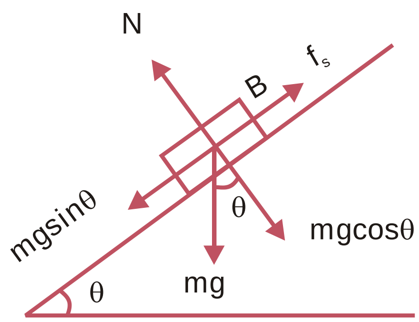



The force down the plane = mg sin $\alpha$ Frictional force(F) = $\mu N = \mu mg \cos\alpha$.

For equilibrium, $mg\sin\alpha = \mu mg\cos\alpha$ $\therefore$ $\tan\alpha = \mu$. ; Note : $\alpha$ will be the same as $\lambda$.

**Motion of a body on an inclined plane:**

Let us consider a block 'B' of mass m, at rest on an inclined plane, which makes an angle '$\theta$' with the horizontal as shown in the figure:




The forces acting on the body are

1. Weight of the body 'mg' acting vertically downwards.
2. Normal reaction 'N' acting perpendicular to the inclined plane.
3. Force of static friction $f_s$ acting tangentially to the inclined plane.

The weight of the body is resolved into two rectangular components:

(i) mg cos$\theta$ perpendicular to the inclined plane.

(ii) mg sin$\theta$ parallel to the inclined plane acting downwards.

This tries to move the block along the plane downwards.

**(a) Body is at rest on the plane**

When the body is at rest on the plane, (one component force of mg) mgsin$\theta$ is equal to static friction acting upwards along the plane. The normal reaction N by the inclined plane on the block is equal to mg cos$\theta$.

i.e., $f_s$ = mgsin$\theta$ and N = mgcos$\theta$. Then the body will be at rest on the plane.

**(b) Angle of repose**

As the angle of inclination '$\theta$' is gradually increased at a particular value of $\theta$ say $\alpha$, the body will be just ready to slide down the plane. This angle of inclination $\alpha$ is called the angle of repose.

The angle of repose ($\alpha$) is defined as the angle of inclination of a plane with respect to horizontal for which the body will be ready to slide down the inclined plane.

**(c) When a body just ready to slide**

If the angle of inclination is gradually increased the value of mg sin$\theta$ increases. At a particular value of inclination say, the angle of repose '$\alpha$', mg sin$\theta$ become equal to the limiting friction ($f_s$). The body tends to slide down the plane.

When $\theta = \alpha$, $f_s$ = mg sin$\alpha$ and N = mgcos$\alpha$

From the law of static friction equation, $f_s = \mu_s N$ or mgsin$\alpha$ = $\mu$ mgcos$\alpha$

$$\mu_s = \frac{mg\sin\alpha}{mg\cos\alpha} = \tan\alpha \quad \text{———— (1)}$$

Thus the coefficient of static friction is numerically equal to the tangent of the angle of repose. In case of inclined plane, the angle of friction is equal to the angle of repose.

**(d) When the body is sliding down**

If $\theta > \alpha$, then $mg\sin\theta > \mu_s mg\cos\theta$ and the body acquires acceleration and slides down.

If $\theta < \alpha$, the body does not slide on the plane.

If $\theta = \alpha$ the body just begins to slide.

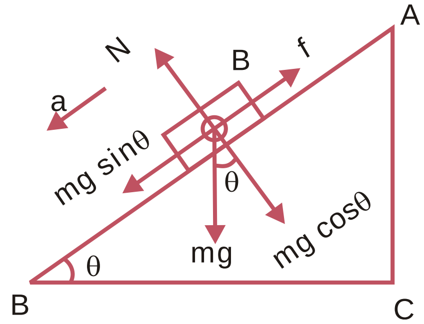
**ILLUSTRATION:**

1. a) A block of mass m slides down on a rough inclined plane of length *l* and height 'h' of an inclined at an angle $\theta$ with the horizontal. If the coefficient of friction is $\mu$ then the find the acceleration of the block.

   b) If the block is placed at the top of the inclination and then released find the
      (i) velocity of the block at the bottom of the inclined plane
      (ii) time taken by the block to the bottom of the inclined plane

   c) If the friction is absent, deduce the parameters of the above problem.

1. (a) Let the block slide down with acceleration with acceleration a (= ?) as shown and draw the FBD of block:




From the FBD of block we have, (i) $N = m g \cos\theta$ and (ii) $mg \sin\theta - f_{max} = ma$

$\Rightarrow mg \sin\theta - \mu N = ma$ [$\because f_{max} = \mu N$) $\Rightarrow mg \sin\theta - \mu mg \cos\theta = ma$ (from(i))

$\Rightarrow g \sin\theta - \mu g \cos\theta = a$ or $a = g[\sin\theta - \mu \cos\theta]$.

(b) (i) The velocity 'v' of the body at point 'B' starting from rest.

$v^2 - 0^2 = 2g(\sin\theta - \mu\cos\theta)l$ Where '$l$' is the length of the inclined plane.

$\therefore v = \sqrt{2gl(\sin\theta - \mu\cos\theta)}$ ——— (iii)

(ii) The time taken by the body to slide down is given by

$s = ut + 1/2\, at^2$ where $s = l$, $u = 0$

$$l = 0(t) + \frac{1}{2}g(\sin\theta - \mu \cos\theta)t^2$$

$$\Rightarrow t^2 = \frac{2l}{g(\sin\theta - \mu\cos\theta)} \qquad \therefore t = \sqrt{\frac{2l}{g(\sin\theta - \mu\cos\theta)}} \text{ ——— (iv)}$$

c) If the friction is absent, then $\mu = 0$

(i) $a = g(\sin\theta - \mu\cos\theta) \Rightarrow a = g \sin\theta$

(ii) $v = \sqrt{2gl(\sin\theta - \mu\cos\theta)} \Rightarrow v = \sqrt{2gl\sin\theta} \Rightarrow v = \sqrt{2gh}$ $(\because l\sin\theta = h)$

(iii) $t = \sqrt{\frac{2l}{g(\sin\theta - \mu\cos\theta)}} \Rightarrow t = \sqrt{\frac{2\ell}{g\sin\theta}}$.

# WORKSHEET-2

CUQ : 1. The angle of inelination of a plane with respect to horizontal for which the body will be ready to slide down the inelined plane is called

1) Angle of repose 2) Angle of inelination
3) Angle of deviation 4) Angle of incidence

2. If there is a block of 'm' is at rest on an inclined plane, then the normal reaction 'N' will be acting
   1) Parallel to the inclined plane
   2) Perpendicular to the inclined plane
   3) Vertically downwards to the inclined plane
   4) It will be zero

3. If the angle of inelination is gradually increased the angle of repose, mg sin$\theta$ becomes equal to
   1) Kinetic friction 2) Dynamic friction
   3) Limiting frinction 4) Frictional force

4. A block of mass 'm' slides down on a rough inclined plane of length 'l' and height 'h' making an angle '$\theta$' with the horizontal. If the coefficient of friction is '$\mu$', then the acceleration is
   1) $a = g(\sin\theta + \mu\cos\theta)$ 2) $a = \sin\theta - \mu\cos\theta$
   3) $a = g(\sin^2\theta + \mu\cos^2\theta)$ 4) $a = g(\sin\theta - \mu\cos\theta)$

5. The time taken by the body to slide down is given by
   1) $t = \sqrt{\dfrac{2l}{g(\sin\theta - \mu\cos\theta)}}$ 2) $t = \dfrac{2l}{g(\sin\theta + \mu\cos\theta)}$
   3) $t = \dfrac{2l}{g(\sin\theta - \mu\cos\theta)}$ 4) $t = \dfrac{2l^2}{g(\sin\theta - \mu\cos\theta)}$

6. When a body slides down on an inclined plane with coefficient of friction $\mu$, and the friction is absent $(\mu = 0)$, then the elocity of the block at the bottom of the inclined plane is
   1) $v = 2gh$ 2) $v = \sqrt{2gh}$ 3) $v = 2g\sin\theta$ 4) $v = 2l\sin\theta$

7. When a body slides down on an inclined plane with coefficient of friction $\mu$, then the velocity of the block at the bottom of an inclined plane si given by

1) $v = \sqrt{2gl(\sin\theta - \mu\cos\theta)}$ 2) $v = 2gl(\sin\theta - \mu\cos\theta)$

3) $v = 2g^2l^2(\sin\theta + \mu\cos\theta)$ 4) $v = \sin\theta + \mu\cos\theta$

8. When the angle of inclination '$\theta$' is less than the angle of repose $(\alpha)$ $[\theta < \alpha]$ then the body

1) Just begins to slide down 2) Slides on the plane

3) Does not slide on the plane 4) The body acquires acceleration

9. When $f_s = mg\sin\theta$ and $N = mg\cos\theta$, then the body will be

1) Starts sliding down the plane

2) Rests on the plane

3) tries to move the block along the plane downwards

4) Just ready to slide down

## JEE MAIN & ADVANCED

**LEVEL-1 Single Correct Choice Type:**

1. If the length of Smooth Inclined plane is 10 m and the value of g is 10 $m/s^2$ and the Angle of Inclination is $30^0$, then the Acceleration of the block is

1) 5 $m/s^2$ 2) 2.5 $m/s^2$ 3) 1 $m/s^2$ 4) 2 $m/s^2$

2. If the length of Smooth Inclined plane is 10 m and the value of g is 10 $m/s^2$ and the Angle of Inclination is $30^0$, then the Velocity of the block is___

1) 15 m/s 2) 20 m/s 3) 25 m/s 4) 10 m/s

3. If the length of Smooth Inclined plane is 10 m and the value of g is 10 $m/s^2$ and the Angle of Inclination is $30^0$, then the Time taken by the block to reach bottom is ____

1) 1s 2) 2s 3) 3s 4) 4s

4. If the length of Rough Inclined plane is 10 m and the value of g is 10 $m/s^2$ and the Angle of Inclination is $30^0$ and Coefficient of Friction is 0.1, then the Acceleration of the block is__

1) 4.13 2) 3.14 3) 1.43 4) 1.34

5. If the length of Rough Inclined plane is 10 m and the value of g is 10 $m/s^2$ and the Angle of Inclination is $30^0$ and Coefficient of Friction is 0.1, then the Velocity of the block is___

1) 7.09 2) 8.09 3) 6.09 4) 9.09

6. If the length of Rough Inclined plane is 10 m and the value of g is 10 $m/s^2$ and the Angle of Inclination is $30^0$ and Coefficient of Friction is 0.1, then Time taken by the block to reach bottom is ____

1) 1.2 2) 2.1 3) 2.2 4) 3.2

7. A brick of mass 2kg just begins to slide down on inclined plane at an angle of 45º with the horizontal. The force of friction will be

1) 19.6 sin 45º 2) 9.8 sin 45º 3) 19.6 cos 45º 4) 9.8 cos 45º

**Statement Type:**

8. *Statement-I:* When a body tends to slide down on an inclined plane, force of friction acts up the plane

*Statement-II:* Friction always opposes the relative motion

1) Both Statements are true.

2) Both Statements are false.

3) Statement I is true, Statement II is false.

4) Statement I is false, Statement II is true.

**LEVEL-2 & 3 Single Correct Choice Type:**

9. The minimum force required to move a body up an inclined plane of inclination 30º is found to be thrice the minimum force required to prevent it from sliding down the plane. The coefficient of friction between the body and the plane is

1) $1/\sqrt{3}$ 2) $1/2\sqrt{3}$ 3) $1/3\sqrt{3}$ 4) $1/4\sqrt{3}$

10. The angle of inclination of an inclined plane is 60º. Coefficient of friction between 10kg body on it and its surface is 0.2, g = 10 $ms^{-2}$. The acceleration of the body down the plane in $ms^{-2}$ is

1) 5.667 2) 6.66 3) 7.66 4) Zero

11. A 30kg block is to be moved up an inclined plane at an angle $30^\circ$ to the horizontal with a velocity of 5 m/s. If the frictional force retarding the motion is 150N, then the horizontal force required to move the block up the plane is $\left(g = 10 m/s^2\right)$

1) 364 N 2) 346 N 3) 326 N 4) 664 N

12. A body is sliding down on an inclined plane having coefficient of friction 0.5. if the normal reaction is twice that of resultant downward force along the inclined plane, then the angle between the inclined plane and the horizontal is

1) $60^\circ$ 2) $30^\circ$ 3) $45^\circ$ 4) $30^\circ$

**Comprehension type :**

The angle of inclination of an inclined plane is $60°$. Coefficient of friction between 10kg body on it and its surface is 0.2, $g = 10 m/s^2$.

13. The resultant force on the body is

1) 56.6 N 2) 66.6 N 3) 76.6 N 4) 86.6 N

14. The frictional force on teh body is

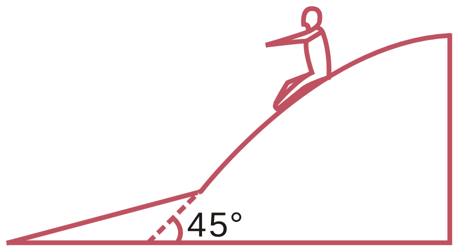
1) Zero 2) 2.5 N 3) 7.5 N 4) 10 N

15. The minimum force required to pull the body up the inclined plane

1) 66.6 N 2) 86.6 N 3) 96.6 N 4) 76.6 N

**LEVEL-4 & 5 Single Correct Choice Type:**

16. In a children-park an inclined plane is constructed with an angle of incline 45° in the middle part (figure). Find the acceleration of a boy sliding on it if the friction coefficient between the cloth of the boy and the incline is 0.6 and g = 10 $m/s^2$.




1) $2\sqrt{2} m/s^2$ 2) $8\sqrt{2} m/s^2$ 3) $6\sqrt{2} m/s^2$ 4) $4\sqrt{2} m/s^2$

17. A block slides down an inclined surface of inclination 30° with the horizontal. Starting from rest it covers 8 m in the first two seconds. Find the coefficient of kinetic friction between the two.

1) 1.19 2) 0.59 3) 0.11 4) 0.99

18. 17. The friction coefficient between road and the tyre of a vehicle is 4/3. Find the maximum incline of the road so that once hard brakes are applied and the wheel starts skidding, the vehicle going down at a speed of 10 m/s is stopped within 5m. ( g = 10 $m/s^2$)

1) $\sin^{-1}\left(\frac{7}{25}\right)$ 2) $\sin^{-1}\left(\frac{16}{7}\right)$ 3) $\sin^{-1}\left(\frac{4}{3}\right)$ 4) $\sin^{-1}\left(\frac{1}{8}\right)$

**Multi Correct Choice Type:**

19. A body of mass 2 kg is lying on a rough inclined plane of inclination 30°. Coefficient of static friction = 0.2. [Take g = 10m/s$^2$).Then the magnitude of the force parallel to the incline needed to make the block move

1) up the incline is 13N
2) down the incline is 18N
3) up the incline is 21N
4) down the incline is zero

20. If the coefficient of friction between an insect and bowl surface is $\mu$ and the radius of the bowl is r, the maximum height to which the insect can crawl in the bowl is

1) $\frac{r}{\sqrt{1+\mu^2}}$
2) $r\left[1-\frac{1}{\sqrt{1+\mu^2}}\right]$
3) $r\sqrt{1+\mu^2}$
4) $r\left[\sqrt{1+\mu^2}-1\right]$

# FRICTION PART-1_KEY

## FRICTION WORKSHEET-1

**CUQ**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1) 2 | 2) 3 | 3) 4 | 4) 3 | 5) 2 | 6) 1 | 7) 3 | 8) 2 |
| 9) 4 | 10) 2 | | | | | | |

### JEE MAIN AND ADVANCED:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1) 4 | 2) 1 | 3) 1 | 4) 4 | 5) 2 | 6) 1 | 7) 1 | 8) 1 |
| 9) 4 | 10) 1 | 11) 3 | 12) 3 | 13) 3 | 14) 2 | 15) 1 | 16) 2 |
| 17) 4 | 18) 3 | 19) 8 | 20) a-3;b-1;c-1;d-1,4 | | 21) 4 | 22) 3 | |

### HINTS & SOLUTIONS

2. $fs = \tan\theta$

3. $\mu = \frac{fs}{R} = \frac{mg}{2F}$

4. $f_s = \mu_s mg$

5. $f = \mu mg$ , $\mu = \frac{f}{mg}$

6. $a = (\mu_s - \mu_K)g$

8. $a_{max} = \mu g$

9. $f = \mu mg\sin\theta$

10. Applied force = frictional force=2.8 N

11. $t = \frac{\mu}{\mu g}$

13. power $= \frac{\text{work}}{\text{time}} = \frac{\mu mgs}{t}$

14. $\frac{a_1}{a_2} = \frac{\frac{F}{m}}{\frac{F - \mu_K mg}{m}}$

15. $v^2 - u^2 = 2as \Rightarrow v^2 - u^2 = -2\mu gs$

$\Rightarrow (5.6)^2 = 2 \times \mu \times 9.8 \times 16 \quad \Rightarrow \mu = \frac{5.6 \times 5.6}{2 \times 9.8 \times 16} = 0.1$

16. W = f.s $= \mu mg.s$ = 0.1 × 10 × 9.8 × 16 = 156.8 joules

17. V = u+at

18. $F_{up} = W + \mu F$

19. $mg = 100N \rightarrow m = 10$ kg;

$a1 = 1m/s^2$, $a_2 = 10\ m/s^{2;}$

$F - f = ma_1$, $2F - f = ma_2$

## FRICTION WORKSHEET-2

**CUQ** 1) 1 2) 3) 3 4) 4 5) 1 6) 2 7) 1
8) 3 9) 2

### JEE MAIN & ADVANCED

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1) 1 | 2) 4 | 3) 2 | 4) 1 | 5) 4 | 6) 3 | 7) 3 | 8) 3 |
| 9) 2 | 10) 3 | 11) 2 | 12) 3 | 13) 3 | 14) 4 | 15) 2 | 16) 1 |
| 17) 3 | 18) 1 | 19) 1,4 | 20) 2 | | | | |

### HINTS & SOLUTIONS

1. $a = g\sin\theta = 10 \times \sin 30° = 5m/s^2$

2. $v = \sqrt{2gl\sin\theta}$; $v = 10m/s$

3. $t = \sqrt{\frac{2l}{g\sin\theta}}$; $t = 2\sec$

4. $a = g(\sin\theta - \mu\cos\theta)$; $a = 4.13$

5. $v = \sqrt{2gl(\sin\theta - \mu\cos\theta)}$ $v = 9.09m/s$

6. $t = \sqrt{\dfrac{2l}{g(\sin\theta - \mu\cos\theta)}}$ ; $t = 2.2\,sec$

7. $\mu = mg\cos\theta$ ; $\mu = 19.6\cos 45°$

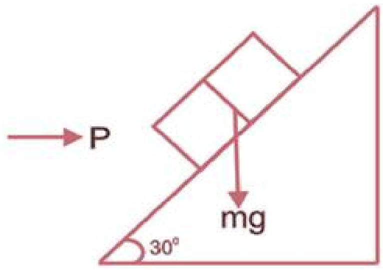
9. $F_{up} = 3 \times F_{down}$

$\sin\theta + \mu\cos\theta = 3\sin\theta - 3\mu\cos\theta$

$\mu = \dfrac{1}{2\sqrt{3}}$

10. $a = g(\sin\theta + \mu\cos\theta)$ ; $a = 7.66$

11. The force requried to move a body up an inclined plane is




$F = mg\sin\theta + f_k$

$f_k = \mu_k(mg\cos\theta + P\sin\theta) = 150N$

$= 30(120)\sin 30° + 150$

$= 300N$

If P is the horizontal force $F = P\cos\theta$

$P = \dfrac{F}{\cos\theta} = \dfrac{300}{\cos 30°} = \dfrac{300 \times 2}{\sqrt{3}} = 346N$

12. $N = 2mg(\sin\theta - \mu\cos\theta)$

$\cos\theta = 2\cos\theta(\tan\theta - \mu)$

$\dfrac{1}{2} = \tan\theta - \dfrac{1}{2} \Rightarrow \tan\theta = 1 \Rightarrow \theta = 45°$

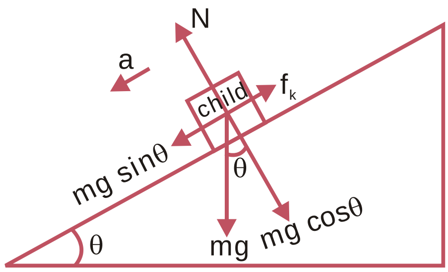
13. $a = g(\sin\theta + \mu\cos\theta)$ ; $a = 7.66 m/s^2$

$F = ma$ ; $F = 76.6N$

14. $f_k = \mu mg\cos\theta$ = 10 N

15. $f_{up} = 76.6 + 10$ = 86.6 N

16.




Given, $\theta = 45°$, $\mu_k = 0.6$ , g = 10m/s², a = ?

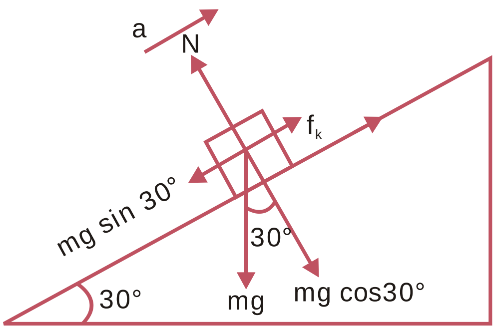
F.B.D. of child: (i) N = mgcos θ (ii)mgsin θ – $f_k$ = ma

$\Rightarrow mg\sin\theta - \mu_k N = ma \Rightarrow mg\sin\theta - \mu_k mg\cos\theta = ma \Rightarrow g\sin\theta - \mu_k g\cos\theta = a$

$\Rightarrow g(\sin\theta - \mu_k\cos\theta) = a \Rightarrow 10\left(\frac{1}{\sqrt{2}} - 0.6\times\frac{1}{\sqrt{2}}\right) = a$

$\Rightarrow \frac{10}{\sqrt{2}}(1-0.6) = a \Rightarrow \frac{10}{\sqrt{2}}\times 0.4 = a \Rightarrow \frac{4}{\sqrt{2}} = a \Rightarrow a = 2\sqrt{2}\ m/s^2$

17.




* = 0, s = 8m, t = 2s, $\mu_s = ?$ Using $s = ut + \frac{1}{2}at^2$, we get

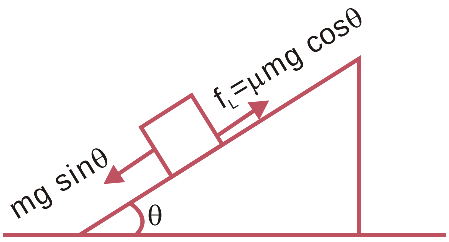
$8 = \frac{1}{2}\times a\times 4 \Rightarrow a = 4m/s^2$ From the F.B.D of block it is clear that,

N = mg cos30° $= \frac{mg\sqrt{3}}{2}$ and mg sin30° – $f_k$ = ma $\Rightarrow \frac{mg}{2} - \mu_k N = m\times 4$

$\Rightarrow \frac{mg}{2} - \mu_k\frac{mg\sqrt{3}}{2} = m\times 4 \Rightarrow 5 - \mu_k 5\sqrt{3} = 4 \Rightarrow 5(1-\mu_k\sqrt{3}) = 4 \Rightarrow \mu_k = \frac{1}{5\sqrt{3}} \approx 0.11$

18. Let $\theta$ be req*ired incline of the road as shown in the fig*re. Since the tyre is coming to rest after travelling some distance the acceleration of the tyre is in the *pward direction.

Maxim*m retardation $a = \frac{f_L - mg\sin\theta}{m} = \frac{\mu mg\cos\theta - mg\sin\theta}{m} = \mu g\cos\theta - g\sin\theta$




Now, *sing v² = *² – 2as, we get $0 = (10)^2 - 2(\mu g\cos\theta - g\sin\theta)(5)$

or $\mu g\cos\theta - g\sin\theta = 10$ or $\left(\frac{4}{3}\right)(10)\cos\theta - (10)\sin\theta = 10$

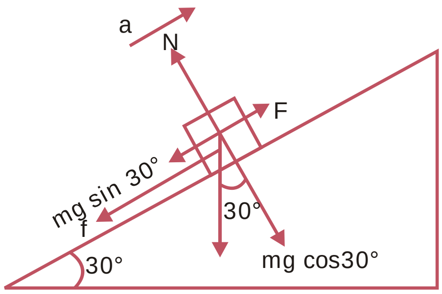
or $4\cos\theta - 3\sin\theta = 3$ or $4\cos\theta = 3(1+\sin\theta)$

or $16\cos^2\theta = 9(1+\sin^2\theta + 2\sin\theta)$ or $16(1-\sin^2\theta) = 9(1+\sin^2\theta+2\sin\theta)$

or $25\sin^2\theta + 18\sin\theta - 7 = 0$

$\sin\theta = \frac{-18 \pm \sqrt{324+700}}{50} = \frac{\sqrt{1024}-18}{50} = \frac{32-18}{50} = \frac{14}{50} = \frac{7}{25}$ $\therefore\ \theta \approx 16°$

19.




Given m = 2kg, $\theta = 30°$, F = ? $\mu_s = 0.2$

a) F.B.D. of block,

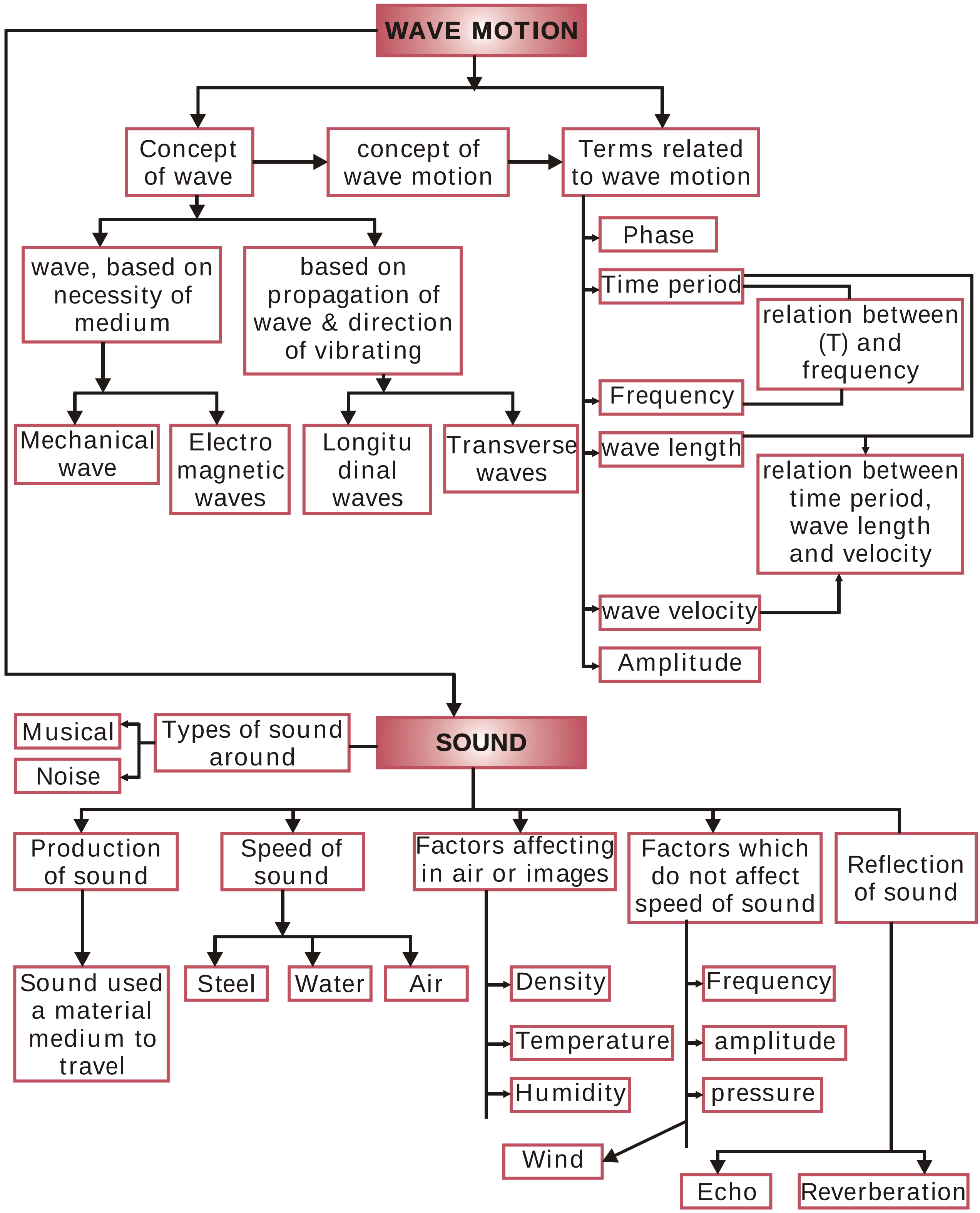
(i) N = mgcos30° $= \frac{mg\sqrt{3}}{2}$ F – (mg sin30 + $f_s$) = 0

F = mg sin30° + $f_s$ = $\frac{mg}{2} + \mu_s N = \frac{mg}{2} + \mu_s\frac{mg\sqrt{3}}{2} = \frac{mg}{2}(1+\mu_s\sqrt{3})$

$\Rightarrow F = \frac{2\times10}{2}(1+0.2\sqrt{3}) = 10 + 2\sqrt{3} \approx 13N$

b) $f_s$ (3N) < F (10N) $\Rightarrow$ No force is req*ired to move the block down the plane.

# MEMO GRAPH


WAVE MOTION
Concept of wave
concept of wave motion
Terms related to wave motion
wave, based on necessity of medium
based on propagation of wave & direction of vibrating
Mechanical wave
Electro magnetic waves
Longitu dinal waves
Transverse waves
Phase
Time period
relation between (T) and frequency
Frequency
wave length
relation between time period, wave length and velocity
wave velocity
Amplitude
SOUND
Musical
Noise
Types of sound around
Production of sound
Speed of sound
Factors affecting in air or images
Factors which do not affect speed of sound
Reflection of sound
Sound used a material medium to travel
Steel
Water
Air
Density
Temperature
Humidity
Wind
Frequency
amplitude
pressure
Echo
Reverberation

# WAVE MOTION AND SOUND

## KNOW YOUR SCIENTIST

Heinrich Rudolf Hertz
(1857 - 1894)

### Heinrich Rudolf Hertz
### (1857 - 1894)

German Physicist who was the first to broadcast and receive radio waves. He produced electromagnetic waves, sent them through space, and measured their wavelength and speed. He showed that the nature of their vibration, reflection and refraction was the same as that of light and heat waves, establishing their identity for the first time. He also pioneered research on discharge of electricity through gases, and discovered the photoelectric effect. In his honour the SI unit of frequency is named as hertz (Hz)

## WAVE MOTION AND SOUND_SYNOPSIS-1

**Introduction of wave :** There are two ways of transfer of energy from one point to other.

1. By the transport of matter and
2. Without the transport of matter (i.e through wave motion]

**Wave :** A wave is produced by the vibrations of the particles of the medium through which it passes.

When a wave passes through a medium, the medium itself does not move along the direction of the wave, only the particles of the medium vibrate about their fixed positions. For example, when a water wave passes over the surface of water in a pond, it does not drive water to one side of the pond, only the water molecules vibrate up and down about their fixed positions. Similarly, when sound waves produced by ringing bell come to us through air, there is no actual movement of the air from the bell to our ears. Only the sound energy travels through the vibrations of the air molecules. The light from the sun also comes to us in the form of light waves, there being no direct contact between the sun and the earth. Thus

A wave is a disturbance by which energy is transferred from one point to the other through vibrations of the medium particles without their actual movements.

**(OR)**

A periodic disturbance produced in a material medium due to the vibrating motion of the particles of the medium is called a wave.

**Wave motion :** The movement of a disturbance produced in one part of a medium to another involving the transfer of energy but not the transfer of matter is called wave motion.

**Examples :** The following are examples of wave motion

(1) Formation of ripples on the water surface.

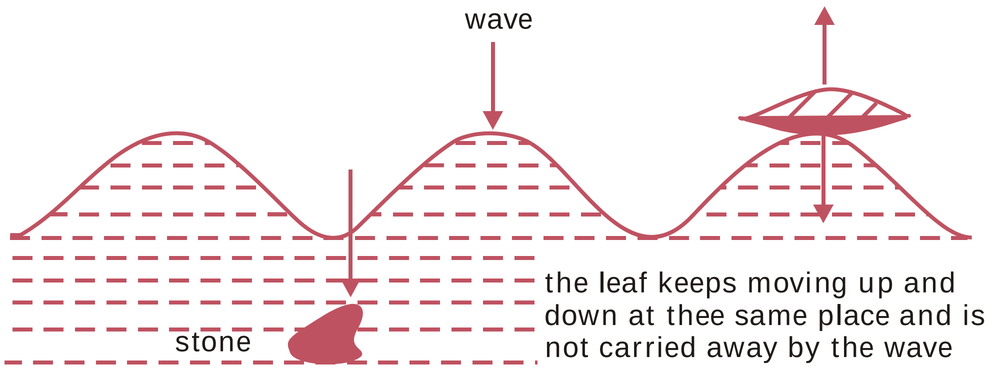
(2) Propagation of sound wave through air or any other material medium.

**Explanation of wave motion:** If we drop a stone into a pond, we see circular water waves (ripples) spreading out in all directions on the surface of the water .

a. Ripples produced in water when a stone is dropped into a pond

If we place a small leaf on the water surface, the leaf moves up and down about its original position but does not move away from or towards the source of disturbance along with the waves.

This shows that the disturbances moves from one place to another but the water is not carried with it. The water particles simply move up and down about their mean positions. The formation of ripples on the surface of water is an example of wave motion. **The material in which the wave motion is produced called a medium.** Water surface is the medium of wave motion in the above example. In the above example, when the stone is




b. Wave motion does not carry matter (material) away from or towards the source of disturbance

thrown into the pond, the energy carried by the stone disturbs the water molecules close to it. By gaining the energy from the stone, the water molecules near the stone start vibrating up and down. These vibrating water molecules transfer some of the energy to the next set of water molecules which also start vibrating, and so on. In this way, water wave is formed. We can now say that, **wave motion is a vibratory disturbance produced in one part of the medium that travels to another part involving the transfer of energy but not the transfer of any matter with it. The disturbance itself is called a wave.**

Sound is also an example of wave motion. The sound energy of our speech reaches the listener's ear through the vibratory motion of the air particles.

## Characteristics of wave motion:

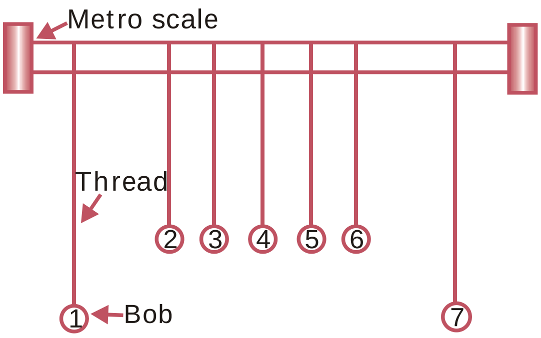
1. In wave motion the particles of the medium vibrate about their mean positions. The particles of the medium don't move from one place to another.
2. During a wave motion, energy is transferred from one point of the medium to another. There is no transfer of matter through the medium.
3. During wave motion, the medium does not move as a whole only the disturbance travels through the medium.
4. A wave motion travels at the same speed in all directions in any medium. The speed of a wave depends upon the nature of the medium through which it travel.

## Experiment to show that a wave motion transfers only energy and not matter:

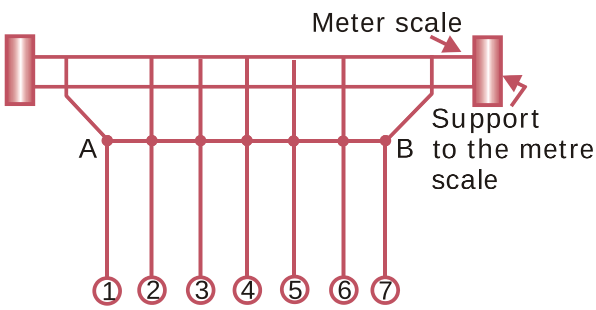
Wave motion involves the transfer of energy from one part of the medium to another. This can be shown by performing the following simple experiment.

Suspend seven pendulum bobs (small metallic balls) from a meter scale as shown in figure. The bobs 2,3,4,5 and 6 are suspended by strings of equal length, and are at equal distances from each other. The bobs 1 and 7 are suspended by slightly longer strings and are placed at a slightly larger distance from their neighbouring bobs.

Now pass a string horizontally through all the seven strings, tying firm knots with each string so that all the seven bobs are at the same level and at equal distances from each other.




An arrangement of seven bobs suspended from a meter scale.

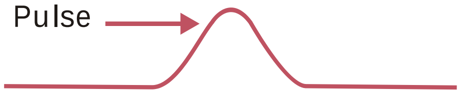



An arrangement of seven bobs to show the transfer of energy during wave motion

Now push bob 1 slightly so that it starts oscillating in the plane of the strings without colliding with its neighbouring bob. After some time, bob 2 starts oscillating, then bob 3 and so on until the last bob starts oscillating. Thus, we see that the oscillations of bob 1 travelled progressively to bob 7 without touching it.How did the disturbance reach from bob 1 to bob 7? This can be explained as follows.

When bob 1 is disturbed by giving it a certain amount of energy, it starts oscillating. These oscillations of bob 1 produce disturbances in the horizontal string (AB). This disturbance in the string AB is progressively transferred to the other bobs, which start swinging. Thus, the energy given to bob 1 is transferred to the other bobs, progressively, but no matter is transferred.

**Pulse:** The wave set up by a single disturbance in the medium is called a pulse




**Ex:** We clap our hands.

**Classification of waves based on necessity of medium:** Depending upon the requirement of medium for the propagation of wave, the waves are classified in the following two groups.

(i) Mechanical waves (or elastic waves]

(ii) Electromagnetic waves.

**(i) Mechanical waves:**

The waves which need a material medium for their propagation are called mechanical waves. The medium may be a solid, a liquid or a gas.

**(OR)**

The wave which required material media for their transmission is called mechanical wave.

**Note:** Mechanical waves cannot travel through vacuum.

**Ex.** Vibrations in a rope, vibrations in a stretched string, vibrations on the surface of water, sound waves in air etc., are the mechanical waves.

**(ii) Electromagnetic waves (non-mechanical waves):**

The waves which don't need a material medium for their propagation are called electromagnetic waves.

**(OR)**

The waves which do not required material media for their transmission is called electromagnetic waves.

**Note:** (i) electromagnetic waves can travel through vacuum.

(ii) electromagnetic waves travel in vacuum with the speed of light i.e $3 \times 10^8$ m/s

**Example :** Light waves, X-rays, Gamma rays, Radio waves, Microwaves etc., are the examples of electromagnetic waves.

**Classification of waves based on propagation of wave & direction of vibrating particles:** On the basis of the relative directions of the propagation of the wave with respect to direction of the periodic changes in the medium (such as displacement, pressure etc.) the waves are classified into the following two groups.

**(i) Longitudinal waves** **(ii) Transverse waves**

**Longitudinal wave:** A wave in which the particles of the medium oscillate (vibrate) to-and-fro (back and forth) in the same direction in which the wave is moving, is called a longitudinal wave.

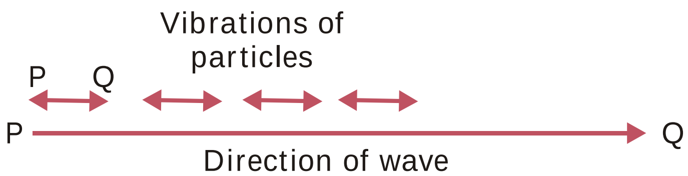
**(OR)**

A wave in which the particle of the medium vibrate up and parallel to the direction of wave is longitudinal wave.

**Ex:** 1. Sound waves in air 2. Waves in spiral spring

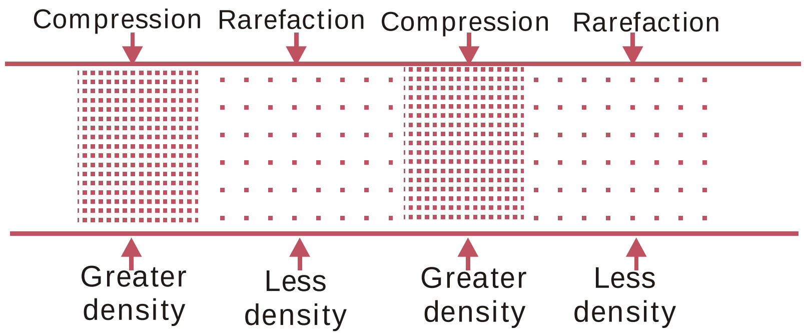
3. The waves produced in air when a sitar wire is plucked.

**Representation of a longitudinal wave :** In this figure, the direction of wave has been shown from P to Q, in the horizontal plane. The direction of vibrations of the particles is also along PQ, parallel to the direction of wave. That is, the particle of the medium vibrate back and forth in the horizontal direction.




**Compression and Rarefactions of a longitudinal wave :**

**Compression :** The part of a longitudinal wave in which the density of the particles of the medium is higher than the normal density is called a compression.




Pictorial representation of a longitudinal wave

**Rarefaction :** The part of a longitudinal wave in which the density of the particles of the medium is lesser than the normal density is called a rarefaction.

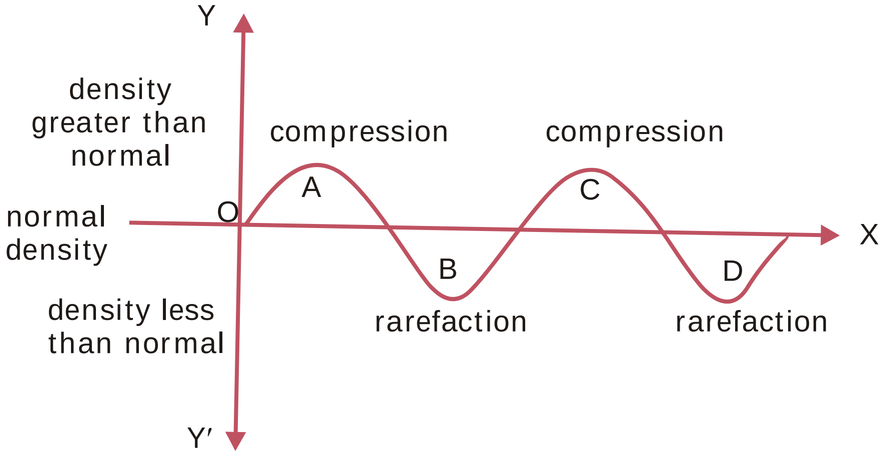
**Note :**

1. Compression and rarefaction are formed alternately in longitudinal wave.
2. The distance between the two successive compressions or between the two successive rarefactions is equal to one wavelength ($\lambda$).
3. Longitudinal waves can travel through all media i.e. solids, liquids and gases.
4. The phase difference between two successive compressions or rarefactions is equal to $2\pi$ radians.

**Graphical representation of longitudinal waves:** Longitudinal waves in air represented graphically by plotting the density of air against distance from the source.

In other words, a longitudinal waves is represented by a density - distance graph.

In figure the horizontal line OX represents the normal density of air. All the points above this line represent greater density. In a compression of a longitudinal wave, the density of the particles is high. So, here A and C represent compressions. All the points below the line OX represent less density (than normal). In a rarefaction, the density of the particles is less than that in the normal. So here B and D represent rarefactions.




Graphical representation of a longitudinal wave by density - distance graph

**Formation of Longitudinal wave in a slinky:**

**Note:** (A long flexible spring which can be compressed or extended easily is called slinky)

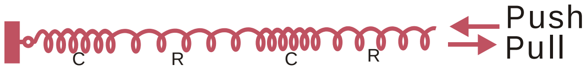
The waves which travel along a spring (or slinky) when it is pushed and pulled at one end, are longitudinal waves.

We will now describe how longitudinal waves are formed on a spring. Figure: (a) shows the normal position of a spring whose one end is fixed.

Now, if the free end of the spring is moved to and fro continuously, then longitudinal waves consisting of alternate compressions and rarefactions travel along the spring.

When a wave travels along the spring, then each turn of the spring moves back and forth by only a small distance in the direction of the wave. Since the particles of the medium (turns of the spring) are moving back and forth in the direction of the wave, the waves which travel across the spring are longitudinal waves.

(a) Normal position of a spring




(b) Longitudinal wave in a spring

In a spring, a compression is that part in which the coil (or turns) are closer together than normal. In figure (b), the regions marked ‘C’ are compressions.

In a spring, a rarefaction is that part in which the coils (or turns) are farther apart than normal. In figure (b), the regions marked R are rarefactions.

**The Sound waves in air are longitudinal waves:** When a sound wave passes through air, the particles of air vibrate back and forth parallel to the direction of sound wave. Thus, when a sound wave travels in the horizontal direction, then the particles of the medium also vibrate back and forth in the horizontal direction.

It should be remember that the waves produced in air when a sitar wire is plucked are longitudinal waves, because those are sound waves.

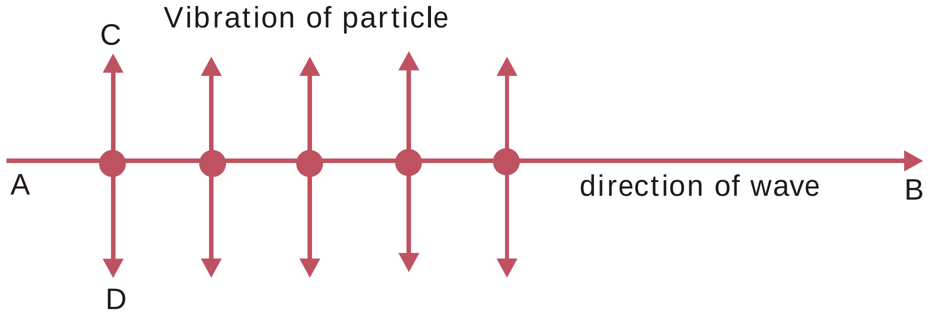
**Transverse wave (T.W) :** A wave in which the particles of the medium oscillate (vibrate) up and down, i.e perpendicular to the direction in which the wave is moving is called a transverse wave.

**(OR)**

If the particles of the medium vibrate perpendicular to the direction of the propagation of the wave then the wave is called transverse wave **(or)** When the particles of a medium oscillate at right angles to the direction of propagation of a wave, then the wave so produced is called a transverse wave.

**Examples :**
1. Waves on the surface of water
2. Waves on a long stretched rubber tube
3. Waves along a stretched string.
4. Even the light waves and radio waves are transverse waves.

**Representation of a transverse wave :** In this figure, the direction of wave is from A to B but the vibration of the particles are along CD which is at right angles to the direction of wave AB. So, this is a transverse wave.



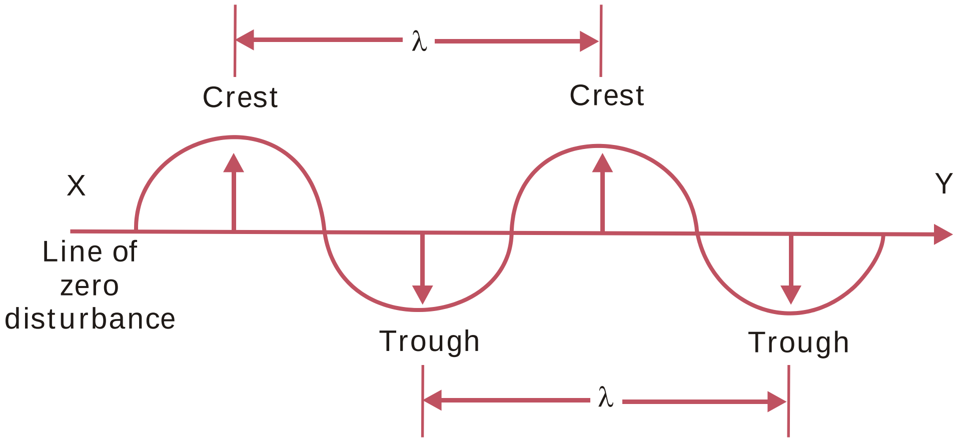

Representation of a transverse wave

**Crest and trough of a transverse wave :**

We know that when a transverse wave travels horizontally in a medium, the particles of the medium vibrate up and down in the vertical direction. When the vibrating particles move upward or above the line of zero disturbance, they form an 'elevation' or 'hump' and when the vibrating particles move downward or below the line of zero disturbance, they form a 'depression' or 'hollow'.

**Crest:** The 'elevation' or 'hump' in a transverse wave is called crest. In other words, a crest is that part of the transverse wave which is above the line of zero disturbance of the medium. In figure XY is the line is zero disturbance.

**Trough :** The 'depression' or 'hollow' in a transverse wave is called trough. In other words, a trough is that part of the transverse wave which is below the line of zero disturbance.




Crest and trough of a transverse wave

**Note :**

(1) The distance between the two consecutive crests or between the two consecutive troughs is equal to one wavelength ($\lambda$).

(2) The distance between a crest and the adjoining trough is $\frac{\lambda}{2}$

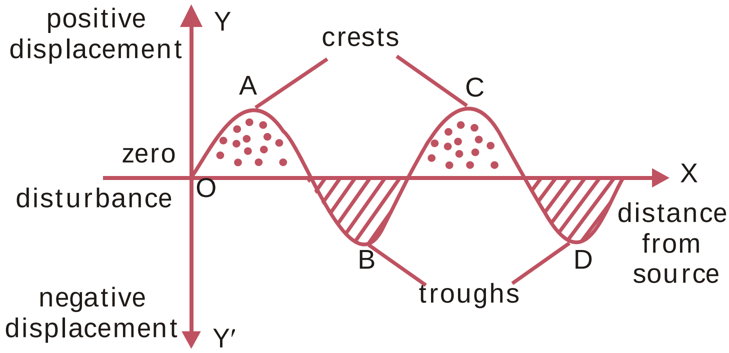
(3) A transverse wave cannot travel inside the liquid and in gases.

(4) Propagation of a transverse wave, through a medium, results in the formation of crests and troughs.

**Graphical representation of transverse waves:** A transverse wave is represented graphically by plotting the displacement of different particles of the medium from the line of zero disturbance against distance from the source. In other words, a transverse wave is represented by a displacement - distance graph.

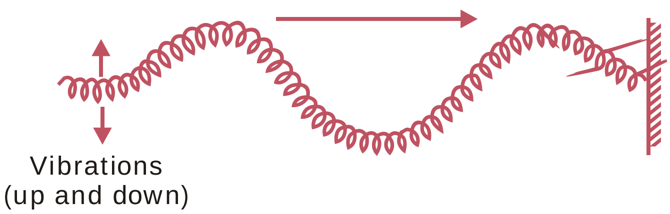
When a transverse wave passes through a medium, then some particles of the medium are displaced above the line of zero disturbance whereas others are displaced below the line of zero disturbance. So a transverse wave is represented graphically by plotting the displacement of different particles of the medium from the line of zero disturbance against the distance from the source.

In figure the horizontal line OX represents the line of zero disturbance of the particles of the medium. All the particles above the line of zero disturbance have positive displacements and those below it have negative displacements. In figure A and C represent two crests and B and D represent two troughs of the transverse wave.




Graphical representation of a transverse wave by displacement - distance graph

**Formation of transfers wave in a slinky:** The waves produced by moving one end of a long spring (or slinky) up and down rapidly, whose other end is fixed, are transverse waves. The transverse wave produced on a slinky is shown in figure. As the wave passes along the slinky in the horizontal direction, the particles of slinky vibrate up and down at right angles to the direction of wave.




Transverse wave on a long spring (or slinky)

**Formation of transfers wave on the surface of water:**

The water waves (or ripples) formed on the surface of water in a pond are also transverse waves. This is because of the fact that in water wave, the molecules of water move up and down in the vertical direction when the wave travels in the horizontal direction along the water surface. Since the water molecules vibrate up and down at the same place therefore, a cork or leaf placed on the surface of water moves up and down at the same place as water wave moves across the surface of the pond.

**Note:** When a stone is dropped in a pond of water, transverse waves are produced on the surface of water.

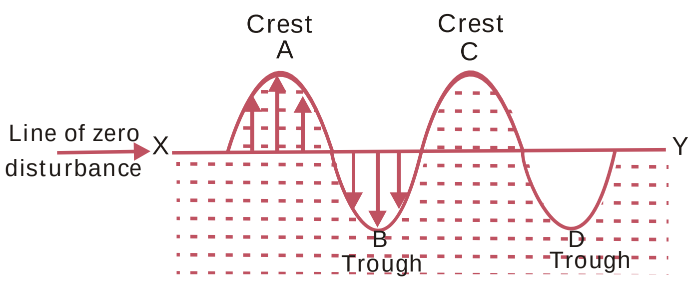



Diagram to show the crests and troughs of a transverse wave.
Here, A and C are crests; B and D are troughs

**Difference between transverse waves and longitudinal waves :**

| Transverse waves | Longitudinal waves |
|---|---|
| 1. In transverse waves, the particles of the medium vibrate at right angles to the direction of wave.<br>2. Transverse waves consist of crest and troughs.<br>3. Transverse waves can be propagated only through a solid or over the surface of a liquid but not in a gas.<br>4. In transverse waves, the distance between the two consecutive crests or between the two consecutive troughs is equal to one wavelength.<br>5. It is represented by displacement distance graph. | 1. In longitudinal waves, the particles of the medium vibrate parallel to the direction of wave.<br>2. Longitudinal waves consist of compression and rarefactions.<br>3. Longitudinal waves can be propagated through solids, liquids, as well as gases.<br>4. In longitudinal waves, the distance between the two successive compressions or between the twp successive rarefactions is equal to one wavelength.<br>5. It is represented by density distance graph. |

## SCIENTIFIC FACT

**Why does voice differ from person to person?**

phonation is the scientific term of voice. It is achieved by larynx which is especially adapted to act as a vibrating element is the vocal folds commonly called the vocal cords. The vocal folds protrude from the lateral walls of the larynx walls of the larynx towards the center of the glottis. These folds are stretched and positioned by several specific muscles of the larynx itself.

During normal breathing the folds are wide open. It allows the easy passage of air. During phonation the folds close together so that the passage of air between these will cause vibration. The pitch of the vibration only determines the quality of the voice. This pitch inturn is determined by the following three factors: the degree of stretch of the folds, how tightly the folds approximated to each other, mass of the edges of the folds. These three aspects vary from person to person to person causing voice difference among persons.

But a person can voluntarily change his voice by changing the resonating qualities of the different strutchers of resonators namely mouth, nose and associated nasal sinuses and even the chest cavity.

Testosterone secretion by the testes in male causes hypertrophy of the laryngeal mucosa and enlargement of the larynx. These effects cause typical adult masculint base voice.

# WAVE MOTION AND SOUND_WORKSHEET-1

CUQ 1. From which of the following ways are used to transfer of energy from one point to other?

1) by the transport of matter

2) without the transport of matter (wave motion)

3) both (1) and (2)

4) neither (1) nor (2)

2. When a wave passes through a medium

1) the medium itself move along the direction of the wave

2) the medium itself does not move along the direction of the wave

3) the medium sometimes move and same times does not move along the direction of the wave.

4) both (1) and (2)

3. When a water wave passes over the surface of water in a pond.

1) it drive water to one side of the pond.

2) it does not drive water to one side of the pond.

3) it some times drive and same times does not drive water to one side of the pond

4) both (1) and (2)

4. The movement of a disturbance produced in one part of a medium to another involving the transfer of energy but not the transfer of matter is called

1) Wave motion 2) Wave length 3) Wave speed 4) Wave frequency

5. Formation of ripple on the water surface is an example of _______

1) Wave motion 2) Periodic motion 3) Rotatory motion 4) Oscillatory motion

6. The wave set up by a single disturbance in the medium is called

1) Frequency 2) Speed 3) Pulse 4) Amplitude

7. When a medium starts vibrating the propagation of energy takes place in

1) horizontal direction 2) vertical direction

3) all direction 4) none of the above

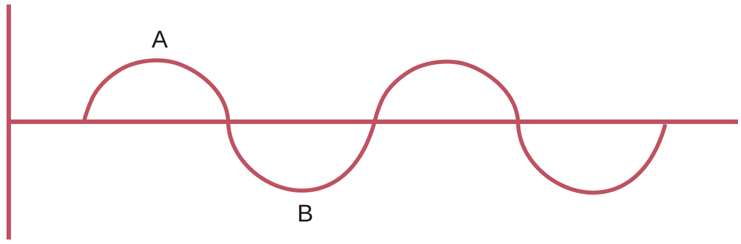
8. When a wave is found on the surface of water, its particles vibrate in the

1) direction of wave propagation

2) direction of right angles to the wave propagation.

3) in all direction

4) none of the above.

9. 


In the above transvers wave the elevation 'A' is called

1) crest 2 trough 3) both (1) and (2) 4) neither (1) nor (2)

10. From the above fig the depression 'B' is called

1) crest 2 trough 3) both (1) and (2) 4) neither (1) nor (2)

## JEE MAINS

### Single Correct Choice Type:

1. _______ energy of our speech reaches to the listener's ear through the vibratory motion of the air particles.

1) Electrical 2) Sound 3) Potential 4) Kinetic

2. The speed of a wave depends upon the ________of the medium through which it travel.

1) nature 2) colour 3) place 4) both (1) and (2)

3. The _____ is a disturbance by which energy is transfer without the transport of matter.

1) light 2) heat 3) wave 4) none of these

4. Wave transfer from one place to another only

1) energy 2) particles 3) mass 4) medium

5. The lowest part of the transverse wave below the line of zero distrubance is the

1) compression 2) crest 3) trough 4) rarefaction

6. In a wave motion, the particles oscillate perpendicular to the direction of wave propagation. The wave is

1) transverse 2) longitudinal 3) both (1) and (2) 4) none of these

7. In a wave motion, the particles oscillate in the direction of wave propagation. The wave is

1) transverse 2) longitudinal 3) both (1) and (2) 4) none of these

8. The highest part of the transverse wave above the line of zero distrubance is the

1) compression 2) crest 3) trough 4) rarefaction

### Comprehension Type:

9. The separation between two successive crests in a wave is

1) $\lambda$ 2) $\lambda/2$ 3) $2\lambda$ 4) $\lambda/4$

10. The distance between a compression and the next rarefaction in a longitudinal wave is
    1) $\lambda/2$  2) $\lambda$  3) $\lambda/4$  4) $2\lambda$
11. The distance between a crest and the next trough in a transverse wave is
    1) $\lambda/2$  2) $\lambda/4$  3) $\lambda$  4) $2\lambda$
12. If the distance between a crest and the adjoining trough is 20 cm. Then the wave - length will be
    1) 30 cm  2) 20 cm  3) 40 cm  4) 25 cm
13. Mechanical (elastic) waves can travel
    1) in a medium but not in a vacuum  2) in a vacuum but not in medium
    3) in a vacuum as well as in a medium.  4) neither in vacuum nor in a medium.
14. Sound waves are
    1) longitudinal  2) transverse
    3) partly longitudinal, partly transverse
    4) sometimes longitudinal, sometimes transverse.

## JEE ADVANCED

**Multi Correct Choice Type:**

15. Which of the following statement is/are false?
    1) both light and sound wave are longitudinal
    2) both light and sound waves can travel in vacuum.
    3) both light and sound waves are transverse
    4) sound waves in air are longitudinal while the light waves are transverse.

**Statement Type:**

16. ***Statement I :*** The wave in which crests and troughs are formed are called longitudinal waves.
    ***Statement II :*** The wave in which crests and troughs are formed are called transverse wave
    1) Both Statements are true
    2) Both Statements are false
    3) Statement - I is true, Statement - II is false.
    4) Statement - I is false, Statement - II is true.

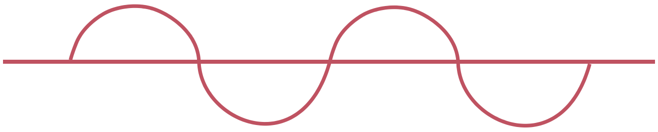
**Matrix Match Type:**

17.

| Column-I | Column-II |
|---|---|
| a) Mechanical waves | 1)Vibrations in a rope |
| b) Electromagnetic waves | 2)Light waves |
| c) Transverese wave | 3)Sound waves in air |
| d) longitudinal wave | 4)Waves on a long stretched rubber tube |

# WAVE MOTION AND SOUND_SYNOPSIS-2

1. **Phase :** The points on a wave which are in the same state of vibration are said to be in the same phase.




In the above figure A, B are in the same phase and C, D are in the same phase.

**Ex :** All points that lie on the crests are in the same phase similarly all points that lie on the trough are in the same phase.

2. **Time period (T) or Periodic time :** The time in which a vibrating body completes one vibration is called time period **or** The time required to produce one complete wave (or cycle) is called time period of the wave.

**Ex :** Suppose two waves are produced in 1 second. Then the time required to produce one wave will be 1/2 second or 0.5 second. In other words, the time period of this wave will be 0.5 second.

**Note :** The S.I unit of time period is second (s).

3. **Frequency (f or $\upsilon$) :** The number of complete waves (or cycles) produced in one second is called frequency of wave **or** The number of vibrations per second is called frequency.

**Unit :** The S.I unit of frequency is hertz (which is written as Hz).

**Hertz:** When a vibrating body produces one vibration in one second, then its frequency is said to be one hertz.

1 hertz is equal to 1 vibration per second.

i.e. $1 \text{ hertz} = \frac{1 \text{ vibration (cycles)}}{1 \text{ second}}$, 1 kHz = 1000 Hz

**Ex :** If 10 complete waves (or vibrations) are produced in one second, then the frequency of the waves will be 10 hertz (or 10 cycles per second).

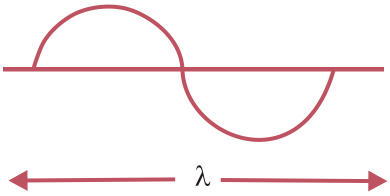
**Note1:** The frequency of a wave is fixed and does not change even when it passes through different substances.

**Note2:** It does not depend on the wave velocity, amplitude and nature of the medium.

**4. Wavelength (λ):** Wavelength of a wave is the length of one wave.

The distance travelled by the wave in one time period is called the wavelength **or** The distance between two nearest particles which are in the same phase is called wave length. It is denoted by the Greek letter λ (lambda).

**Ex:**




Here the length of the wave is λ.

**Unit :** The S.I unit of wave length is meter (m).

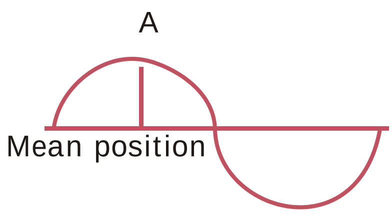
**5. Wave velocity (velocity of wave) :** The distance travelled by a wave in one second is called the velocity of wave. The wave velocity is denoted by v.

$$\text{Wave velocity} = \frac{\text{distance travelled by the wave}}{\text{time taken}}$$

**Unit :** The S.I unit of wave velocity is m/s or $ms^{-1}$.

**6. Amplitude (A) :** The maximum displacement of the particles of a medium from their mean positions during the propagation of a wave is called amplitude of the wave. It is represented by the letter 'A'.

**Ex:**




Here A is the amplitude of the wave.

**Unit :** The S.I unit of amplitude is metre (m).

**Note :** The amplitude of a wave is a measure of its energy. Thus the greater the amplitude of a wave, the greater the energy carried by the wave.

**Relation between time Period (T) and frequency (f):**

The frequency of a wave is the reciprocal of its time–period.

i.e., $\text{frequency} = \frac{1}{\text{time period}} \Rightarrow f = \frac{1}{T} \Rightarrow T = \frac{1}{f}$

Where f = frequency of the wave and T = time period of the wave.

**Relationship between, time period, wavelength and velocity:**

We know that, velocity of the wave = $\frac{\text{distance travelled by the wave}}{\text{time taken}}$

Suppose a wave travels a distance $\lambda$ in time T, then $v = \frac{\lambda}{T}$

[i.e. the relation between wave velocity, wavelength and time period]

Hence T is the time taken by one wave. We know that $\frac{1}{T}$ becomes the number of waves per second and this is known as frequency (f) of the wave.

So the above equation can also be written as, $v = f \times \lambda$

Where, v = velocity of the wave   f = frequency   $\lambda$ = wavelength

In other words velocity of a wave = frequency × wavelength

Thus, the velocity (or speed) of a wave in a medium is equal to the product of its frequency and wavelength. The formula $v = f \times \lambda$ is called wave equation.

1. The time period of a wave is equal to reciprocal of the frequency of a wave.

   i.e., $T = \frac{1}{f} \quad \Rightarrow f = \frac{1}{T}$

2. Wave velocity $= \frac{\text{distance travelled by the wave}}{\text{time taken}} \quad \Rightarrow v = \frac{\lambda}{T}$

3. Velocity of a wave = frequency × wavelength

   $v = f \times \lambda \quad \Rightarrow f = \frac{v}{\lambda} \quad \Rightarrow \lambda = \frac{v}{f}$

## DID YOU KNOW?

**How is sound caused by cracking of knuckles?**

Sound is caused by the cracking of knuckles when a bubble which has formed in the lubricating fluid ( the synovial fluid )which surrounds the two bones of the knuckles bursts.

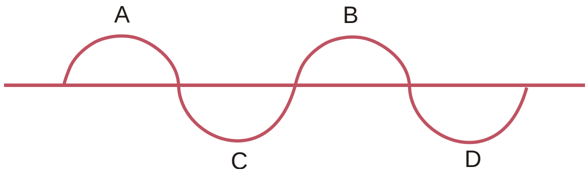
Normally there is a lubricating fluid which prevents the two bones which forms the knuckles from grating against each other. This fluid contains tiny bubbles which cannot be seen by naked eye.

So when the knuckle bone remain close together the fluid is under pressure and the bubbles remain tiny.

But when one clenches his fist, the bones are pulled apart which causes the bubbles to join together to form one large bubble. Then when one works the fist, the pressure changes and makes the bubble to burst which then produces a loud cracking noise. Doctors are not sure whether cracking them is harmful or not, thought there is a belief that cracking ones knuckles could make them large and unsightly.

# WAVE MOTION AND SOUND_WORKSHEET-2

CUQ 1.




In the above figure which of the following are in the same phase?

1) A, B 2) B,C 3) B,D 4) A,C

2. In the above figure which of the following are not in the same phase?

1) A,B 2) C,D 3) A,C 4) both (1) and (2)

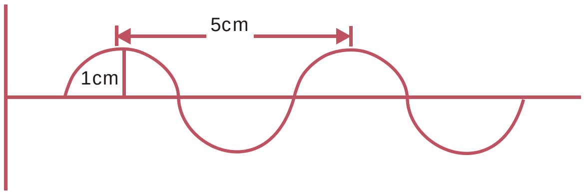
3. If two waves are produced in 2s, then the time period of the wave is

1)1s 2) 2s 3) 0.5s 4) 0.2s

4. If 2 complete waves are produced in one second then the frequency of the waves will be

1) 2Hz 2) 3Hz 3) 4Hz 4) Hz

5.




From the above figure what is the wave length of the wave?

1) 10cm 2) 20cm 3) 5cm 4) 1cm

6. If distance travelled by the wave is x m and time taken is y s, then the wave velocity is

1) $\frac{x}{y}$ m/s 2) $\frac{y}{x}$ m/s 3) xy m/s 4) $\frac{x^2}{y^2}$ m/s

7. The amplitude of a wave measure of its_________

1) velocity 2) force 3) frequency 4) energy

8. If x is the time period of a wave, then the frequency of the wave is

1) x 2) $\frac{1}{x}$ 3) $x^2$ 4) $\frac{1}{x^2}$

## JEE MAINS

**Single Correct Choice Type:**

1. The points on a wave which are in the same state of vibrations are said to be in the same

1) phase 2) time period 3) frequency 4) wave velocity

2. The reciprocal of time period of the wave is called __________of the wave.

1) wave velocity 2) frequency 3) wave length 4) amplitude

3. One vibration per second is called

1) one frequency 2) one hertz 3) one time period 4) one kilo Hertz

4. The S.I unit of wave velocity is

1) cm/s 2) m/s 3) $m/s^2$ 4) $cm/s^2$

5. The distance travelled by the wave in one time period is called the _________ of the wave.

1) frequency 2) amplitude 3) wave length 4) wave velocity

6. The relation among wave velocity (v), wavelength ($\lambda$) and time period (T) is

1) $v = \lambda T$ 2) $v = \frac{\lambda}{T}$ 3) $vT = \frac{\lambda}{2}$ 4) $v\lambda = T$

7. The number of cycles in one second is called_________ of the wave.

1) wavelength 2) velocity 3) frequency 4) time period

8. The time period of an electromagnetic waves is $10^{-15}$ seconds. What is the frequency of wave in hertz ?

1) $10^{15}$ 2) $10^{5}$ 3) $10^{2}$ 4) $10^{7}$

9. The wavelength of a sound wave is 66m, calculate the frequency of the wave if the velocity of sound is 330 m/s.

1) 1.5 Hz 2) 10 Hz 3) 5 Hz 4) 7 Hz

10. Calculate the wavelength of radio waves of frequency $10^9$Hz . The speed of waves is $3\times10^8$ m/s.

1) 20 cm 2) 35 cm 3) 30 cm 4) 25 cm

11. A tuning fork produces 256 waves in 2 seconds. Calculate the frequency of the tuning fork.

1) 1.120 Hz 2) 125 Hz 3) 128Hz 4) 130Hz

12. The frequency of a source of sound is 100 Hz. How many times it vibrates in a minute?

1) 5000 2) 6000 3) 5400 4) 5500

## JEE ADVANCED

### Multi Correct Choice Type:

13. which of the following statements are true :

1) The time interval between the formation of two successive crests (or) troughs is called time period
2) The S.I unit of frequency is Hz
3) The S.I unit of wave length is m
4) The S.I unit of wave velocity is m/s or $ms^{-1}$.

**Statement Type:**

14. *Statement I :* The frequency of a vibratory particles decreases as its time period increases .

*Statement II :* Frequency $f = \frac{1}{T}$.

1) Both Statements are true
2) Both Statements are false
3) Statement - I is true, Statement - II is false.
4) Statement - I is false, Statement - II is true.

**Matrix Match Type:**

| | Column-I | Column-II |
|---|---|---|
| 15. | a) Time period | 1) m/s |
| | b) Frequency | 2) m |
| | c) Wave length | 3) Hz |
| | d) Wave velocity | 4) second |

**Integer Answer Type:**

16. The frequency of a wave travelling at a speed of 500m/s is 25 Hz. Its wave length is_______

**Multi Correct Choice Type:**

17. A sound of wavelength 68 cm travels 850m in 2.5s.Then
1) The velocity of the sound is 340m/s
2) The frequency of the sound is 500Hz
3) The velocity of the sound is 320m/s
4) The frequency of the sound is 400Hz

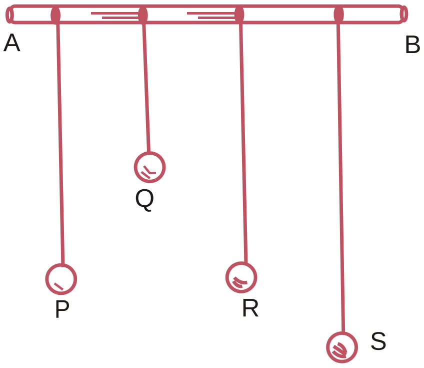
## WAVE MOTION AND SOUND_SYNOPSIS-3

1. **Natural or free vibrations** :When a body is set into vibration and then left to itself, the vibrations are called natural or free vibrations.
2. **Frequency:** Number of vibrations made by body in one second is called frequency of the body.
3. **Natural frequency:** Every body has its own frequency and every time it makes same number of vibrations in a given interval of time when it is set into state of free vibrations.
4. **Damped vibrations:** Periodic vibrations of decreasing amplitude are called damped vibrations.
5. **Forced Vibrations:** When a body executes vibrations under the action of an external periodic force, then the vibrations of the body are called forced vibrations.
6. **Resonance:** Is the phenomenon in which if one of the two bodies of the same natural frequency is set into vibration, the other body also vibrates with larger amplitude under the influence of the first body.

**Experiment to demonstrate resonance and forced vibrations:**

Consider four simple pendulums P,Q,R and S suspended from horizontal stretched rubber card AB. Let the lengths of pendulums p and R be equal Q is shorter and S is longer than P. If the pendulum P is pulled aside perpendicular to AB and released, after a short time, the pendulum R starts vibrating with increasing amplitude while Q and S remain at rest. The length of pendulums P and R being equal, their frequencies are equal. hence they are in resonance.

The length of P.Q.S are different. Their amplitudes are also different. Hence Q and S are said to be in a state of forced vibrations since they differ in frequencies among themselves and also with P.




**A few incidents of resonance phenomenon observed in your day–to–day life.**

i) A pronounced rattling sound may occur in a car when it is travelling at a particular speed, but if it travels faster or slower than this speed no rattle occurs. This is due to the resonance between the car engine and the body of the car.

ii) When we tune a transistor radio, we are actually adjusting its natural frequency to that incoming carrier electromagnetic waves from a radio transmitter. When two frequencies match, a clear sound with appreciable amplitude is heard.

iii) Consider a swing on which child sits. When the swing is given a series of pushes by mother with the frequency equal to that of natural frequency of the swing, the swing oscillates with larger amplitude. In this case, resonance occurs between force applied by mother and natural frequency of the swing.

iv) When a band group plays a tune in the street at times vessels in the house vibrate with the same frequency due to resonance.

7. **Progressive wave:** A wave originating in a source and travelling forward in a medium is called a progressive wave.

8. **Transverse wave:** When the displacement of particles of a medium is at right angles of the direction of transverse wave, the wave is said to be a transverse wave.

9. **Longitudinal wave:** When the displacement of particles of a medium is parallel to the direction of propagation of the wave, the wave is said to be longitudinal wave.

10. **Stationary wave:** Stationary wave (or a standing wave) is defined as a resultant wave formed when two waves of equal frequency and amplitude travel in opposite directions along the same path.

Stationary waves are characterised by nodes and antinodes.

11. **Nodes:** The points in a standing wave where the particles undergo minimum displacement are called nodes.

12. **Antinodes:** The points in a standing wave, where; the particles undergo maximum displacement are called antinodes.

13. The distance between two successive nodes (or antinodes) is $\lambda/2$

14. The distance between a node and successive antinode is $\lambda/4$.

| Progressive waves | Stationary waves |
|---|---|
| 1. These waves are produced by vibrating source and continuously travel forward in the medium. | 1. These are formed when two waves of equal frequency and equal amplitude travel in opposite directions along the same path. |
| 2. These waves travel in the form of crests and troughs (or compressions and rarefactions) through the medium in all directions. | 2. These waves are confined to a fixed region of the medium were they form nodes and antinodes. |
| 3. All the particles have same amplitude and frequency every where in the medium. Every particle undergoes the maximum displacement at one time or the other. | 3. Amplitudes of different particles in the medium are different at different points. It varies from a minimum at nodes to a maximum at antinodes |
| 4. Distance between successive crests (compressions) or troughs (rarefactions) is $\lambda$ | 4. Distance between successive nodes or antinodes is $\lambda/2$. |
| 5. Energy is carried continuously by forward moving waves throughout the medium. | 5. Energy is trapped in a fixed region of medium. |
| 6. Every particle undergoes maximum displacement at one time or other | 6. The particles at nodes undergo only minimum displacement, while at antinodes, they undergo only maximum displacement. |

## AMAZING FACT

**When I heat water for coffee, the sound of the water tells me when it has begun to boil. First there is a hissing the grows and then dies out as a harsher sound takes over. Just as the water begins really to boil, the sound becomes softer. Can you explain these sounds, especially the softening as the water beings to boil?**

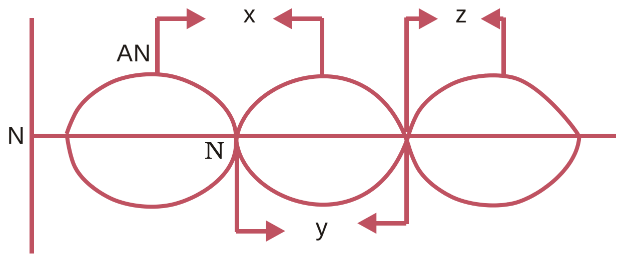
The first sound comes when the bottom of the pan is heated and small bubbles form, each with a click and collectively with a hiss. With further heating, the bubbles detach from the bottom, rise into the cooler water and then collapse, creating a louder noise. This noise continues until the water is sufficiently hot for the bubbles to reach the surface to break. Then the water is in full boil, and the noise of the bubbles reaching the surface is a softer, splashing sound.

# WAVE MOTION AND SOUND_WORKSHEET-3

CUQ 1. The phenomenon in which if one of the two bodies of the same natural frequency is set into vibration the other body also vibrates with larger amplitude under the influence of the first body is called ___________
1) Sound 2) Wave 3) Resonance 4) Wave length

2. When a band group plays a tune in the street at times vessels in the house vibrate with the same frequency due to_____________
1) Sound 2) Wave 3) Resonance 4) Wave length

3. In the progressive waves ________ is carried continuously by forward moving waves through out the medium.
1) Energy 2) Sound 3) Frequency 4) Time period

4. 


The above figure the distance between antinode to next antinode x =
1) $\frac{\lambda}{2}$ 2) $\lambda$ 3) $2\lambda$ 4) $\frac{\lambda}{4}$

5. From the above figure the distance between node to next node y is equal to
1) $\frac{\lambda}{2}$ 2) $\lambda$ 3) $2\lambda$ 4) $\frac{\lambda}{4}$

6. From the above figure the distance between node to antinode z is equal to
1) $\frac{\lambda}{2}$ 2) $\lambda$ 3) $2\lambda$ 4) $\frac{\lambda}{4}$

7. _______ waves energy is trapped in a fixed region of medium.
1) Stationary 2) Longitudinal 3) Transverse 4) All of these

8. _______ waves energy is carried continuously by forward moving throughout the medium.
1) Stationary 2) Progressive 3) both(1)&(2) 4) neither(1) nor (2)

## JEE MAINS

### Single Correct Choice Type:

1. The vibrations produced in a body, on being slightly distrubuted from its mean position are called
1) Free vibrations 2) natural vibrations
3) both (1) & (2) 4) Neither (1) nor (2)

2. The time period of a body executing natural vibrations is called

1) natural velocity 2) natural speed
3) mass 4) natural time period

3. The numbers of vibrations executed per second by freely vibrating body is called

1) natural time period 2) natural speed
3) natural frequency 4) both (1) and (2)

4. If the distance between two consecutive nodes is 10cm in a statianary wave then the wavelength of the wave is

1) 5 cm 2) 40 cm 3) 20 cm 4) 10 cm

5. Stationary waves are characterised by ___________.

1) nodes 2) antinodes 3) Both (1) and (2) 4) Neither (1) nor (2)

6. Particles undergo minimum displacement at________________in a stationary wave.

1) nodes 2) antinodes 3) Both (1) and (2) 4) Neither (1) nor (2)

7. In a stationary wave, the point at which the displacement is maximum is called

1) node 2) antinode 3) crest 4) trough

8. In the absence of any resistance (such as resistance of air etc) the amplitude of free vibration

1) increases 2) decreases 3) remans constant 4) None

9. The special case of forced vibrations is

1) damped vibrations 2) free vibrations
3) resonance 4) periodic vibrations

10. The loud sound heared at resonance because of

1) sound box
2) the body vibrates with larger amplitude
3) the body vibrates with smaller amplitude
4) the body doesnot vibrates

11. If the distance between two succesive antinodes is 10cm in a statianary wave then the wavelength of the wave is

1) 5 cm 2) 40 cm 3) 20 cm 4) 10 cm

12. If the distance between a node and the next antinode in a stationary wave is 10 cm then the wavelength of the wave is

1) 40 cm 2) 5 cm 3) 20 cm 4) 10 cm

# JEE ADVANCED

**Multi Correct Choice Type:**

13. Choose the correct statements:
    1) A wave orignating from a source and travelling foward in a medium is called progressive wave.
    2) Periodic waves of decreasing amplitude are called damped vibrations.
    3) Periodic waves of increasing amplitude are called damped vibrations
    4) A wave orignating from a source and drapped in a particular region is called progressive wave.

**Reasoning Type:**

14. *Statement*-I: When the displacement of particles of a medium is at right angles of the direction of transverse wave, the wave is said to be a transverse wave.

    *Statement-II*: When the displacement of particles of a medium is parallel to the direction of propagation of the wave, the wave is said to be longitudinal wave.
    1) Both Statements are true
    2) Both Statements are false
    3) Statement - I is true, Statement - II is false.
    4) Statement - I is false, Statement - II is true.

**Matrix Match Type:**

15.

| Column-I | Column-II |
|---|---|
| a) The waves are produced by vibrating source and contineously travel on the medium | 1) Progressive waves |
| b) The waves are confirned top a fixed region of the medium were they from nodes and antinodes | 2) Stationary waves |
| c) Energy is carried continuously by forward moving waves throughout the medium | 3) Resonance |
| d) Energy is trapped in a fixed region of medium | 4) None of these |

**Integer Answer Type:** 16. If the distance between two successive antinodes is 5 cm in a stationary wave, then the wave lenght of the wave is ______

1) 5 cm 2) 40 cm 3) 20 cm 4)10cm

**Multi Correct Choice Type:**

17. The phenomenon of resonance occurs in
    1) machine parts 2) radio and TV reception
    3) air columns 4) cradle

**Comprehension Type:**

Vibrations are classified into forced vibration and natural vibrations.

18. Vibrations of stringed instrument in air
    1) free vibrations 2) damped vibrations

3) natural vibrations 4) both (1) and (3)

19. Vibrations of simple pendulum in air is
1) damped vibrations 2) free vibrations
3) natural vibrations 4) periodic vibrations

20. The vibrations which takes place under the influence of an external periodic force are called
1) damped vibrations 2) free vibrations
3) forced vibrations 4) periodic vibrations

# WAVE MOTION AND SOUND_SYNOPSIS-4

**Sound:**

Sound is a form of energy which emitted by a vibrating body that travels in the form of waves and causes the sensation of hearing. Sound cannot travel through vacuum.

**Production of sound:**

We hear many sounds every day such as the sound of our school bell, an alarm clock, a barking dog and so on. We talk and communicate with others by producing sounds.

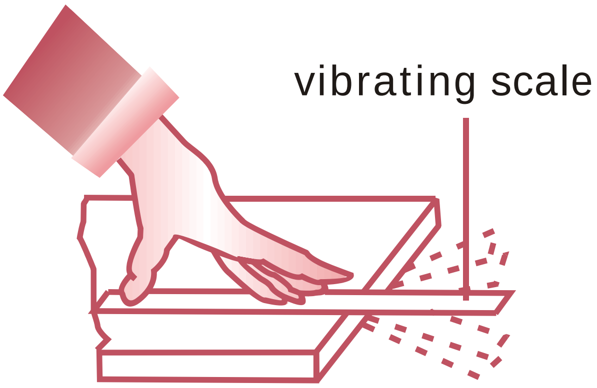
Sound is produced when a body vibrates (moves back and forth rapidly). In other words, sound is produced by vibrating bodies.

**Note:** Vibrations of the bodies produces sound so vibrations are the causes of sound.

The following experiments demonstrate this fact.

**Activity (1):** Take a drum and beat it. Observe what happens you will hear the sound of the drum. Now, touch the membrane of the drum. What do you feel? You will be able to feel its vibrations. When the sound stops, touch the membrane of the drum again, you will not feel any vibrations so we can say that only vibrating membrane of the drum produces sound.

**Activity (2):** Take a metre scale and place it on a table end press the metre scale with one hand and cause it to vibrate as shown in figure you will see that the vibrating metre scale produces sound. Now stop the metre scale from vibrating (by touching it with your five-finger). It will also stop producing sound.




A vibrating object

**Activity (3):** Stretch a string by holding one end in your mouth under the teeth and the other end in one hand pluck it near the middle you will notice that the string starts vibrating and a sound is heard. After some time when the string stops vibrating, so sound is heard.

Stretching string produces sound

**EXPERIMENT:**

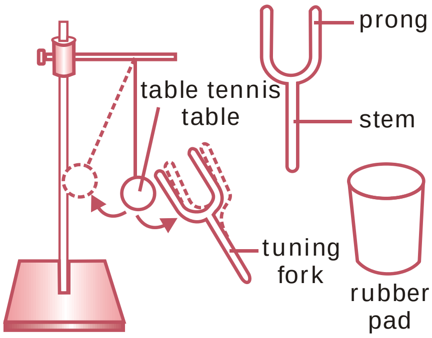
**Aim:** To demonstrate that sound is produced by a vibrating body.

**Aids**: Tuning fork, Rubber pad, Table tennis ball, Thread, Stand

**Method:** Hit the tuning fork hard against a rubber pad. It produces sound. If you look at the prongs of the tuning fork closely, they look hazy because they are vibrating. Suspend the table tennis ball with the thread tied to the stand. Bring the prong of the vibrating tuning fork near the ball.

**Observation:** The ball jumps to-and-fro. This shows that the prongs of the tuning fork are vibrating.

**Result:** Every source of sound is a vibrating body.




Tuning fork produces sound.

**Sound need a material medium to travel:**

The substance through which sound travels is called a medium. The medium can be a solid substance, a liquid or a gas.

Solids, liquids and gases are called material media. Sound needs a material medium like solid, liquid or gas to travel.

In other words, sound can travel through solids, liquids and gases but it cannot travel through vacuum (or empty space). Sound waves are called mechanical waves because they need a material medium (like solid, liquid or gas) for their propagation.

**Note:** Sound travels about 15 times faster in steel than in air.

**EXPERIMENT :**

**Aim:** To demonstrate that sound requires a medium for propagation.

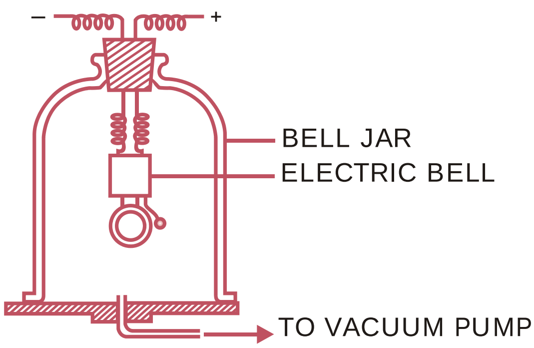
**Aids**: Electric bell, Glass bell jar, Vacuum pump, Battery

**Method:** Place the electric bell inside the glass bell jar and connect it to a battery. When the circuit is closed, you can hear the bell ring. The jar contains air and sound travels through this air.

Now, remove the air from the jar with the help of the vacuum pump connected to the bell jar. As the air is taken out, the loudness of the sound slowly decreases until the sound becomes too faint. Finally you cannot hear the bell even though the hammer of the bell is seen striking the gong as before.

Allow air to enter the jar gradually. You will hear the sound slowly increasing.

**Result:** Sound cannot propagate in the absence of a material medium like solid, liquid or gas.



**The speed of sound:** Sound takes some time to travel from the sound producing body to our ears.

The speed of sound tells us the rate at which sound travels from the sound travels from the sound producing body to our ears.

**Speed of sound in different media:** The speed of sound is different in different media. The speed of sound is more in solids, less in liquids and least in gases (since solids are much more elastic than liquids and gases). The speed of sound is nearly 5100 m/s in steel, 1450 m/s in water and 330 m/s in air at $0^0$C.

**Mathematical formula for speed of sound:**

1. Speed of sound $= \dfrac{\text{Distance travelled by sound}}{\text{time taken}}$

2. Laplace's formula for velocity of sound, $v = \sqrt{\dfrac{\gamma P}{\rho}}$

Where $\gamma = \dfrac{C_p}{C_v} = \dfrac{\text{specific heat at constant pressure}}{\text{specific heat at constant volume}}$

P = Pressure of the medium. $\rho$ = density of the medium.

**Factors affecting the speed of sound in air or in a gas:**

i) Density ii) Temperature

iii) Humidity iv) Direction of wind

1. **Effect of density:** as $v \propto \dfrac{1}{\sqrt{\rho}}$ i.e

The speed of sound is inversely proportional to the square root of the density of the medium.

**Ex:** The density of oxygen is 16 times the density of hydrogen, therefore the speed of sound in hydrogen is four times the speed of sound in oxygen.

2. **Effect of temperature:** The speed of sound increases with the increase in temperature of a gas.

It is found that the velocity of sound in a gas is directly proportional to the square root of its absolute temperature i.e

$v \propto \sqrt{T}$ , The reason is that with the increase in temperature there is a decrease in the density and consequently, the speed of sound increases.

The speed of sound in air increases by about 0.6 m/s (or 60 cm per second) for each degree Celsius rise in temperature i.e $V_t = V_0 + 0.6\ t$

**Ex:** Speed of sound in dry still air at 0°C is 330 m/s. At 25°C, the speed of sound in dry still air will be $V_{25} = V_0 + 0.6t = 330 + 0.6 \times 25 = 345$ m/s

3. **Effect of humidity:** The speed of sound increases with the increase in humidity.

The presence of water vapour in the air changes its density. The presence of water vapour reduces the density of air i.e density of moist air < Density of dry air therefore, velocity of sound in moist air > velocity of sound in dry air.

$$\left[\text{since } v \propto \frac{1}{\sqrt{\rho}}\right]$$

Hence, the velocity of sound in moist air is greater than the velocity of sound in dry air. That is why sound travels faster on a rainy day than on a dry day.

4. **Effect of wind:** The speed of sound increases or decreases according to the direction of wind.

(i) If the wind blows in the same direction in which the sound travels, the velocity of sound increases i.e velocity of sound = velocity of sound in still air + velocity of wind.

i.e $v = v_s + v_w$

(ii) If the wind blows in the opposite direction in which the sound travels, the velocity of sound decreases i.e velocity of sound = velocity of sound in still air – velocity of wind

i.e $v = v_s - v_w$

**Factors which do not affect the speed of sound in air:**

There is no effect on the speed of sound in air due to the following factors.

i) Change in frequency

ii) Change in amplitude

iii) Changein pressure

iv) Change in factors like phase, loudness, pitch, quality of sound etc.

**Effect of pressure:**

We know, $v = \sqrt{\frac{\gamma P}{\rho}}$

Thus, if the temperature of a gas remains constant, a change in pressure of the gas remains constant, a change in pressure of the gas changes its density in the same ratio i.e if pressure P of the gas is doubled, the volume becomes half, so density (m/v) gets doubled. So $P/\rho$ remains unchanged.

Consequently, the velocity of sound is independent of the pressure of the gas provided the temperature remains constant.

**Comparison of speed of sound with speed of light:** The speed of light in air is $3 \times 10^8$ $ms^{-1}$ which is about a million times larger as compared to the speed of sound in air i.e 330 m $s^{-1}$ at $0^0C$.

Apart from this, the speed of light decreases in a denser medium (speed of light in water is 2.25 × 108 $ms^{-1}$, in glass is 2 × 108 $ms^{-1}$ ), while the speed of sound is more in solids, less in liquids and still less in gases (speed of sound in steel is nearly 5100 m $s^{-1}$, in water is nearly 1450 m $s^{-1}$ and in air is nearly 330 m $s^{-1}$).

**Lightning is seen much earlier than the thunder is heard:** In thundering, the light is seen much earlier than the sound of thunder is heard although they are produced simultaneously, as light takes almost negligible time in comparison to sound in reaching us from the thunder.

**STAR FACT**

**How is sound produced when the thumb and the middle finger are clicked?**
A small amount of air is trapped and pressurised when the fixed end of the thumb and the free end of the middle finger are brought together. When this pressurised air is released suddenly during the clicking it produces a sound as in the case of a ballon burst.

## WAVE MOTION AND SOUND_WORKSHEET-4

CUQ 1. In which of following the speed of sound is more?

1) Gas 2) Water 3) Iron 4) Kerosene

2. If the distance travelled by sound is x, time taken is y, then the speed of the sound is

1) $\frac{x}{y}$ 2) xy 3) $\frac{y}{x}$ 4) $\frac{x^2}{y^2}$

3. If the density of medium increases then the speed of the sound

1) decreases 2) increases 3) remain same 4) both (1) and (2)

4. If the temperature of a gas increases then the speed of sound

1) decreases 2) increases 3) remain same 4) both (1) and (2)

5. If the humidity decreases the speed of sound

1) decreases 2) increases 3) remain same 4) both (1) and (2)

6. In which of the following the speed of sound is more?

1) In dry air 2) in moist air 3) both (1) and (2) 4) neither (1) nor (2)

7. If the velocity of wind is $V_W$, velocity of sound in still air is $V_s$, then the velocity of sound if the wind blows in the same direction in which the sound travels is

1) $V_s + V_w$ 2) $V_s - V_w$ 3) $\frac{V_s}{V_w}$ 4) $\frac{V_w}{V_s}$

8. If the velocity of wind is $V_W$, velocity of sound in still air is $V_s$, then the velocity of sound if the wind blows in the opposite direction in which the sound travels is

1) $V_s + V_w$ 2) $V_s - V_w$ 3) $\frac{V_s}{V_w}$ 4) $\frac{V_w}{V_s}$

9. The speed of sound in air does not effect due to

1) change in frequency 2) change in amplitude

3) change in pressure 4) all of these

## JEE MAINS

### Single Correct Choice Type:

1. Vibrating bodies produce ____________

1)light 2) heat 3) sound 4) none of these

2. Sound cannot travel through

1) vacuum 2) liquids 3) Solids 4) gases

3. The instrument used in the labaratory to produce sound of fixed frequency is ______

1) Simple pendulum 2) tuniny fork

3) thermoments 4) meter scale

**Comprehension Type:**

Sound takes some time to travel from the sound producing body to our ears.

4. Formula for speed of sound

1) Distance travelled by sound × time taken

2) Distance travelled by sound / time taken

3) Distance travelled by sound – time taken

4) Distance travelled by sound + time taken

5. The speed of sound in air at 0°C is approximately

1) 330 m/s 2) 1450 m/s 3) 5100 m/s 4) $3 \times 10^8$ m/s

6. The velocity of sound in air increases for every 1°C rise in temperature apporcimately by _______

1) 0.6 m/s 2) 0.061 m/s 3) 6.1 m/s 4) 61.0 m/s

7. Velocity of sound is highest in the case of _____

1) Solids 2) Liquids 3) Gasses 4) Vacuum

8. If $V_g$, $V_l$, and $V_s$ represent the speed of sound in a gas, liquid and solid respectively, then

1) $V_g > V_l > V_s$ 2) $V_s > V_l > V_g$ 3) $V_l > V_s > V_g$ 4) $V_g > V_s > V_l$

9. During thunder storm, the flash of lightning is seen before the thunder is heard because,

1) Sound travels much faster than light.

2) Light travels much faster than sound.

3) Both sound and light have equal speed.

4) none of these.

# WAVE MOTION AND SOUND

## WAVE MOTION AND SOUND_WORKSHEET-1_KEY

**CUQ'S:**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1)3 | 2)2 | 3)2 | 4)1 | 5)1 | 6) 3 | 7) 3 | 8) 2 |
| 9) 1 | 10) 2 | | | | | | |

**JEE MAINS AND ADVANCED:**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1) 2 | 2) 1 | 3) 3 | 4) 1 | 5) 3 | 6) 1 | 7) 2 | 8)2 |
| 9) 1 | 10) 1 | 11) 1 | 12) 3 | 13) 1 | 14) 1 | 15) 1,2,3 | |

16) 4 17) a-1,3,4;b-2;c-1,2,4;d-1,3,4

## WAVE MOTION AND SOUND_WORKSHEET-2_KEY

**CUQ'S:**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1) 1 | 2) 3 | 3) 1 | 4) 1 | 5) 3 | 6) 1 | 7) 4 | 8) 2 |

**JEE MAINS AND ADVANCED:**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1) 1 | 2) 2 | 3) 1 | 4) 2 | 5) 4 | 6) 2 | 7) 3 | 8) 1 |
| 9) 3 | 10) 3 | 11) 3 | 12)2 | 13) 1,2,3,4 | | 14) 1 | |

15) a-4;b-3;c-2;d-1 16) 20 17) 1,2

**HINTS AND SOLUTIONS:**

8. Given (T) = $10^{-15}$ s, f = ?, We know, $f = \frac{1}{T} = \frac{1}{10^{-15}s} = 10^{15}\,Hz$

9. $\lambda = 66\,m$, $f = ?$, v = 330 m/s, We know that $v = f\lambda$

$\Rightarrow f = \frac{v}{\lambda} = \frac{330}{66} = 5\,Hz$

10. $\lambda = ?$, f = $10^9$Hz, v = 3 × $10^8$ m/s, We know, $v = f\lambda$ $\Rightarrow \lambda = \frac{v}{f} = \frac{3\times10^8}{10^9}\,m$

= 3 × $10^{-1}$m = 0.3 m = 30 cm.

11. In 2 seconds, the number of waves produced = 256

$\therefore$ in 1 second the number of waves produced $= \frac{265}{2} = 128$

$\therefore$ Frequency of the tuning fork = 128 Hz.

12. Frequency = 100 Hz

$\therefore$ Number of vibrations in 1 second= 100

$\therefore$ Number of vibrations in 60 second = 100 × 60 = 6000

17. $\lambda = 68\text{ cm} = 0.68\text{ m}$, Distance travelled by the wave = 850 m

time taken = 2.5 s

i) $\therefore$ velocity of the wave

$$v = \frac{\text{Distance travelled by the wave}}{\text{Time taken}} = \frac{850\text{ m}}{2.5\text{ s}} = 340\text{ m/s}$$

ii) Again we know that, $v = f\lambda \Rightarrow f = \frac{v}{\lambda} = \frac{340}{0.68}\text{Hz} = 500\text{ Hz}$

According to question, $10\lambda = 20\text{cm} \Rightarrow \lambda = 2\text{cm}$

## WAVE MOTION AND SOUND_WORKSHEET-3_KEY

**CUQ'S:**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1) 3 | 2)3 | 3) 1 | 4) 1 | 5) 1 | 6) 4 | 7) 1 | 8) 2 |

**JEE MAINS AND ADVANCED:**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1)3 | 2)4 | 3)3 | 4)3 | 5)3 | 6)1 | 7)2 | 8) 3 |
| 9) 3 | 10)2 | 11)3 | 12)1 | 13)1,2 | 14)1 | 15)a-1;b-2;c-1;d-2 | |
| 16)4 | 17) 1,2,3,4 | | | | | | |
| 18) 2 | 19) 1 | 20) 3 | | | | | |

## WAVE MOTION AND SOUND_WORKSHEET-4_KEY

**CUQ'S:**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1) 3 | 2)1 | 3)1 | 4)2 | 5)1 | 6)2 | 7)1 | 8)2 |
| 9)4 | | | | | | | |

**JEE MAINS AND ADVANCED:**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1)3 | 2)1 | 3) 2 | 4) 2 | 5)1 | 6)1 | 7)1 | 8)2 |
| 9)2 | | | | | | | |

# CIRCULAR MOTION PART-1

## SYNOPSIS - 1

In day to day life, we encounter several cases of circular motion i.e., motion along a circle. Practical situations encountered, are however, more of a combination of rotatory and translatory motion. In this chapter, we would restrict our discussion to purely circular motion.

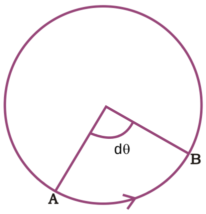
If a particle moves along a circle, with constant speed, it is referred to as "uniform circular motion". Otherwise "non uniform circular motion".

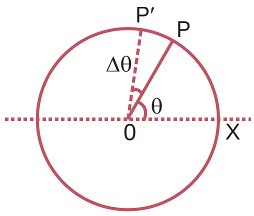
**Angular Variables :**

**Radius vector:** Suppose a particle P is moving in a circle of radius r (see figure )(1). The line joining the centre of the circle to the position of the particle is called radius vector.

**Angular displacement :** Let O be the centre of the circle. Let O be the origin and OX the X – axis. The position of the particle P at a given instant may be described by the angle $\theta$ between OP and OX. We call $\theta$ the angular position of the particle. The angle turned by the radius vector in a given time interval is called angular displacement $\theta$

$$d\theta = \frac{arc\ length}{radius}$$




**SI unit radian**

⇒Small angular displacements are vectors

⇒Large angular displacements are scalar as it does not obey commutative law

⇒The direction of angular displacement is along the axis of rotation and it is given by right hand screw rule.

⇒When a particle completes one revolution the angular displacement is $\theta = 2\pi$ radian

⇒When a particle completes N revolutions in a circle the angular displacement is $\theta = 2\pi$ N.

**Angular Velocity :** As the particle moves on the circle, its angular position $\theta$ changes. Suppose the particle goes to a nearby point P$^{I}$ in time $\Delta t$ so that $\theta$ increases to $\theta + \Delta\theta$. The rate of change of angular position is called angular velocity. Thus, the angular velocity is $\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$

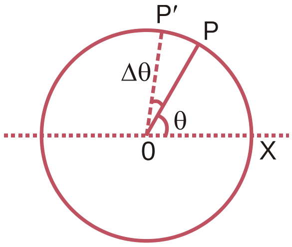
⇒Unit of angular velocity is rad/s.

⇒It's Dimensional Formula is $\left[M^0L^0T^{-1}\right]$.

⇒Angular velocity is a axial vector. ( The vector which is along the axis of rotation is called axial vector)

⇒Its direction is given by right hand screw rule

⇒ Its direction is along the axis of rotation




## Angular acceleration

The rate of change of angular velocity is called angular acceleration. Thus, the angular acceleration is $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

⇒ Unit of angular accelaration is $rad/s^2$.

⇒ It's Dimensional Formula is $\left[M^0L^0T^{-2}\right]$.

⇒ Angular acceleration is a axial vector.

⇒ Its direction is given by right hand screw rule

⇒ Its direction is along the axis of rotation

⇒ If the angular acceleration $\alpha$ is constant, we have equations of motion in angular variable.

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \text{ ------ -(1)}$$

$$\omega = \omega_0 + \alpha t \text{ ------ -(2)} \quad \text{and} \quad \omega^2 = \omega_0{}^2 + 2\alpha\theta \quad \text{------ -(3)}$$

where $\omega_0$ and $\omega$ are the angular velocities at t = 0 and at time t and $\theta$ is the angular position at time t. realtion among linear and angular variables,The linear distance PP$^{I}$ travelled by the particle in time $\Delta t$ is

$$\Delta s = r\,\Delta\theta \text{ or, } \frac{\Delta s}{\Delta t} = r\frac{\Delta\theta}{\Delta t} \quad \text{or,} \quad v = r\omega \quad \text{---- -(4)}$$

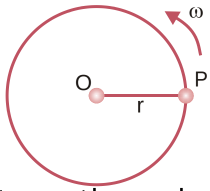
where v is the linear speed of the particle. Differentiating equation (4) with respect to time, the rate of change of speed is $a_t = \frac{dv}{dt} = r\frac{d\omega}{dt}$ or, $a_t = r\,\alpha$ ------ -(5)

Remember that $a_t = \frac{dv}{dt}$ is the rate of change of speed and is not the rate of the change of velocity. It is, therefore, not equal to the net acceleration.

We shall show that $a_t$ is the component of acceleration along the tangent and hence we have used the suffix t. It is called the tangential acceleration.

## Period of Revolution and Frequency :

Let a particle move along a fixed circular path of radius 'r' with a uniform angular velocity '$\omega$'.




The time taken by the particle to go through the circular path once completely, is known as its period of revolution.

Now, by definition, the particle rotates by '$\omega$' angle in a time of 1 sec.

$\therefore$ time taken by the particle to rotate by an angle of $2\pi$ would be $\frac{2\pi}{\omega}$ sec.

$\therefore$ Period of revolution $T = \frac{2\pi}{\omega}$

Alternatively, If 'v' be its linear speed, then $v = r\omega$. Now, from time = $\frac{\text{distance}}{\text{speed}}$

$\Rightarrow$ Period of revolution = $\frac{\text{distance covered in 1 revolution}}{\text{linear speed}} = \frac{2\pi r}{V} \Rightarrow \frac{2\pi r}{\omega r} = \frac{2\pi}{\omega}$

$\Rightarrow$ The number of revolutions performed in 1 sec is known as its frequency.

$\Rightarrow$ By, definition, of period of revolution, in T time, number of revolutions per formed = 1

$\therefore$ In time 1 sec. no. of revolutions performed = $\frac{1}{T}$ $\therefore$ frequency '$f$' = $\frac{1}{T} = \frac{\omega}{2\pi}$

**Note :**

When a body makes 'N' revolutions in 't' sec's then its average angular velocity is $\omega = \frac{2\pi N}{t}$

If a particle makes 'n' rotations per sec its angular velocity is $\omega = 2\pi n$

**Angular velocity of hands of a clock:**

Angular velocity of seconds hand $\omega = \frac{2\pi}{T} = \frac{2\pi}{60} = \frac{\pi}{30} rad\ S^{-1}$

Angular velocity of minutes hand $\omega = \frac{2\pi}{60 \times 60} = \frac{\pi}{1800} rad\ S^{-1}$

Angular velocity of hours hand $\omega = \frac{2\pi}{12 \times 3600} = \frac{\pi}{21600} rad\ S^{-1}$

In case of self rotation of earth about its own axis $\omega = \frac{2\pi}{24 \times 60 \times 60} rad/\sec$

**Uniform Circular Motion :**

If the particle moves in the circle with a uniform speed, we call it a uniform circular motion. In this case, $\frac{dv}{dt} = 0$

Thus, the acceleration of the particle is towards the centre. The magnitude of the acceleration is $a = \omega^2 r = \frac{v^2}{r^2} r = \frac{v^2}{r}$ - - - - - -(9)

Thus, if a particle moves in a circle of radius r with a constant speed v, its acceleration is $\frac{v^2}{r}$ directed towards the centre. This acceleration is called centripetal acceleration.

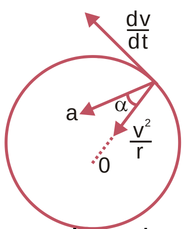
**Note** that the speed remains constant, the direction continuously changes and hence the "velocity" changes and there is an acceleration during the motion.

**Nonuniform Circular Motion:**

If the speed of the particle moving in a circle is not constant, the acceleration has both the radial and the tangential components. The radial and the tangential accelerations are

$a_r = -\omega^2 r = -\frac{v^2}{r}$ and $a_t = \frac{dv}{dt}$ - - - - - - - - - - - - (10)

Thus, the component of the acceleration towards the centre is $\omega^2 r = \frac{v^2}{r}$ and the component, along the tangent (along the direction of motion) is $\frac{dv}{dt}$. The magnitude of the acceleration is $a = \sqrt{a_r^2 + a_t^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}$




The direction of this resultant acceleration makes an angle $\alpha$ with the radius (figure ) where $\tan\alpha = \left(\frac{dv}{dt}\right) / \left(\frac{v^2}{r}\right)$

**Following three points are important regarding the above discussion:**

1. In uniform circular motion, speed (v) of the particle is constant, i.e., $\frac{dv}{dt} = 0$.

   Thus, $a_t = 0$ and $a = a_r = r\omega^2$

2. In accelerated circular motion, $\frac{dv}{dt} = \text{positive}$, i.e., tangential acceleration of particle is parallel to velocity $\vec{v}$.

3. In decelerated circular motion, $\frac{dv}{dt}$ = negative and hence, tangential acceleration is anti – parallel to velocity $\vec{v}$.

**Note:** On any curved path ( not necessarily a circular one) the acceleration of the particle has two components $a_t$ and $a_n$ in two mutually perpendicular directions. Component of $\vec{a}$ along $\vec{v}$ is $a_t$ and perpendicular to $\vec{v}$ is $a_n$.

Thus, $\left|\vec{a}\right| = \sqrt{a_t^2 + a_n^2} = \sqrt{a_x^2 + a_y^2}$ (For 2–D motion )

Here $a_t$ = rate of change of speed $\frac{dv}{dt} = \frac{d\left|\vec{v}\right|}{dt}$

$$= \frac{d}{dt}\left(\sqrt{v_x^2 + v_y^2}\right) = \frac{v_x \frac{dv_x}{dt} + v_y \frac{dv_y}{dt}}{\sqrt{v_x^2 + v_y^2}} = \frac{v_x a_x + v_y a_y}{\sqrt{v_x^2 + v_y^2}} = \frac{\vec{a} \bullet \vec{v}}{v}$$

or $a_t$ = component of $\vec{a}$ along $\vec{v}$

If a and $a_t$ are known, $a_n$ can be found by the relation

$$a_n = \sqrt{a^2 - a_t^2} = \sqrt{a_x^2 + a_y^2 - a_t^2} \quad \text{or} \quad a_n = \sqrt{\left(\frac{dv_x}{dt}\right)^2 + \left(\frac{dv_y}{dt}\right)^2 - \left(\frac{dv}{dt}\right)^2}$$

# WORKSHEET-1

CUQ 1. A body exhibiting circular motion moves in
1) Straight line path 2) Circular path
3) Zig-zag path 4) None of these

2. The relation between linear velocity and angular velocity is
1) $v = r\omega$ 2) $r = \frac{\theta}{\omega}$ 3) $\omega = v + r$ 4) $v = \frac{\omega}{r}$

3. Angle described by the minute hand of clock in 60 minutes is
1)360° 2)180° 3)270° 4)90°

4. Uniform circular motion is accelerated motion because
1) Speed changes continuously
2) Velocity changes continuously
3) Both speed and velocity changes continuously
4) None of these

5. A particle covers equal distance around a circular path, in equal intervals of time. Which of the following quantity connected with the motion of the particle remains constant with time?
1) Velocity 2) Acceleration 3) Speed 4) Displacement

6. A particle perfoming uniform circular motion has
   1) a radial velocity and radial acceleration
   2) a radial velocity and transverse acceleration
   3) transverse velocity and radial acceleration
   4) transverse velocity and transverse acceleration
7. In uniform circular motion
   1) both the angular velocity and the angular momentum vary
   2) The velocity varies but the momentum remains constant
   3) Magnitude of both the velocity and the momentum stay constant
   4) The momentum varies but the velocity remains constant
8. Velocity vector and acceleration vector in a uniform circular motion are related as
   1) both in the same direction 2) perpendicular to each other
   3) both in opposite direction 4) not related to each other
9. A stone of mass m is tied to a string of length $l$ and rotated in a circle with a constant speed $v$. If the string is released, the stone files:
   1) radially outward 2) radially inward
   3) tangentially outward 4) with an accelration $\frac{mv^2}{l}$
10. In uniform circular motion, speed (V) of the particle is constant then $\frac{dv}{dt}$ = ____________
   1) Zero 2) $r\omega^2$ 3) $r\omega$ 4) $\omega^2$

## JEE MAIN & ADVANCED

**LEVEL-1 Single Correct Choice Type:**

1. The line joining the centre of the circle to the position of the particle is called _______________.
   1) Radius vector 2) Velocity 3) Circular path 4) Acceleration
2. The angular displacement $d\theta$ = _______________.
   1) $\frac{\text{Radius}}{\text{arc length}}$ 2) Radius × arc length 3) $\frac{\text{arc length}}{Radius}$ 4) arc length - Radius
3. Angular velocity = _____________.
   1) $\frac{d\theta}{dt}$ 2) axiial vector 3) unit $\frac{rad}{S}$ 4) All of these

4. Period of revolution T = _______________.

1) $\frac{2\pi}{\omega}$ 2) $2\pi\omega$ 3) $\frac{\omega}{2\pi}$ 4) $\omega^2 r$

5. Angular velocity of seconds hand $\omega$ = ______________.

1) $\frac{2\pi}{60}$ 2) $\frac{30}{2\pi}$ 3) $\frac{2\pi}{30}$ 4) Both (1) & (3)

**Comprehension Type:**

If the speed of the particle moving in a circle is not constant, the acceleration has both the radial and the tangential components. The radial and the tangential accelerations are

$a_r = -\omega^2 r = -\frac{v^2}{r}$ and $a_t = \frac{dv}{dt}$.

6. The magnetude of acceleration, in non uniform circular motion.

1) $\sqrt{\left(\frac{v^2}{r}\right)^2 - \left(\frac{dv}{dt}\right)^2}$ 2) $\sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}$ 3) $\frac{v^2}{r}$ 4) $\frac{dv}{dt}$

7. In accelerated circular motion $\frac{dv}{dt}$ = positive i.e.. tangention acceleration of the particle is ________

1) Perpendicular to $\vec{v}$ 2) Anti parallel to $\vec{v}$

3) Parallel to $\vec{v}$ 4) Towards the centre.

8. In accelerated circular motion $\frac{dv}{dt}$ = negative i.e.. tangention acceleration of the particle is ________

1) Perpendicular to $\vec{v}$ 2) Anti parallel to $\vec{v}$

3) Parallel to $\vec{v}$ 4) Towards the centre.

**Multi Correct Choice Type:**

9. When a particle completes one revolution, the angular displacement is $\theta$= _____.

1) $\frac{\pi}{2}$ 2) $2\pi$ 3) $360^0$ 4) $180^0$

10. Right hand screw is applicable for _______________.

1) Angular displacement 2) Angular velocity

3) Angular acceleration 4) Tangential acceleration.

**LEVEL-2 & 3** **Single Correct Choice Type:**

11. A particle moves in a circle of radius 20 cm with a linear speed of 10m/s. Find the angular velocity.

1) 50 rad/s 2) 75 rad/s 3) 90 rad/s 4) 25 rad/s

12. A particle travels in a circle of radius 20 cm at a speed that uniformly increases. If the speed changes from 5.0 m/s to 6.0 m/s in 2.0 s, find the angular acceleration.

1) 9.5 rad/$s^2$ 2) 7.8 rad/$s^2$ 3) 5.7 rad/$s^2$ 4) 2.5 rad/$s^2$

13. Find the magnitude of the linear acceleration of a particle moving in a circle of radius 10 cm with uniform speed completing the circle in 4 s.

1) $7.9\pi^2 cm/s^2$ 2) $5.6\pi^2 cm/s^2$ 3) $2.5\pi^2 cm/s^2$ 4) $3.2\pi^2 cm/s^2$

14. A car is travelling at an acceleration $2m/s^2$. If the diameter of car wheel is 50 cm, the angular acceleration of wheel is

1) $2.5 rad/s^2$ 2) $4 rad/s^2$ 3) $5 rad/s^2$ 4) $8 rad/s^2$

15. A particle is moving at uniform speed 2 $ms^{-1}$ along a circle of radius 0.5m. The centripetal acceleration of particle is

1) $1ms^{-2}$ 2) $2ms^{-2}$ 3) $4ms^{-2}$ 4) $8ms^{-2}$

16. A particle is moving along a circle of radius 1.5 m. If centripetal acceleration of particle is $6ms^{-2}$ then angular velocity of that particle is

1) $\sqrt{3} rads^{-1}$ 2) $2rads^{-1}$ 3) $3rads^{-1}$ 4) $4rads^{-1}$

17. A particle is moving with uniform speed 0.5 m/s along a circle of radius 1m then the angular velocity of particle is

1) 2 rad $s^{-1}$ 2) 1.5 rad $s^{-1}$ 3) 1 rad $s^{-1}$ 4) 0.5 rad $s^{-1}$

18. Find the frequency of hour hand of a clock.

1) $2.31 \times 10^{-5}$ per sec 2) $6.13 \times 10^{-5}$ per sec

3) $5.71 \times 10^{-4}$ per sec 4) $9.81 \times 10^{-5}$ per sec

**Multi Correct Choice Type:**

19. An object follows a curved path. The following quantities may remain constant during the motion.

1) Speed 2) Velocity 3) Acceleration 4) Magnitude of acceleration

**Statement Type:**

20. *Statement-I:* If a particle moves in a circle with a uniform speed, then its velocity and acceleration both change.

*Statement-II:* To keep any particle in uniform circular motion, a resultant force must act towards the centre.

1) Both Statement I and II are true.
2) Both Statement I and II are false.
3) Statement I is true, Statement II is false.
4) Statement I is false, Statement II is true.

**LEVEL-4 & 5** **Single Correct Choice Type:**

21. A stationary wheel starts rotating about its own axis at uniform angular acceleration $8 rad/s^2$. The time taken by it to complete 77 rotations is

1) 5.5 sec 2) 7 sec 3) 11 sec 4) 14 sec

22. A particle moves uniformly in a circle of radius 10 cm, covering a linear distance of 5 cm in a time 2.5 sec. Find the time taken for 10 revolutions.

1) 0.2 sec 2) 18.34 sec 3) 79.12 sec 4) 314.15 sec

23. A particle is moving along a circle and its linear velocity is 0.5 $ms^{-1}$ when its angular velocity is 2.5 $rads^{-1}$. Its tangential acceleration is 0.2 $ms^{-2}$. Then its angular acceleration is

1) 1 $rads^{-2}$ 2) 2 $rads^{-2}$ 3) 2.5 $rads^{-2}$ 4) 3 $rads^{-2}$

**Multi Correct Choice Type:**

24. A body is revolving round in a circular path with constant angular velocity. Then

1) It experiences an acceleration which is not constant.
2) It is moving with variable velocity.
3) The work done by centripetal force is zero.
4) There is no tangential acceleration.

**Matrix Match Type:**

25.

| Column - I | Column - II |
|---|---|
| a) Angular velocity of seconds hand of a clock | 1) $\frac{\pi}{12}$ rad / hr |
| b) Angular velocity of minutes hand of a clock | 2) $\frac{\pi}{6}$ rad / hr |
| c) Angular velocity of hours hand of a clock | 3) $\frac{\pi}{1800}$ rad / s |
| d) Angular velocity of self rotation of earth | 4) $\frac{\pi}{30}$ rad / s |

# SYNOPSIS - 2

## KINEMATICS EQUATIONS OF CIRCULAR MOTION AND COMPARISION WITH EQUIATIONS OF TRANSLATIONAL MOTION

| Circular motion | Translational motion |
|---|---|
| 1) $\omega_2 = \omega_1 \pm \alpha t$ | 1) $v = u \pm at$ |
| 2) $\theta = \omega_1 t \pm \frac{1}{2}\alpha t^2$ | 2) $s = ut \pm \frac{1}{2}at^2$ |
| 3) $\omega_2^2 - \omega_1^2 = \pm 2\alpha\theta$ | 3) $V^2 - u^2 = \pm 2as$ |
| 4) $\theta_n = \omega_1 \pm \frac{\alpha}{2}(2n-1)$ | 4) $S_n = u \pm \frac{a}{2}(2n-1)$ |
| 5) $\frac{\theta}{t} = \frac{\omega_1+\omega_2}{2}$ | 5) $\frac{S}{t} = \frac{u+v}{2}$ |

## DYNAMICS OF CIRCULAR MOTION

### Centripetal force :

If a particle moves in a circle as seen from an inertial frame, a resultant nonzero force must act on the particle. That is because a particle moving in a circle is accelerated and acceleration can be produced in an inertial frame only if a resultant force acts on it.

If the speed of the particle remains constant, the acceleration of the particle is towards the centre and its magnitude is $v^2/r$. Here v is the speed of the particle and r is the radius of the circle.

The resultant force must act towards the centre and its magnitude F must satisfy

$$a = \frac{F}{m} \text{ or } \frac{v^2}{r} = \frac{F}{m} \text{ or } F = \frac{mv^2}{r}$$

Since this resultant force is directed towards the centre, it is called centripetal force. Thus, a centripetal force of magnitude $mv^2/r$ is needed to keep the particle in uniform circular motion.

It should be clearly understood that “centripetal force” is another word for “force towards the centre”. This force must originate from some external source such as gravitation, tension, friction, coulomb force etc. Centripetal force is not a new kind of force, just as an “upward force” or a “downward force” is not a new kind of force.

### Centrifugal force:

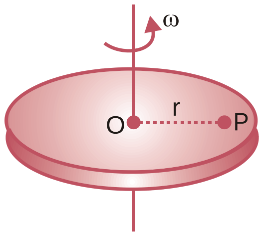
We have learnt in the chapter Newton’s Laws of motion, that when a situation

is analysed from a non–inertial frame of reference, Newton’s laws of motion are applicable only if, we allow to include an additional “pseudo force”, on the system.

Let us analyse, the case of a body rotating with a uniform speed.

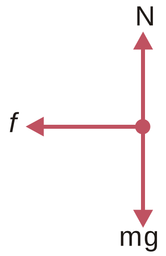
Consider a particle ‘P’ placed on a rough circular table, which rotates about its vertical axis passing through its centre with uniform angular velocity ‘$\omega$’ see figure (1). Let the particle ‘P’ be at rest, (relative to the table) at a distance ‘r’ from the axis of rotation.

As seen from ground (inertial reference frame) the F.B.D. of particle appears to be as shown in figure.




For the vertical equilibrium, weight mg of particle acting downwards is balanced by the normal reaction 'N' offered by the table surface on the particle.

Since the particle revolves about the axis of rotation, it is being accelerated, by a force directed towards the axis. This force is supplied by the frictional force. Had there been no friction, the particle would not have stayed at rest relative to the table.

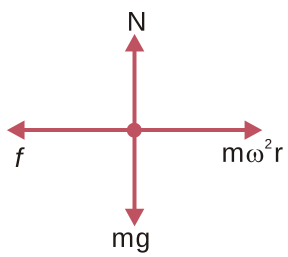



Thus $f = m\omega^2 r$ - - - - - - - - -(1)

Next, let us attach a reference frame to the circular table, which is

non–inertial reference frame, since it would be in an acceleration. When we start analysing the situation with respect to the circular table, the particle appears to be at rest. But before applying Newton's laws of motion, we should allow for "pseudo force" to act on the particle.

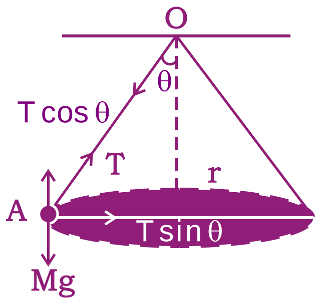
Figure shows, the F.B.D of the particle w.r.t non inertial frame. Considering the horizontal equilibrium $f = m\omega^2 r$ which is same as the result arrived in equation (1).




This pseudo force is equal but opposite to centripetal force and is known as centrifugal force ("centrifugal" means "fleeing away from centre"). Thus centrifugal force is always equal but opposite to the centripetal force. No, centrifugal force exists in reality. However, it is a fictitious or pseudo or virtual (something not real) force, which should be applied on a system executing circular motion, when the reference frame is itself performing circular motion.

**Conical Pendulum:** The bob is given a horizontal push a little through angular displacement $\theta$ and arranged such that the bob describes a horizontal cirlce with uniform angular velocity $\omega$ in such a way that the string always makes an angle $\theta$ with the vertical. As the string traces the surface of the cone, the arrangement is called a conical pendulum.

Let T be the tension in the string of length $l$ and $r$ be the radius of circular path. The vertical component of tension T balances the weight of the bob and horizontal component provides the necessary centripetal force.




$T\cos\theta = Mg \rightarrow (1)$ $\quad$ $T\sin\theta = Mr\omega^2 \rightarrow (2)$

From eq. (1) and (2) , we get $\tan\theta = \frac{r\omega^2}{g}$ i.e., $\omega = \sqrt{\frac{g\tan\theta}{r}}$

But $r = l\sin\theta$ and $\omega = \frac{2\pi}{T}$ Time period of evolution $T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$

## WORK SHEET-2

**CUQ**

1. A car moves on a curved but level road. The necessary centripetal force on the car is provided by
   1) Inertia 2) Gravity
   3) Friction between the tyres and the road
   4) Normal reaction of the car
2. The magnitude of centripetal acceleration is given by
   1) $a = \frac{v}{r}$ 2) $a = \frac{v^2}{r}$ 3) $a = r\omega$ 4) $a = v^2 r$
3. In revolution of earth round the sun, centripetal force is
   1) Electrical force 2) Magnetic force
   3) Gravitational force 4) None of these

4. A stone of mass m is tied to a string of length l and rotated in a circle with a constant speed v, if the string is released, the stone flies:

1) radially outwards 2) radially inwards

3) tangentially outwards 4) with an acceleration $\frac{mv^2}{l}$

5. A motor cyclist going round the circular path at constant speed has:

1) constant linear velocity

2) constant acceleration

3) acceleration of constant magnitude and changing direction

4) constant centripetal force

6. When milk is churned, cream gets seperated due to

1) centripetal force 2) centrifugal force

3) frictional force 4) gravitational force

7. The normal component of acceleration of a particle in circular motion is due to

1) speed of the particle 2) change in direction of velcoity

3) change in the magnitude of velocity 4) rate of change of acceleration

8. Time period of conical penudlum

1) $T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$ 2) $T=2\pi\sqrt{\frac{\log\tan\theta}{l}}$ 3) $T = 2\pi\sqrt{\frac{l\ \sin\theta}{g}}$ 4) $T = 2\pi\sqrt{\frac{l}{g}}$

## JEE MAIN & ADVANCED

### LEVEL-1 Single Correct Choice Type:

1. A body of mass 0.2 kg is rotated along a circle of radius 0.5m in horizontal plane with uniform speed 3 $ms^{-1}$. The centripetal force on the body is

1) 1.8 N 2) 3.6 N 3) 18 N 4) 36 N

2. The centripetal force required by a 1000 kg car that takes a turn of radius 50 m at a speed of 36 kmph is

1) 1000 N 2) 3500 N 3) 1600 N 4) 2000 N

3. A wheel which is initally at rest is subjected to an angular acceleration and it completes 10 rotations in time 't'. Then the time take by it to complete the next 10 rotations is

1) 2t 2) $\sqrt{2}$ t 3) $(\sqrt{2}-1)t$ 4) $(\sqrt{2}+1)t$

4. The angular velocity of particle increases from O to $\omega$ as it completes $x$ rotations. Then number of rotations completed by it when its angular velocity becomes 2$\omega$.

1) $x$ 2) 2$x$ 3) 3$x$ 4) 4$x$

**Statement Type:**

5. ***Statement I:*** A cyclist bends inwards from his vertical position, while turning to secure the necessary centripetal force.

***Statement II:*** Friction between the tyres and road provides him the necessary centripetal force.

1) Both Statement I and II are true.
2) Both Statement I and II are false.
3) Statement I is true, Statement II is false.
4) Statement I is false, Statement II is true.

**Matrix Match Type:**

6.

| Column - I | Column - II |
|---|---|
| a) Centrifugal force | 1) Along the axis of rotation |
| b) Centipetal force | 2) Towards the centre of rotation |
| c) Tangential force | 3) Away from the centre of rotation |
| d) Angular velocity | 4) Changes the angular velocity. |

**LEVEL-2 & 3**

7. A stone of mass 0.5 kg is attached to a string of length 2m and is whirled in a horizontal circle. If the string can with stand a tension of 9N, the maximum velocity with which the stone can be whirled is

1) $6ms^{-1}$ 2) $8ms^{-1}$ 3) $4ms^{-1}$ 4) $12ms^{-1}$

8. The centripetal force required for a 1000 kg car travelling at 36 kmph ot take a turn by $90^0$ in travelling along an arc of length 628 m is

1) 250 N 2) 500 N 3) 1000 N 4) 125 N

9. A stationary wheel starts rotating about its own axis at constant angular acceleration. If the wheel completes 50 rotations in first 2 seconds, then the number of rotations made by it in next two seconds is

1) 75 2) 1000 3) 125 4)150

10. A stationary wheel starts rotating about its own axis at an angular acceleration 5.5 rad/s$^2$. To acquire an angular velocity 420 revolutions per minute, the number of rotations made by the wheel is

1) 14 2) 21 3) 28 4) 35

11. A circular disc is rotating about its own axis at constant angular acceleration. If its angular velocity increases from 210 rpm to 420 rpm during 21 rotations then the angular acceleration of disc is

1) 5.5 rad/s$^2$ 2) 11 rad/s$^2$ 3) 16.5 rad/s$^2$ 4) 22 rad/s$^2$

12. A circular disc is rotating about its own axis at uniform angular velocity $\omega$. The disc is subjected to uniform angular retardation by which its angular velocity is decreased to $\frac{\omega}{2}$ during 120 rotations. The number of rotations further made by it before coming to rest is

1) 120 2) 60 3) 40 4) 20

13. A point moves along a circle with a velocity v = at, where a = 0.50 $m/s^2$. Find the total acceleration of the point at the moment when it has covered the $n^{th}$ (n = 0.10) fraction of the circle after beginning of the motion.

1) 0.8 $m/s^2$ 2) 0.3 $m/s^2$ 3) 0.2 $m/s^2$ 4) 0.50 $m/s^2$

14. Two particles of masses in the ratio 1:2 are moving in circles of radii in the ratio 2:3 with time periods in the ratio 3:4. The ratio of their centripetal forces is

1) 9:4 2) 1:4 3) 9:16 4) 16:27

**Multi Correct Choice Type:**

15. A person applies a constant force $\vec{F}$ on a particle of mass m and finds that the particle moves in a circle of radius r with a uniform speed v as seen from an inertial frame of reference.

1) This is not possible
2) There are other forces on the particle
3) The resultant of the other forces $\frac{mv^2}{r}$ towards the centre
4) The resultant of the other forces varies in magnitude as well as in direction.

**LEVEL-4 & 5** 16. A spot light S moves in a horizontal plane with a constant angular velocity of 0.1 rad/s. The spot light P moves along the wall at a distance 3m. When $\theta = 45^0$, the velocity of spot P is.

1) 0.3 $ms^{-1}$ 2) 0.6 $ms^{-1}$ 3) 0.8 $ms^{-1}$ 4) 1.2 $ms^{-1}$

17. A car is moving in a circular horizontal track of radius 10 m with a constant speed of 10 $ms^{-1}$. A plumb bob is suspended from the roof of the car by a light rigid rod of length 1m. The angle made by the rod with the track is g = 10 $ms^{-2}$.

1) Zero 2) $30^0$ 3) $45^0$ 4) $60^0$

18. A string is wrapped several times round a solid cylinder. Then free end of the string is held stationary. If the cylinder is released to move down, then the acceleration of that cylinder is

1) g/3 2) g / 2 3) 3g / 2 4) 2g / 3

**Statement Type:**

19. Statement-I: Two small spheres are suspended from same point O on roof with strings of different lengths. Both spheres move along horizontal circles as shown. Then both spheres may move along circles in same horizontal plane.

Statement-II: For both spheres in statement 1 to move in circular paths in same horizontal plane, their angular speeds must be same.

1) Both Statement I and II are true

2) Both Statement I and II are false

3) Statement I is true, Statement II is false.

4) Statement I is false, Statement II is true.

# SYNOPSIS - 3

When vehicles go through turnings, they travel along a nearly circular arc. There must be some force which will produce the required centripetal acceleration. If the vehicles travel in a horizontal circular path, this resultant force is also horizontal. The necessary centripetal force is being provided to the vehicles by following three ways.

1. By friction only
2. By banking of roads only
3. By friction and banking of roads both.

In real life the necessary centripetal force is provided by friction and banking of roads both. Now let us write equations of motion in each of the three cases separately and see what are the constraints in each case.

**By Friction only :**

Suppose a car of mass m is moving at a speed v in a horizontal circular arc of radius r. In this case, the necessary centripetal force to the car will be provided by force of friction *f* acting towards centre.

Thus , $f = \frac{mv^2}{r}$ Further, limiting value of *f* is $\mu N$ or $f_L = \mu N = \mu mg \quad (N = mg)$

Therefore, for a safe turn without sliding $\frac{mv^2}{r} \le f_L$ or $\frac{mv^2}{r} \le \mu mg$

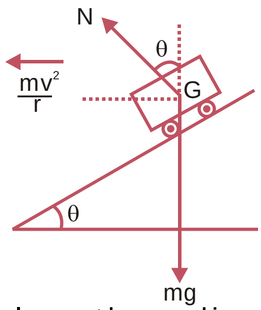
or $\mu \ge \frac{v^2}{rg}$ or $v \le \sqrt{\mu rg}$

Here, two situations may arise. If $\mu$ and r are known to us, the speed of the vehicle should not exceed $\sqrt{\mu rg}$ and if v and r are known to us, the coefficient of friction should be greater than $\frac{v^2}{rg}$.

**Note:** You might have seen that if the speed of the car is too high, car starts skidding outwards. With this radius of the circle increases or the necessary centripetal force is reduced (centripetal force $\propto \frac{1}{r}$).

## By Banking of Roads Only :

Friction is not always reliable at circular turns if high speeds and sharp turns are involved. To avoid dependence on friction, the roads are banked at the turn so that the outer part of the road is some what lifted compared to the inner part.



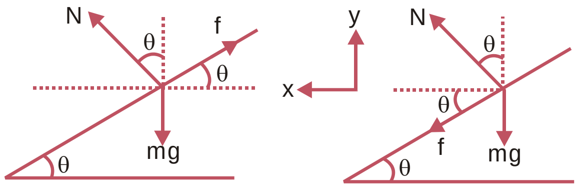

Applying. Newton's second law along the radius and the first law in the vertical direction. $N\sin\theta = \frac{mv^2}{r}$ and $N\cos\theta = mg$

From these two equations, we get $\tan\theta = \frac{v^2}{rg}$ or $v = \sqrt{rg\tan\theta}$

**Note:** This is the speed at which car does not slide down even if track is smooth. If track is smooth and speed is less than $\sqrt{rg\tan\theta}$, vehicle will move down so that r gets decreased and if speed is more than this vehicle will move up.

## By Friction and Banking of Road Both

If a vehicle is moving on a circular road which is rough and banked also, then three forces may act on the vehicle, of these the first force, i.e., weight (mg) is fixed both in magnitude and direction.




The direction of second force, i.e., normal reaction N is also fixed (perpendicular to road) while the direction of the third force, i.e., friction *f* can be either inwards or outwards while its magnitude can be varied upto a maximum limit $(f_L = \mu N)$.So the magnitude of normal reaction N and direction plus magnitude of friction *f* are so adjusted that the resultant of the three forces mentioned above is $\frac{mv^2}{r}$ towards the centre. Of these m and r are also constant. Therefore, magnitude of N and direction plus magnitude of friction mainly depends on the speed of the vehicle v. Thus, situation varies from problem to problem. Even though we can see that:

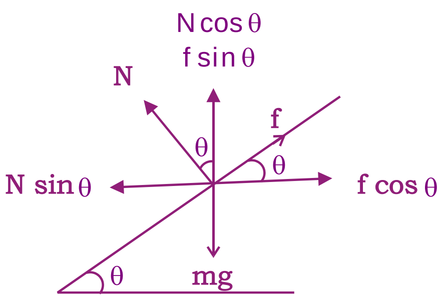
i) Friction *f* is outwards if the vehicle is at rest or v = 0. Because in that case the component of weight mg sin$\theta$ is balanced by *f*.

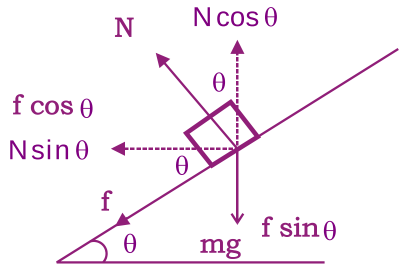
ii) Friction *f* is inwards if $v > \sqrt{rg \tan\theta}$

iii) Friction *f* is outwards if $v < \sqrt{rg \tan\theta}$

iv) Friction *f* is zero if $v = \sqrt{rg \tan\theta}$

By using above four equations and drawing F.B.D of vehicle and applying Newton's law's we can solve any problem of this type.



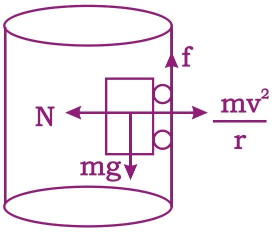

To find max. safe speed, we have to consider figure (ii)

$$N\sin\theta + f\cos\theta = \frac{mv_{max}^2}{r} \ldots\ldots\ldots(1); \qquad N\cos\theta - f\sin\theta = mg \ldots\ldots\ldots(2)$$

From (1) and (2) we get $v_{max} = \sqrt{\frac{rg(\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)}}$

To find minimum speed we use fig (i), and $v_{min} = \sqrt{\frac{rg(\sin\theta - \mu\cos\theta)}{(\cos\theta + \mu\sin\theta)}}$

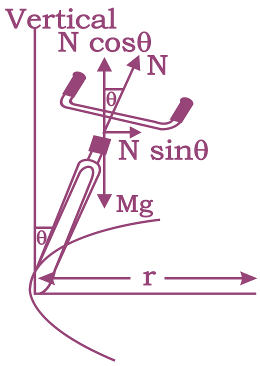
## Death Well:



For the body incontact with the vertical wall of death well not to fall, the condition is $mg \le f_{ms}$ ; $mg \le \mu_s N$ ; But $N = mr\omega^2$ ; Hence $mg \le \mu_s mr\omega^2$ .

**Turning of cyclist on the Road:**

A cyclist provides himself the necessary centripetal force by leaning inward on a horizontal track, while going round a curve.




Let N be the reaction of the ground on cyclist. It will act along a line-making angle $\theta$ with the verticle. The verticle component N cos$\theta$ of normal reaction N will balance the weight of the cyclist, while the horizontal component N sin$\theta$ will provide the necessary centripetal force tothe cyclist.

$N\sin\theta = \frac{mV^2}{r}$ and N cos$\theta$ = mg; $\tan\theta = \frac{V^2}{rg}$. Therefore, the cyclist should bend through an angle $\theta = \tan^{-1}\left(\frac{V^2}{rg}\right)$ to get the necessary centripetal force.

## WORKSHEET-3

**CUQ** 1.What will be the maximum velocity with which a vehicle can negotiate a turn of radius r safely, when the coefficient of friction between the tyres and the road is $\mu$ .

1) $\sqrt{\mu rg}$ 2) $\sqrt{\mu r}$ 3) $\sqrt{2\mu rg}$ 4) $\frac{1}{\sqrt{\mu rg}}$

2. A car moving on a horizontal road may be thrown off the road in taking a turn
   1) by the gravitational force
   2) due to lack of proper centripetal force
   3) due to rolling frictional force between the tyres and the road
   4) due to reaction on the ground
3. The outer rail of the curved railway track is raised above the inner one
   1) to provide centripetal force 2) to overcome the frictional force
   3) to balance the gravity 4) for some reason other than those mentioned above

4. A cyclist bends while taking turn to
   1) reduce friction
   2) generate required centripetal force
   3) reduce apparent wiehgt
   4) reduce speed

5. Banking of the road is $v_{\max} =$
   1) $v_{\max} = \sqrt{\frac{rg(\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)}}$
   2) $v_{\max} = \sqrt{\frac{rg(\sin\theta - \mu\cos\theta)}{(\cos\theta + \mu\sin\theta)}}$
   3) $v_{\max} = \sqrt{\frac{rg(\cos\theta - \mu\sin\theta)}{(\sin\theta + \mu\cos\theta)}}$
   4) $v_{\max} = \sqrt{\frac{rg(\cos\theta + \mu\sin\theta)}{(\sin\theta - \mu\cos\theta)}}$

6. Banking of the road $v_{\min} =$
   1) $v_{\min} = \sqrt{\frac{rg(\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)}}$
   2) $v_{\min} = \sqrt{\frac{rg(\sin\theta - \mu\cos\theta)}{(\cos\theta + \mu\sin\theta)}}$
   3) $v_{\min} = \sqrt{\frac{rg(\cos\theta - \mu\sin\theta)}{(\sin\theta + \mu\cos\theta)}}$
   4) $v_{\min} = \sqrt{\frac{rg(\cos\theta + \mu\sin\theta)}{(\sin\theta - \mu\cos\theta)}}$

7. Skidding occurs when the maximum frictional force of a flat road on a car is
   1) less than centripetal force required
   2) more than centripeal force required
   3) equal to centripetal force required
   4) independent of centripetal force

8. A car is traveling in banked curved road of radius 125m. If the coefficient of friction between the tyres and road is 0.5 and g = 10 m/s². The maximum speed to avoid skidding is
   1) 50 m/s 2) 25 m/s 3) 20 m/s 4) 10 m/s

9. A body incontact with the vertical wall of deapth well, not fall condition is
   1) $mg \le \mu_s mr\omega^2$ 2) $mg \le \mu_s mr^2\omega$ 3) $mg \le \mu_s mr\omega$ 4) $mg \le \mu_s m$

## JEE MAIN & ADVANCED

**LEVEL-1 Single Correct Choice Type:**

1. The angle through which the outer edge is raised above the inner edge is called
   1) Angle of banking
   2) Angle of repose
   3) Angle of friction
   4) None of these

2. A cyclist riding at a speed of $14\sqrt{3}\ ms^{-1}$ takes a turn around a circular road of radius $20\sqrt{3}\ m$. What is the inclination to the vertical ?
   1) 30° 2) 60° 3) 45° 4) 90°

3. Calculate the maximum speed with which a car can be driven safely along a curved road of radius 30 m and banked at 30º with the horizontal.Given, $g = 9.8\ ms^{-2}$.

1) $13.03\ ms^{-1}$ 2) $12.08\ ms^{-1}$ 3) $15.08\ ms^{-1}$ 4) $14.02\ ms^{-1}$

4. Find the maximum speed at which a car can turn round a curve of 30m radius on a level road. (Where the coefficient of friction between the tyre and the road is 0.4)

1) 13.84 m/s 2) 10.84 m/s 3) 11.84m/s 4) 12.84m/s

5. A park has a radius of 10m. If a vehicle goes round it at an average speed of 18 km/hr, what should be the proper angle of banking [$g=10m/s^2$]

1) $\tan^{-1}(1/4)$ 2) $\tan^{-1}(3/2)$ 3) $\tan^{-1}(1/2)$ 4) $\tan^{-1}(2/3)$

6. A curved road of 50m in radius is banked to correct angle for a given speed. If the speed is to be doubled keeping the same banking angle, the radius of curvature of the road to be changed to

1) 250 m 2) 100 m 3) 150 m 4) 200 m

7. A car is moving in a circular horizontal track of radius 10 m with a constant speed of $10ms^{-1}$. A plumb bob is suspended from the roof of the car by a string o length 1m. The angle made by the string with vertical is $\left(g = 10ms^{-2}\right)$

1) $0^0$ 2) $30^0$ 3) $45^0$ 4) $60^0$

8. A boy sitting on a horizontal platform of joy wheel at a distance of 5m

from its centre. The joy wheel begins to rotate and when the angular speed exceeds 10 revolutions per minute, the boy just slips, the coefficient of friction between the boy and the platform is ($g=10m/s^2$)

1) $\frac{\pi}{6}$ 2) $\frac{\pi^2}{18}$ 3) $\frac{\pi}{6}$ 4) $\frac{\pi}{2}$

9. A car of mass 'm' moving on a horizontal circular path of radius 'r' at an instant its speed is 'v' and increasing at a rate 'a'

1) The acceleration of the car is towards the centre of path

2) The magnitude of frictional force on car is greater than $\frac{mv^2}{r}$

3) The coefficient of friction between the ground and car is not less than a/g

4) The coefficient of friction '$\mu$' between the ground and car is $\tan^{-1}\left[\frac{v^2}{rg}\right]$

**Statement Type:**

10. Statement I: A car is negotiating a curved road of radius r. If the coefficient of friction between the tyres and the road is $\mu$ the car will skid if its speed exceeds $\sqrt{\mu rg}$

Statement II: Banking of road reduces the wear and tear of tyres of vehicle taking turn on curved roads

1) Both Statement I and II are true
2) Both Statement I and II are false
3) Statement I is true, Statement II is false.
4) Statement I is false, Statement II is true.

**LEVEL-2 & 3 Single Correct Choice Type:**

11. A car is traveling in banked curved road of radius 125m. If the coefficient of friction between the tyres and road is 0.5 and g = 10 m/s$^2$. The maximum speed to avoid skidding is

1) 50 m/s 2) 25 m/s 3) 20 m/s 4) 10 m/s

12. In a rotor, a hollow vertical cylindrical structure rotates about its axis and a person rests against the inner wall. At a particular speed of the rotor, the floor below the person is removed and the person hangs resting against the wall without any floor. If the radius of the rotor is 2 m and the coefficient of friction between the wall and the person is 0.2, find the minimum speed at which the floor may be removed. [Take g = 10 m/s$^2$]

1) 7.2 m/s 2) 10 m/s 3) 5.5m/s 4) 4.4 m/s

13. A person is in contact with the inner wall of a vertical hollow cylinder of radius 1m, remains in equilibrium without slipping down as the cylinder is rotated about its own vertical axis with an angular velocity 3.5 rad/s. The minimum coefficient of static friction between person and wall of cylinder such that the person does not slip down is

1) 0.2 2) 0.4 3) 0.6 4) 0.8

14. The road at a circular turn of radius 10m is banked by an angle of 10°, with what speed should vehicle move on the turn so that the normal contact force is able to provide the necessary centripetal force [tan 10° = 0.18]

1) 6 m/s 2) 3.2 m/s 3) 5 m/s 4) 4.2 m/s

15. A person with his hand in his pocket is skating on the ice at the speed of 10 m/s and describes a circle of radius 50m. Then what is inclination to the vertical

1) $\tan^{-1}(3/5)$ 2) $\tan^{-1}(1/5)$ 3) $\tan^{-1}(1/2)$ 4) $\tan^{-1}(1/10)$

16. A car is moving along a circular track of radius $10\sqrt{3}$ m with a constant speed of 36kmph. A plumb bob is suspended from the roof of the car by a light rigid rod of length 1m. The angle made by the rod with the track is

1) Zero    2) 30°    3) 45°    4) 60°

17. A particle is placed on a rough horizontal circular table rotating with a constant angular speed of $\pi$ rad/sec. It comes to rest relative to table at a distance of 0.5 m from the axis of rotation. Then the value of '$\mu$' between particle and table surface is ________× $10^{-1}$.

**Matrix Match Type:**

18.

| Column - I | Column - II |
|---|---|
| a) $V = \sqrt{rg\tan\theta}$ | 1) Friction = 0 |
| b) $V > \sqrt{rg\tan\theta}$ | 2) Can't decide the direction of 'f' |
| c) $V < \sqrt{rg\tan\theta}$ | 3) Friction (f) acts outwards |
| d) Vehicle at rest on a banked roads | 4) Friction (f) acts inwards |

**Statement Type:**

19. *Statement I:* A cyclist always bends in wards while negotiating a curve

*Statement II:* By bending cyclist lowers his centre of gravity

1) Both Statement I and II are true.
2) Both Statement I and II are false.
3) Statement I is true, Statement II is false.
4) Statement I is false, Statement II is true.

**Multi Correct Choice Type:**

20. Skidding of cyclist on curved road is caused if

1) His velocity is large    2) Friction between road and tyres is small
3) Friction between road and tyres is large    4) Curve is sharp

**LEVEL-4 & 5 Comprehension Type:**

A turn of radius 20 m is banked for the vehicle of mass 200kg going at a speed of 10m/s. ( g = 10m/$s^2$)

21. Find the banking angle

1) $\tan^{-1}\left(\frac{1}{4}\right)$    2) $\tan^{-1}\left(\frac{1}{3}\right)$    3) $\tan^{-1}\left(\frac{2}{3}\right)$    4) $\tan^{-1}\left(\frac{1}{2}\right)$

22. Find the direction and magnitude of frictional force acting on a vehicle. If vehicle moves with speed 5m/s (assume that friction is sufficient to prevent slipping.)

1) $300\sqrt{5}$ N (outwards)
2) $200\sqrt{5}$ N (downwards)
3) $400\sqrt{5}$ N (downwards)
4) $100\sqrt{5}$ N (outwards)

23. Find the direction and magnitude of frictional force acting on a vehicle. If vehicle moves with speed15m/s (assume that friction is sufficient to prevent slipping.)

1) $300\sqrt{5}$ N (outwards)
2) $500\sqrt{5}$ N (downwards)
3) $400\sqrt{5}$ N (outwards)
4) $200\sqrt{5}$ N (downwards)

**Single Correct Choice Type:**

24. A vehicle is moving with a velocity $v$ on a curved road of width $b$ and radius of curvature $R$. For counteracting the centrifugal force on the vehicle the difference in elevation required in between the outer and inner edges of the road is

1) $\frac{v^2 b}{Rg}$
2) $\frac{rb}{Rg}$
3) $\frac{vb^2}{Rg}$
4) $\frac{vb}{R^2 g}$

**Multi Correct Choice Type:**

25. A circular road of radius r is banked for a speed v = 40 km/hr. A car of mass m attempts to go on the circular road. The friction coefficient between the tyre and the road is negligible.

1) The car cannot make a turn without skidding
2) If the car turns at a speed less than 40 km/hr, it will slip down.
3) If the car turns at the correct speed of 40 km/hr, the force by the road on the car is equal to $\frac{mv^2}{r}$
4) If the car turns at the correct speed of 40 km/hr, the force by the road on the car is greater than mg as well as greater than $\frac{mv^2}{r}$.

******

# CIRCULAR MOTION PART-1

## WORKSHEET-1

**CUQ:**

| | | | | | |
|---|---|---|---|---|---|
| 1) 2 | 2) 1 | 3) 1 | 4) 2 | 5) 3 | 6) 3 |
| 7) 3 | 8) 2 | 9) 3 | 10) 1 | | |

### JEE MAIN & ADVANCED

| | | | | | |
|---|---|---|---|---|---|
| 1) 1 | 2) 3 | 3) 4 | 4) 1 | 5) 1 | 6) 2 |
| 7) 3 | 8) 2 | 9) 2,3 | 10) 1,2,3 | 11) 1 | 21) 4 |
| 13) 3 | 14) 4 | 15) 4 | 16) 2 | 17) 4 | 18) 1 |
| 19) 1, 4 | 20) 1 | 21) 3 | 22) 4 | 23) 1 | 24) 1,2,3,4 |
| 25) 4,3,2,1 | | | | | |

### KEY & SOLUTIONS

11. The angular velocity is $\omega = \frac{v}{r} = \frac{10\ m/s}{20\ cm} = 50\ rad/s$

12. The tangential acceleration is given by $a_t = \frac{dv}{dt} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{6.0 - 5.0}{2.0} m/s^2 = 0.5\ m/s^2$

    The angular acceleration is $\alpha = a_t / r = \frac{0.5\ m/s^2}{20\ cm} = 2.5\ rad/s^2$

13. The distance covered in completing the circle is $2\pi r = 2\pi \times 10 cm$.

    The linear speed is $v = 2\pi\ r/t = \frac{2\pi \times 10 cm}{4s} = 5\pi\ cm/s$

    The linear acceleration is $a = \frac{v^2}{r} = \frac{(5\pi cm/s)^2}{10\ cm} = 2.5\ \pi^2\ cm/s^2$

    This acceleration is directed towards the centre of the circle.

14. Angular acceleration $a = r\alpha$

15. Centripetal acceleration $a = \frac{v^2}{r}$. 6.Centripetal acceleration $a = r\omega^2 \Rightarrow \omega = \sqrt{\frac{a}{r}}$

17. Angular velocity $\omega = \frac{v}{r}$

18. For an hour hand : Period of revolution = 12 hrs. = 43200 sec.

    $f = \frac{1}{T} = \frac{1}{43200} = 2.31 \times 10^{-5}$ per sec.

21. Given $\omega_i = 0$; $\alpha = 8\ rad s^{-2}$ $\theta = 77(2\pi) = 154\pi\ rad$ $t = \sqrt{\frac{2\theta}{\alpha}}$

22. Given $\Delta s = 5 cm$; $\Delta t = 2.5 sec$; r = 10 cm

Linear speed $v = \frac{\Delta s}{\Delta t} = \frac{5}{2.5} = 2\,cm/sec$ = $2 \times 10^{-2}$ m/sec $\because$ $v = r\omega$

$\therefore$ $\omega = \frac{v}{r} = \frac{2 \times 10^{-2}}{10 \times 10^{-2}} = 0.2$ rad/sec

10 revolutions, would mean a total angular displacement of $10 \times 2\pi$ rad.
Now, by definition, of $\omega$ 0.2 rad is rotated by the particle in 1 sec

$\therefore$ $20\pi$ rad is rotated by the particle in $\frac{20\pi}{0.2} = 100\pi\ sec = 314.15\ sec$.

23. $v = r\omega \Rightarrow r = \frac{v}{\omega} = \frac{0.5}{2.5} = 0.2m$ ;

$a = r\alpha \Rightarrow \alpha = \frac{a}{r} = \frac{0.2}{0.2} = 1 rad/sec^2$

# WORK SHEET-2_KEY

**CUQ:**

| | | | | | |
|---|---|---|---|---|---|
| 1) 3 | 2) 2 | 3) 3 | 4) 3 | 5) 3 | 6) 2 |
| 7) 2 | 8) 1 | | | | |

## JEE MAIN & ADVANCED

| | | | | | | |
|---|---|---|---|---|---|---|
| 1) 2 | 2) 4 | 3) 3 | 4) 4 | 5) 1 | 6) a-3; b-2; c-4; d-1 | |
| 7) 1 | 8) 1 | 9) 4 | 10) 3 | 11) 1 | 12) 3 | |
| 13) 1 | 14) 4 | 15) 2,4 | 16) 2 | 17) 3 | 18) 4 | 19) 1 |

# WORK SHEET-3_KEY

**CUQ:**

| | | | | |
|---|---|---|---|---|
| 1) 1 | 2) 2 | 3) 1 | 4) 2 | 5) 1 |
| 6) 2 | 7) 4 | 8) 1 | 9) 1 | |

## JEE MAIN & ADVANCED

| | | | | |
|---|---|---|---|---|
| 1) 1 | 2) 2 | 3) 1 | 4) 2 | 5) 1 |
| 6) 4 | 7) 3 | 8) 2 | 9) 2,3 | 10) 1 |
| 11) 2 | 12) 2 | 13) 4 | 14) 4 | 15) 2 |
| 16) 2 | 17) $5 \times 10^{-1}$ | 18) 1,4,3,3 | 19) 1 | 20) 1,2,4 |
| 21) 4 | 22) 1 | 23) 2 | 24) 1 | 25) 2,4 |

# OPTICS

## SYNOPSIS-1

## Reflection of light at plane surfaces

### Introduction

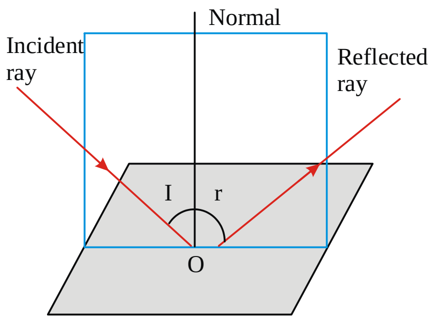
Nature has endowed the human eye (retina) with the sensitivity to detect electromagnetic waves within a small range of the electromagnetic spectrum.

Electromagnetic radiation (Wavelength from 400 nm to 750 nm) is called light. It is mainly through light and the sense of vision.

Light travels along straight line with enormous speed. The speed of light in vacuum is the highest speed attainable in nature. The speed of light in vacuum is $c = 2.99792458 \times 10^8 ms^{-1} \approx 3 \times 10^8 ms^{-1}$

The wavelength of light is very small compared to the size of ordinary objects that we encounter commonly (generally of the order of a few cm or larger). A light wave can be considered to travel from one point to another, along a straight line joining them. The path is called a **ray of light**, and a bundle of such rays constitutes a **beam of light**.

- **Fermat's principle** states that "light travels between two points along the path that requires the least time, as compared to other nearby paths."
- The phenomena of reflection, refraction and dispersion of light are explained using the ray picture of light. We shall study the image formation by plane and spherical reflection and refracting surfaces, using the basic laws of reflection and refraction. The construction and working of some important optical instruments, including the human eye are also explained.
- **Reflection of Light :** When a light ray strikes the boundary of two media such as air and glass, a part of light is turned back into the same medium. This is called reflection of light.




In case of reflection at the point of incidence 'O', the angle between incident ray and normal to the reflecting surface is called the angle of incidence (i). The angle between reflected ray and normal to the reflecting surface is called angle of reflection (r).

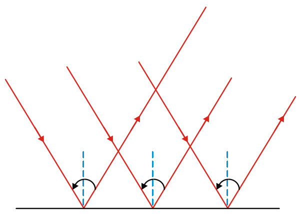
The plane containing incident ray and normal is called plane of incidence.

- **Laws of reflection** : The incident ray, the reflected ray and the normal to the reflecting surface at the point of incidence, all lie in the same plane.
- The angle of incidence is equal to the angle of reflection $\angle i = \angle r$

**Types of reflections**

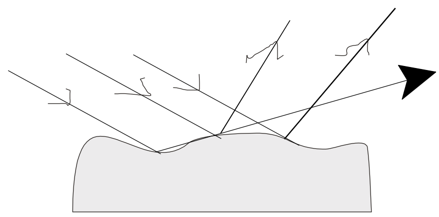
- **Regular reflection:**When the reflection takes place from a perfect smooth plane surface, then the reflection is called regular reflection (or) specular reflection.

  In this case, a parallel beam of light incident will remain parallel even after reflection as shown in the figure.



In case of regular reflection, the reflected light ray has large intensity in one direction and negligibly small intensity in other direction. Regular reflection of light is useful in determining the property of mirror.

- **Diffused reflection:** If the reflecting surface is rough (or uneven), parallel beam of light is reflected in random directions. This kind of refletion is called diffused reflection.

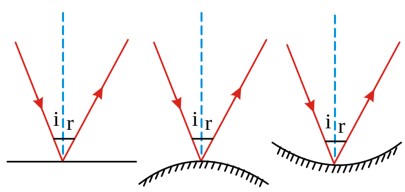


As shown in the above figure if the reflecting surface is rough, the normal at different points will be in different directions, so the rays that are parallel before reflection will be reflected in random directions.

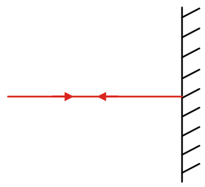
We see non-luminous objects by diffused reflection.

**Important points regarding reflection**

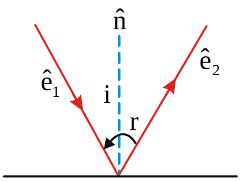
- Laws of reflection are valid for all reflecting surfaces either plane or curved.




- If a light ray is incident normally on a reflecting surface, after reflection it retraces its path i.e., if $\angle i = 0$ then $\angle r = 0$

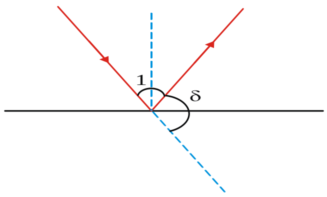


- In case of reflection of light frequency, wavelength and speed does not change. But the intensity of light on reflection will decreases.
- If the reflection of light takes place from a denser medium, there is a phase change of $\pi$ rad.
- If $\hat{I}, \hat{N}$ and $\hat{R}$ are vectors of any magnitude along incident ray, the normal and the reflected ray respectively then $\hat{R}.(\hat{I}\times\hat{N}) = \hat{N}.(\hat{I}\times\hat{R}) = \hat{I}.(\hat{N}\times\hat{R}) = 0$ This is because incident ray, reflected ray and the normal at the point of incidence lie in the same plane.
- **Vector form of law of reflection:**




If $\hat{e}_1$ is unit vector along the incidnet ray $\hat{e}_2$ is the unit vector along the reflected ray $\hat{n}$ is the unit vector along the normal then, $\hat{e}_2 = \hat{e}_1 - 2(\hat{e}_1.\hat{n})\hat{n}$

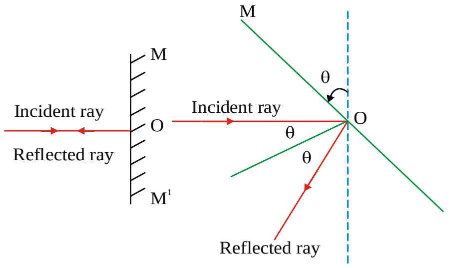
- **Deviation of a ray due to reflection:** The angle between the direction of incident ray and reflected light ray is called the angle of deviation ($\delta$).




From the above figure $\delta = \pi - (i + r)$ But $i = r$

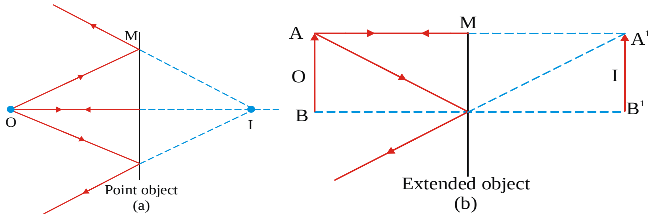
Hence angle of deviation in the case of reflection is $\delta = \pi - 2i$

- By keeping the incident ray fixed, the mirror is rotated by an angle '$\theta$', about an axis in the plane of mirror, the reflected ray is rotated through an angle '$2\theta$'.

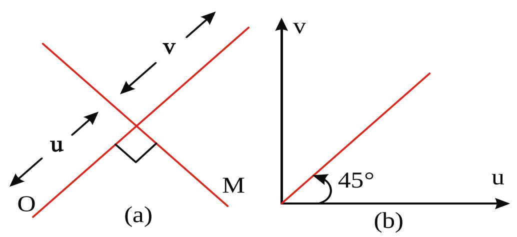



### Reflection from Plane Surface

- When you look into a plane mirror, you see an image of yourself that has three properties.
- The image is up right.
- The image is the same size as you are
- The image is located as far behind the mirror as you are infront of it. This is shown in the figure(b).




- A plane mirror always form virtual image to a real object and vice versa and the line joining object and image is perpendicular plane mirror as shown in figure (a).

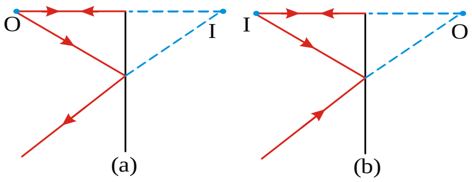



The graph between image distance (v) and object distance (u) for a plane mirror is a straight line as shown in figure (b).

The ratio of image height to the object height is called lateral magnification (m). Thus in case of plane mirror 'm' is equal to one.

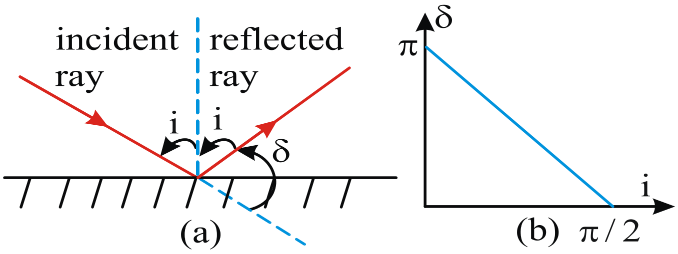
- The principle of reversibility states that rays retrace their path when their direction is reversed. In accordance with the principle of reversibility object and image positions are interchangable. The points corresponding to object and image are called conjugate points.

This is illustrated in figure.

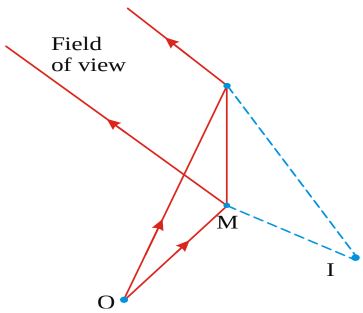



- A mirror whatever may be the size, it forms the complete image of the object lying infront of it. Large mirror gives more bright image than a smaller one. It is seen that the size of reflector must be much larger than the wavelength of the incident light otherwise the light will be scattered in all directions.
- The angle between directions of incident ray and reflected or refracted ray is called deviation ($\delta$).

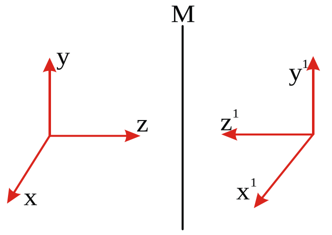
  A plane mirror deviates the incident light through angle $\delta = 180 - 2i$ where 'i' is the angle of incidence. The deviation is maximum for normal incidence, hence $\delta_{max} = 180^0$.




  It is noted that, generally anti - clock wise deviation is taken as positive and clock wise deviation as negative.
- Every object has its own **field of view** for the given mirror. The field of view is the region between the extreme reflected rays and depends on the location of the object infornt of the mirror. If our eye lies in the filed of view then only we can see the image of the object other wise not. This is illustrated in figure.

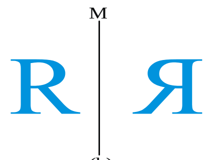



- A plane mirror produces front - back reversal rather than left - right reversal. It must be kept in mind that the mirror produces the reversal effect in the direction perpendicular to plane of the mirror. The figure (a) shows that the right handed co-ordinate system is converted into left handed co-ordinate system.




(a)

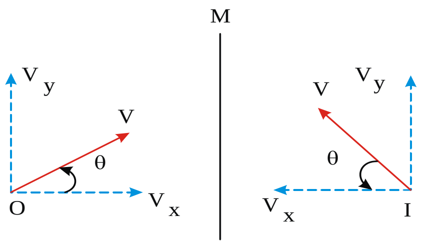
i.e., the image formed by a plane mirror left is turned into right and vice versa with respect to object as shown in figure (b).



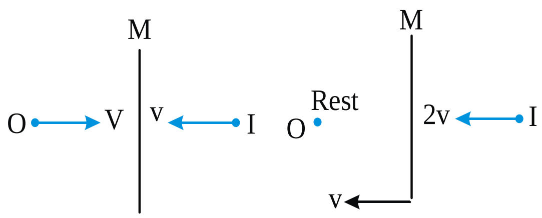

(b)

Ø When the object moves infront of stationary mirror, the relative speed between object and its image along the plane of the mirror is zero and in perpendicular to plane of mirror relative speed is twice that of the object speed.

$(V_{IO})_y = 0$ and $(V_{IO})_x = 2v_x$




Ø If an object moves towards (or away from) a plane mirror at speed v, the image will also approach (or recede) at the same speed v, and the relative velocity of image with respect to object will be 2v as shown in figure (a). If the mirror moved towards (or away from) the stationary object with speed v, the image will also move towards (or away from) the object with a speed 2v, as shown figure (b).

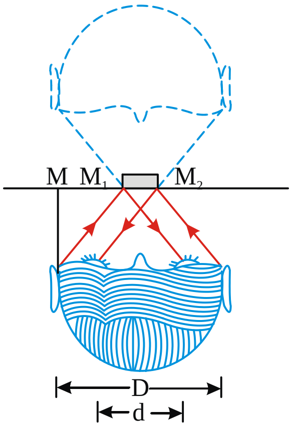



Ø a) A person of height ‘h’ can see his full image in a mirror of minimum length

$$l = \frac{h}{2}$$

b) A person standing at the centre of room looking towards a plane mirror hung on a wall, can see the whole height of the wall behind him if the length of the mirror is equal to one-third the height of the wall.

- The minimum width of a plane mirror required for a person to see the complete width of his face is $(D-d)/2$, where, D is the width of his face and d is the distance between his two eyes.

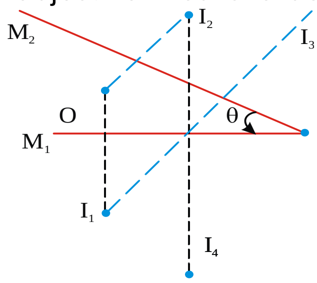



$MM_1 = \frac{1}{2}\left[D - \frac{1}{2}(D-d)\right]$ $MM_1 = \frac{(D+d)}{4}$ .... (i) and $MM_2 = D - \frac{(D+d)}{4}$

$MM_2 = \frac{(3D-d)}{4}$ ... (ii) $\therefore$ Width of the mirror = $M_1M_2 = MM_2 - MM_1$

$= \frac{2D-2d}{4}$ {From (i) and (ii)] $= \frac{2(D-d)}{4} = \frac{D-d}{2}$

- If two plane mirrors inclined to each other at an angle $\theta$, the number of images of a point object formed are determined as follows




- If $\frac{360}{\theta}$ is even number (say m) Number of images formed $n = (m-1)$, for all positions of objectes in between the mirrors.

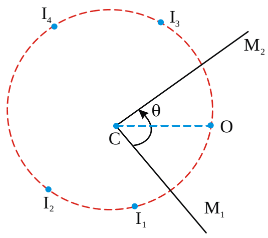
- If $\frac{360}{\theta}$ is odd integer (say m) number of images formed $n = m$, if the object is not on the bisector of mirrors. $n = (m-1)$, if the object is on the bisector of mirrors.

- If $\frac{360}{\theta}$ is a fraction (say m). The number of images formed will be equal to its integer part i.e., $n=[m]$.

  Ex: If m=4.3, the total number of images $n=[4.3]=4$

| $m=\frac{360}{\theta}$ | Position of the object | Number of images (n) |
|---|---|---|
| Even | Any where | $m-1$ |
| Odd | Symmetric | $m-1$ |
| | Asymmetric | M |
| Fraction | Any where | [m] |

- All the images lie on a circle whose radius is equal to the distance between the object 'O' and the point of intersection of mirrors C. If $\theta$ is less more number of images on circle with large radius.

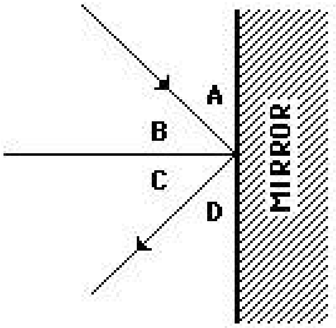



- If the objects is placed in between two parallel mirrors $\theta=0^0$, the number of images formed is infinite but of decreasing intensity in according with $I \alpha r^{-2}$.
- If '$\theta$' is given n is unique but if 'n' is given $\theta$ is not unique. Since same number of images can be formed for different $\theta$.
- The number of images seen may be different from number of images formed and depends on the position of the observer relative to object and mirror.
- When a light ray vector incident on a mirror, only the component vector which is parallel to normal of the mirror changes its sign without change of its magnitude on reflection. It is noted that a mirror can reflects entire energy incident on it, hence the magnitude of reflected vector is same as that of incident vector. Incident vector corresponding to an object and reflected vector corresponds to an image. This vector may be position, velocity or acceleration.

  Example: If a plane mirror lies on x-z plane, a light vector $2\hat{i}+3\hat{j}-4\hat{k}$ on reflection becomes $2\hat{i}-3\hat{j}-4\hat{k}$.

# OPTICS WORKSHEET-1

CUQ 1.Consider the following diagram.



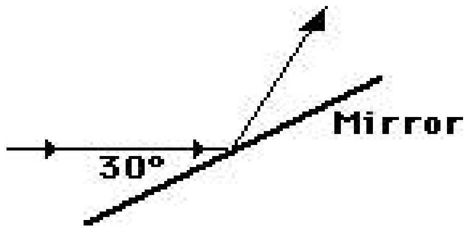

Which one of the angles (A, B, C, or D) is the angle of incidence? ______ Which one of the angles is the angle of reflection? ______

1) A,B  2) B, C  3) A, D  4) A, C

2. As the angle of incidence is increased for a ray incident on a reflecting surface, the angle between the incident and reflected rays ultimately approaches what value?

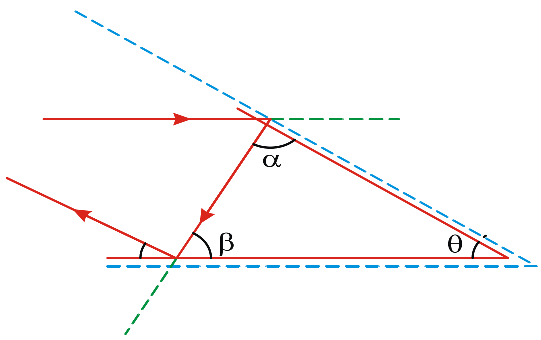
1) zero  2) 45 degrees  3) 90 degrees  4) 180 degrees

3. A ray of light is incident towards a plane mirror at an angle of 30-degrees with the mirror surface. What will be the angle of reflection?




1) $60^0$  2) $70^0$  3) $80^0$  4) $90^0$

4. If two plane mirrors are inclined at angle $\theta$ to each other as shown, then angle of deviation of incident ray is




1) $360-2\theta$  2) $360+2\theta$  3) $180-2\theta$  4) $180+2\theta$

5. Which of the following best describes the image formed by a plane mirror?

   1) virtual, inverted and enlarged

   2) real, inverted and reduced

   3) virtual, upright and the same size as object

   4) real, upright and the same size as object

6. If you stand three feet in front of a plane mirror, how far away would you see yourself in the mirror?

   1) 1.5 ft 2) 3.0 ft 3) 6.0 ft 4) 12.0 ft

7. When the image of an object is seen in a plane mirror, the distance from the mirror to the image depends on ____.

   1) the wavelength of light used for viewing.

   2) the distance from the object to the mirror.

   3) the distance of both the observer and the object to the mirror.

   4) all of these

8. If a man wishes to use a plane mirror on a wall to view both his head and his feet as he stands in front of the mirror, the required length of the mirror ____.

   1) is equal to the height of the man.

   2) is equal to one half the height of the man.

   3) depends on the distance the man stands from the mirror.

   4) depends on both the height of the man and the distance from the man to the mirror.

9. When an object is placed between two parallel mirrors, then number of images formed are

   1) 2 2) 4 3) 8 4) infinite

10. Two plane mirrors are attached to form a dual mirror system with an adjustable angle. As the angle between the mirrors increases, the number of images _____.

   1) increase 2) decreases 3) remains the same 4) all of thses

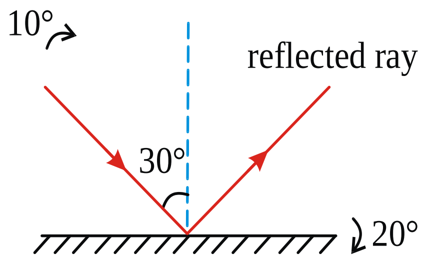
## JEE MAIN & ADVANCED

### LEVEL-1 Single Correct Choice Type:

1. A small object is placed 10 cm infront of a plane mirror. If you stand behind the object 30 cm from the mirror and looks at its image, the distance focussed for your eye will be

   1) 60 cm 2) 20 cm 3) 40 cm 4) 80 cm

2. Fig. Shows a plane mirror onto which a light ray is incident. If the incidenting light ray is turned by $10^0$ and the mirror by $20^0$ as shown, then the angle turned by the reflected ray is




1) $30^0$ clockwise
2) $30^0$ anticlock wise
3) $50^0$ clockwise
4) $50^0$ anticlock wise

3. If a plane mirror is rotated in its ownplane through an angle of $20^0$ keeping the incident ray direction fixed, then the angle through which the reflected ray turns is

1) $40^0$ 2) $0^0$ 3) $20^0$ 4) $10^0$

4. Two plane mirrors are at $45^0$ to each other. If an object is placed between them, then the number of images will be

1) 5 2) 9 3) 7 4) 8

5. A ray reflected successively from two plane mirrors inclined at a certain angle undergoes a deviation of $300^0$. The number of observable images

1) 60 2) 12 3) 11 4) 5

**Multi Correct Choice Type:**

6. Right angle mirrors produce three images of objects. Which of the following is statements is/are true of the middle image?

1) The middle image is the same size as the object.
2) The middle image is the same distance from the mirror as the object.
3) The middle image will exhibit left-right reversal relative to the object.
4) The middle image is a real image.
5) The middle image is an upright image.
6) The magnification of the middle image is -1.
7) When sighting at the middle image, light will reflect twice prior to reaching one's eye.

## Comprehension Type:

If both object and mirror moves away from each other with a speed of 20 m/s, then

7. The speed of the image

1) 60 m/s 2) 80 m/s 3) 40 m/s 4) 20m/s

8. Speed of the image with respect to object

1) 60 m/s 2) 80 m/s 3) 40 m/s 4) 20m/s

9. speed of the image with respect to mirror.

1) 60 m/s 2) 80 m/s 3) 40 m/s 4) 20m/s

## Matrix Match Type:

10. An incident ray makes an angle $35^0$ with the surface of a plane mirror. Then

| Column-1 | Column-2 |
|---|---|
| a) angle of incidence is | p) $45^0$ |
| b) angle of reflection is | q) $25^0$ |
| c) glancing angle of incidence | r) $35^0$ |
| d) glancing angle of reflection | s) $55^0$ |
| | t) $65^0$ |

## LEVEL-2 & 3 Single Correct Choice Type:

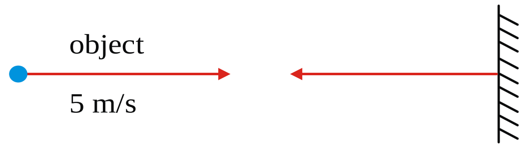
11. A ray of light is incident at $50^0$ on the middle of one of the two mirrors arranged at an angle of $60^0$ between them. The ray then touches the second mirror, get reflected back to the first mirror, making an angle of incidence of

1) $50^0$ 2) $60^0$ 3) $70^0$ 4) $80^0$

12. Two vertical plane mirrors are inclined at an angle of $60^0$ with each other. A ray of light travelling horizontally is reflected first from one mirror and then from the other mirror. then the resultant deviation is

1) $60^0$ 2) $120^0$ 3) $180^0$ 4) $240^0$

13. An object moves with 5 m/s towards right while the mirror moves 1 m/s towards the left as shown. Then the velocity of image.




1) 7 m/s towards left 2) 7m/s towards right

3) 5 m/s towards right 4) 5 m/s towards left

## Statement Type:

14. Statement-I: When an object is placed between two plane parallel mirrors, then all the images found are of equal intensity.

Statement-II: In case of plane parallel mirrors, only two images are possible.

1)Statement – 1 is True, Statement – 2 is True.

2) Statement – 1 is false, Statement – 2 is false.

3) Statement – 1 is True, Statement – 2 is False.

4) Statement – 1 is False, Statement – 2 is True.

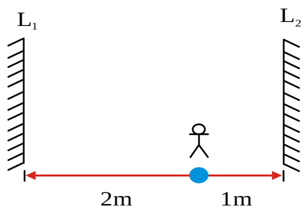
## Matrix Match Type:

15. An object is placed symmetrically between two plane mirrors inclined at an angle $\theta$ and if n is number of images seen then

| **Column-1** | **Column-2** |
|---|---|
| a) $\theta = 60^0$ | p) n = 7 |
| b) $\theta = 45^0$ | q) n = 3 |
| c) $\theta = 90^0$ | r) n = 5 |
| d) $\theta = 20^0$ | s) n = 9 |
| | t) n = 17 |

## LEVEL-4 & 5 Single Correct Choice Type:

16. Two mirrors labelled $L_1$ for left mirror and $L_2$ for right mirror in the figure are parallel to each other and $3.0$m apart. A person standing 1.0 m from the right mirror $(L_2)$ looks into this mirror and sees a series of images. The second nearest image in the right mirror is situated at a distance



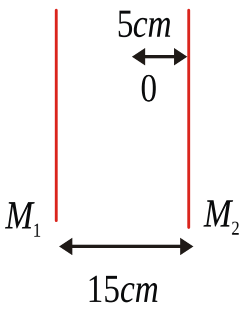

1) $2.0$ m from the person

2) 4.0 m from the person

3) 6.0 m from the person

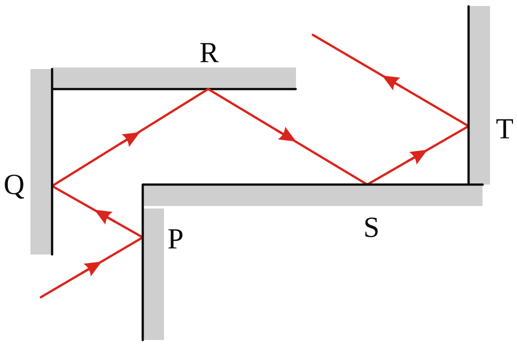
4) 8.0 m from the person

17. Two plane mirrors parallel to each other and an object O parallel between them. Then the distance of the first three images from the mirror $M_2$ will be ( $(in\,cm)^{-}$




1) 5, 10, 15
2) 5, 15, 30
3) 5, 25, 35
4) 5, 15, 25

18. Following figure shows the multiple reflections of a light ray along a glass corridor where the walls are either parallel or perpendicular to one another. If the angle of incidence at point P is 30°, what are the angles of reflection of the light ray at points Q, R, S and T respectively




1) 30°, 30°, 30°, 30°
2) 30°, 60°, 30°, 60°
3) 30°, 60°, 60°, 30°
4) 60°, 60°, 60°, 60°

19. A ray of light is incident on a plane mirror along a vector $\hat{i}+\hat{j}-\hat{k}$. The normal to the mirror at the point of incidence is along $\hat{i}+\hat{j}$. Then unit vector along the reflected ray is

1) $\frac{1}{\sqrt{3}}\left(\hat{i}+\hat{j}+\hat{k}\right)$
2) $-\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$
3) $-\frac{1}{\sqrt{3}}(-\hat{i}-\hat{j}+\hat{k})$
4) $-\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}-\hat{k})$

20. A plane mirror is placed at origin parallel to y-axis, facing the positive x-axis. An object starts from (2m,0,0) with a velocity of $\left(2\hat{i}+2\hat{j}\right)m/s$. The relative velocity of image with respect to object is along

1) Positive x - axis
2) Negative x - axis
3) Positive y - axis
4) Negative y - axis

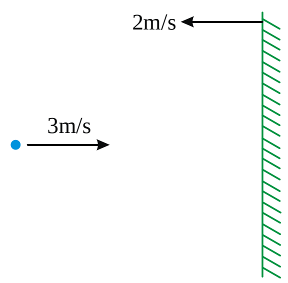
## Multi Correct Choice Type:

21. A plane mirror M is arranged parallel to a wall W at a distance $\ell$ from it. The light produced by a point source S kept on the wall is reflected by the mirror and produces a light spot on the wall. The mirror moves with velocity n towards the wall. Then.

1) The spot of light will move with the speed n on the wall.
2) The spot of light will not move on the wall.
3) As the mirror comes closer, the spot of light will becomes larger and shift ways from the wall with speed larger then n.
4) The size of the light spot on the wall remains the same

## Matrix Match Type:

22. A point object is placed in front of a plane mirror as shown and moving with velocity 3 m/s towards mirror. Mirror is moving with speed 2 m/s towards object, then




| Column – I | Column – II |
|---|---|
| a) speed of image w.r.t. ground | p) 10 m/s |
| b) speed of image w.r.t. mirror | q) 5 m/s |
| c) speed of image w.r.t. object | r) 14 m/s |
| d) speed of mirror w.r.t. object | s) 7 m/s |

# OPTICS_SYNOPSIS-2

## Reflection of light at curved surfaces

### Mirrors:

A smooth, highly polished reflecting surface is called a **mirror**. One surface of the mirror is made opaque by silvering followed by a thin coat of red lead oxide paint.

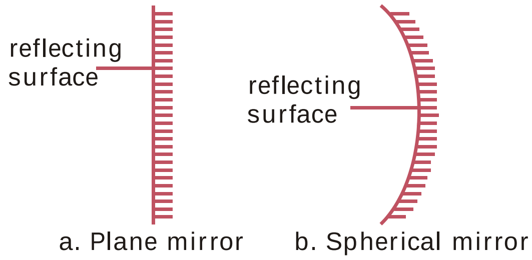
There are two types of mirrors.

#### Plane mirrors:

A highly polished plane surface is called a plane mirror.

#### Spherical mirrors:

A mirror in which the reflecting surface is curved is called a spherical mirror.




In spherical mirrors the polished reflecting surface is a part of a hollow sphere of glass. Depending upon the nature of the reflecting surface of the mirror, spherical mirrors are of two types.

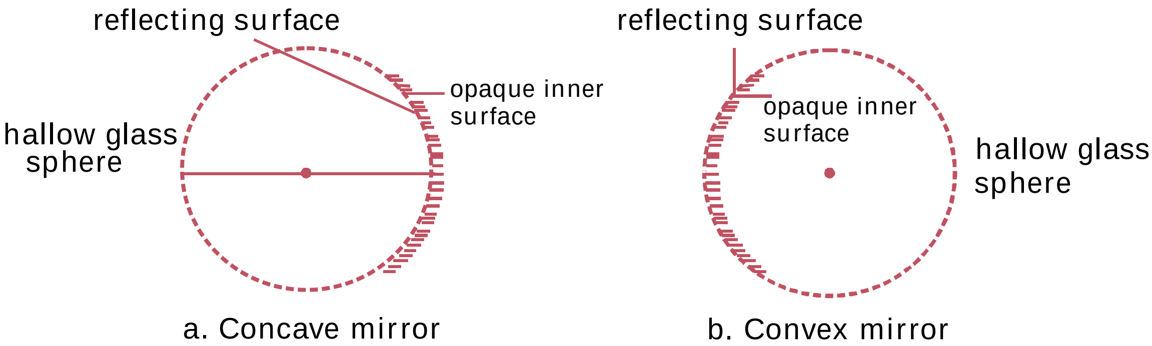
#### Different types of spherical mirrors:

#### Concave mirror:

A spherical mirror whose inner hollow surface is the reflecting surface is called a concave mirror.

#### Convex mirror:

A spherical mirror whose outer surface is the reflecting surface is called a convex mirror.



## Different types of spherical mirrors

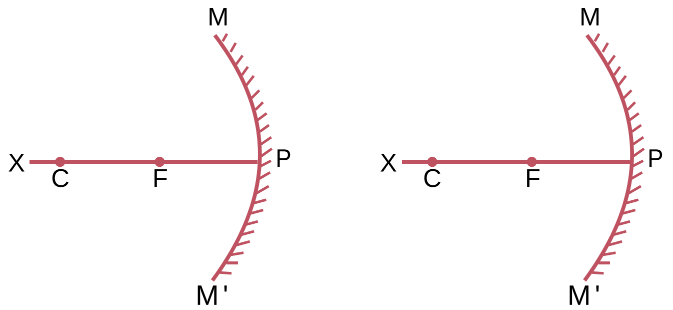
### Terms related to spherical mirrors:

→ **Aperture:** The width (distance) of the spherical mirror from which reflection can take place is called its aperture. It is denoted by $MM'$

→ **Pole:** The centre of a spherical mirror is called its pole. It is denoted by P.

→ **Centre of curvature:** The geometric centre of the hollow sphere of which the spherical mirror is a part is called the centre of curvature of the spherical mirror. It is denoted by C.

→ **Radius of curvature:** The radius of the hollow sphere of which the spherical mirror is a part is called the radius of curvature of the spherical mirror. In other words, the



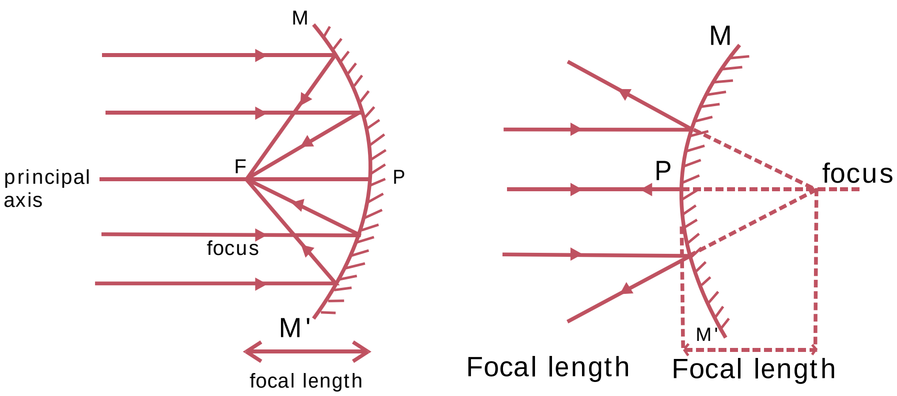

a.Concave mirror b.Convex mirror

distance between the pole and centre of curvature of the spherical mirror (PC) is called its radius of curvature. It is denoted by r.

→ **Principal axis:** The straight line passing through the centre of curvature and the pole of a spherical mirror is called its principal axis (PX).

→ **Focus:** If a beam of light parallel to the principal axis falls on a concave mirror, all the rays after reflection meet at a point. This point is called the focus (F) of the concave mirror.

If a beam of light parallel to the principal axis falls on a convex mirror, all the rays after reflection diverge. If the reflected rays are extended backwards, they appear to come from a point on the principal axis. This point is called the focus of the convex mirror.



→ **Focal length:** The distance between the pole (P) and focus (F) is called the focal length (f). It is denoted by f.

$\therefore f = PF$

**Note:** By sign convention, for concave mirror $f = -ve$ and for convex mirror $f = +ve$

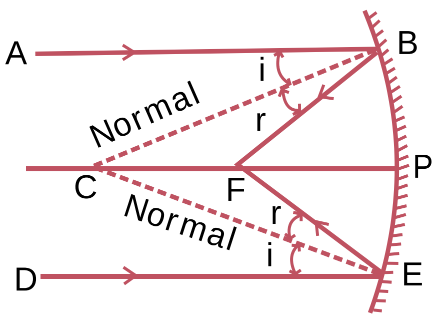
## DIFFERENCES BETWEEN CONCAVE MIRROR AND CONVEX MIRROR

| CONCAVE MIRROR | CONVEX MIRROR |
|---|---|
| Reflection takes place at the concave surface (or bent in surface) | Reflection takes place at the convex surface (or bulging out surface) |
| A parallel beam of light falling on this mirror converges at a point after reflection | A parallel beam of light falling on this mirror appears to diverge from a point after reflection |
| It is a converging mirror | It is a diverging mirror |
| It has a real focus | It has a virtual focus |

**Concave Mirror As Converging Mirror:** Consider two rays of light AB and DE travelling parallel to the principal axis of a concave mirror, meeting the surface of mirror at point B and E respectively. C is the centre of curvature of the mirror. Join CB and CE . CB and CE are perpendiculars to the surface of mirror at points B and E, because any line drawn from the centre of a sphere to its circumference is always at right angles to the circumference.

Thus, $\angle$ABC and $\angle$DEC are the angles of incidence. Applying laws of reflection, make $\angle$CBF and $\angle$CEF, such that the reflected rays meet at point F on the principal axis. Then, point F is the principal focus.

It is found that all the rays coming parallel to principal axis meet at point 'F. In a way the rays converge at a single point. Thus, concave mirror acts as a converging mirror.



**Conclusion:**

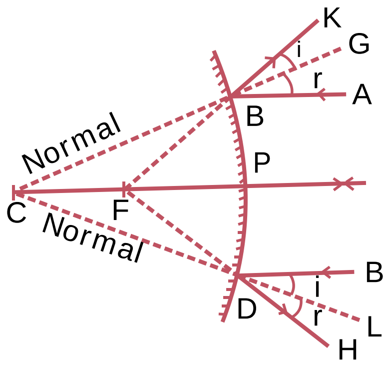
1. Any ray of light travelling parallel to the principal axis of a concave mirror, after reflection passes through the principal focus of the mirror.

2. A ray of light which first passes through principal focus, after reflection, will travel parallel to the principal axis of a concave mirror.

3. A ray of light which first passes through the centre of a curvature of concave mirror, after reflection, will retrace its path. It is because the ray strikes the mirror surface at right angles.

**Convex Mirror As Diverging Mirror:** Consider a convex mirror, having centre of curvature at point C, such that AB and ED are the two rays of light, travelling parallel to the principal axis, meeting the mirror surface at points B and D respectively. Join CB and CD and produce them forward to G and L respectively such that BG and DL act as normal at the points of incidence B and D respectively.

Draw reflected rays BK and DH, such that the angle of incidence is equal to the angle of reflection. Produce BK and DH backward. It is seen that these rays meet the principal axis at point F. This point is called principal focus of the convex mirror.




It is found that all rays coming parallel to principal axis, after reflection, appear to meet at point F when extend in backward. In other words, the reflected rays appear to diverge out from point F. Hence, convex mirror is called a diverging mirror.

**Conclusion:**

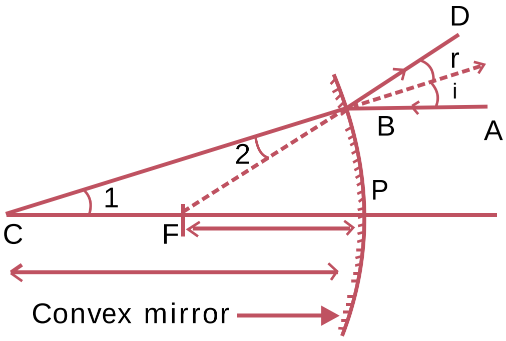
1. Any ray of light travelling parallel to principal axis of a convex mirror, after reflection appears to diverge from the principal focus of the convex mirror.

2. Any ray of light which travels along principal focus of a convex mirror, after reflection from it, will travel parallel to the principal axis.

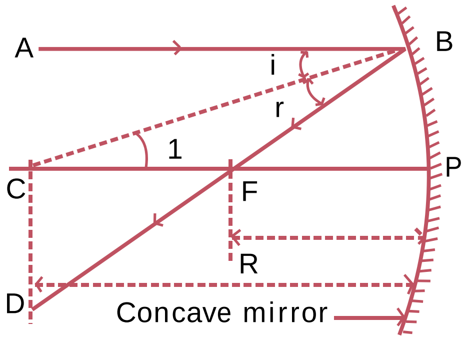
3. Any ray of light which travels along centre of curvature of a convex mirror, after reflection from it, retraces its path. It is because, it strikes the mirror at right angle.

**Relation between focal length (f) and radius of curvature (R):**

Consider a spherical mirror, such that 'C' is its centre of curvature. AB is a ray of light, incident at point B and is travelling parallel to principal axis. After reflection this ray is reflected along BD, cutting principal axis at point F (In case of convex mirror BD is produced backward), which is the principal focus of spherical mirror.




**(a)**




**(b)**

**For convex mirror: (Figure – a)**

$\angle i = \angle r$ (By the laws of reflection)

But, $\angle i = \angle 1$ (Pair of corresponding angles)

and, $\angle r = \angle 2$ ( Pair of vertically opposite angle)

$\therefore \angle 1 = \angle 2$

$\therefore$ CF = BF ––––– (i)

If the point B is very close to the point P, i.e., the linear aperture of the mirror is very small then; (i.e., only for paraxial rays)

BF = PF ––––(ii)

Comparing (i) and (ii); CF = PF, where PF =f

Also, PC = CF + PF $\Rightarrow$ PC = PF + PF ($\therefore$ PF = CF)

$\Rightarrow$ PC = 2PF $\therefore$ R = 2f or $f = \frac{R}{2}$

**For concave mirror: (Figure – b)**

$\angle i = \angle r$ ( By the laws of reflection)

But, $\angle i = \angle 1$ (Pair of alternate angles)

$\therefore \angle 1 = \angle r$

Now, In $\Delta CFB$, $\angle 1 = \angle r$ $\therefore$ CF = FB

If the point B is very close to the point P, i.e., the linear aperture of the mirror is very small then ; FB = PF. CF = PF, where PF = f

Aslo PC = CF + PF $\Rightarrow$ PC = PF + PF

$\therefore$ R = f + f ( $\therefore$ PF = CF)

$$R = 2f \text{ or } f = \frac{R}{2}$$

Thus, we can say that focal length of spherical mirror is half of its radius of curvature **or**

Principal focus of a spherical mirror lies midway, between the pole and centre of curvature of that mirror.

## Images formed by a concave mirror for different positions of the object:

| Position of the object | Position of the image | Nature and size of the image |
|---|---|---|
| At infinity | At the focus (F) | Real, inverted and very small (highly diminished) |
| Beyond the centre of curvature (C) | Between the focus(F) and centre of curvature(C) | Real, inverted and diminished |
| At the centre of curvature (C) | At the centre of curvature (C) | Real, inverted, same size as the object |
| Between the centre of curvature (C) and focus(F) | Beyond the centre of curvature (C) | Real, inverted, bigger than the object(magnified) |
| At the focus (F) | At infinity | Real, inverted and enlarged (highly magnified) |
| Between the focus (F) and Pole (P) | Behind the mirror | Virtual, erect and enlarged (magnified) |

## Images formed by a convex mirror for different positions of the object

| Position of the object | Position of the image | Nature and size of the image |
|---|---|---|
| Between the focus(F) and infinity | Behind the mirror between P and F | Virtual, erect and diminished |
| At the focus(F) | Behind the mirror at a distance of f/2 from the pole | Virtual, erect and diminished |
| Between the focus(F) and the pole (P) | Behind the mirror between P and F | Virtual, erect and diminished |
| At infinity | Behind the mirror at the focus (F) | Virtual, erect and highly diminished (point size) |

## STAR FACT

**Why is the shadow bigger than the object ?**

The shadow is bigger than the object only when the object (o)is nearer to the source of light. Let us consider a point source(P) in which the light emanates from almost a single point and goes out radially in all directions. As light travels only in straight line paths if obstructed by any object it creates a shadow. If we consider the light source to be at infinite distance, the light rays reaching the object will be parallel to one another and cause a shadow with the same size as that of the object.

If the light source is nearer to the object, only the light rays emitted at an angle greater than the angle made by the line joining the point source and a point on the corner of the object would go unobstructed by the object. So the rays travelling within this angle gets obstructed and a bigger shadow is cast. As the distance between the object and the source decreases, the angle of the rays to go unobstructed also increases ie the size of the shadow also increases.

**Mirror Formula – Relation Between Focal Length, Object Distance And Image Distance of Mirror:**

**Definition:** The equation relating the object distance, the image distance and focal length of the mirror is called the mirror formula.

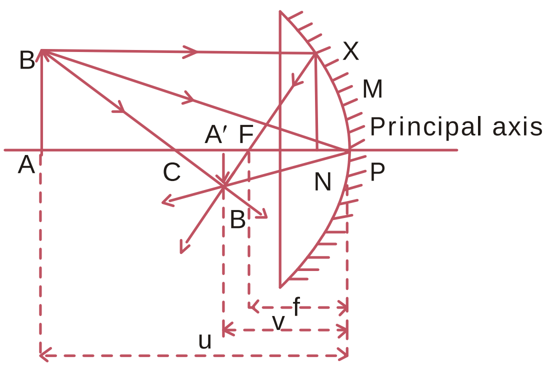
**Spherical mirror formula:** It is a relation between object distance 'u' image distance 'v' and focal length of 'f' of a spherical mirror. The relation is

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}.$$

**Derivation:**

**Mirror Formula for Concave Mirrors:**

**Description:** The diagram shows the principal section of a concave mirror M forming a real and inverted image $A'\ B'$ of a real and erect object AB. The object is beyond C, while the image is between F and C.




ray diagram for a concave mirror real image formula

**Calculation.**

Here, Object distance (measure from P to A)

PA = –u (object on the left of the mirror)

Image distance (measured from P to $A'$)

$PA' = -v$ (image on the left of the mirror)

Focal length (measured from P to F)

PF = – f (focus on the left of the mirror)

In similar $\Delta$s $A'B'F$ and NXF

$$\frac{A'B'}{NX}=\frac{FA'}{FN}=\frac{A'P-FP}{FP}$$ (For mirror of small aperture N is near P, FN = FP)

Putting values, with proper sign

$$\frac{A'B'}{NX}=\frac{-v-(-f)}{-f}=\frac{v-f}{f} \quad \text{---(1)}$$

In similar triangles $A'B'P$ and ABP ($\angle A'PB'=\angle APB$)

$$\frac{A'B'}{AB}=\frac{A'P}{AP}=\frac{-v}{-u}=\frac{v}{u} \quad \text{--(2)}$$

But since, NX = AB $\frac{A'B'}{NX}=\frac{A'B'}{AB}$

Hence, from equation (1) and (2), $\frac{v-f}{f}=\frac{v}{u}$

Cross – multiplying $uv-uf=vf$

Transposing, $-vf-uf=-uv$

Changing sign and dividing by *uvf,* we get $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}=\frac{2}{R}$ as $f=\frac{R}{2}$

**Note:** Mirror formula same for both concave and convex mirror

**Linear Magnification:**

**Definition:** The ratio of the size of the image, as formed by reflection from the mirror to the size of the object, is called linear magnification produced by the mirror. It is represented by the symbol m. If I be the size of the image and O be the size of the object,

Then $m = \frac{I}{O}$

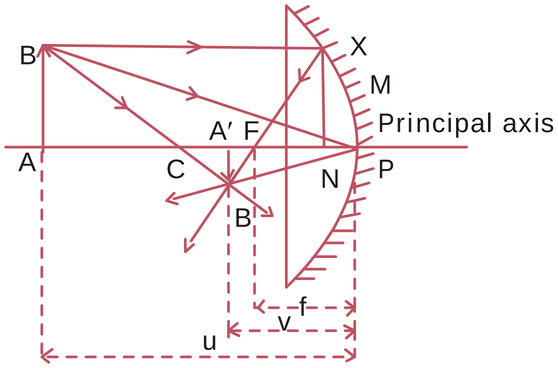
If we represent size of the object by $h_1$ and size of the image by $h_2$

Then, $I = h_2$ and $O = h_1$ Hence, $m = \frac{h_2}{h_1}$

**Expression:**

(i) For concave mirror forming real image

$I = A'B' = -h_2$ (inverted image, $O = AB = +h_1$(erect object)




ray diagram for a concave mirror real image formula

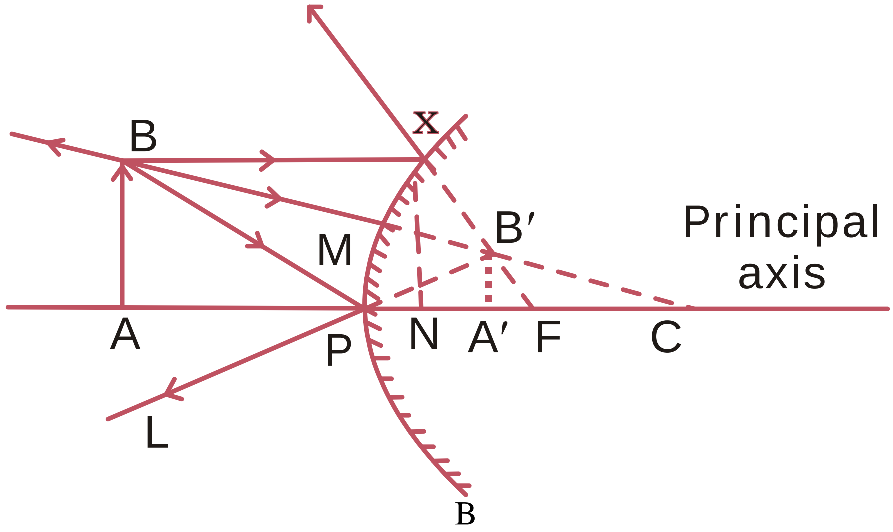
Then, $m = -\frac{h_2}{h_1} = -\frac{A'B'}{AB}$, In similar $\Delta$s $A'B'P$ and ABP

$\frac{A'B'}{AB} = \frac{PA'}{PA}$ Then, $m = -\frac{PA'}{PA} = -\frac{-v}{-u}$ i.e $m = -\frac{v}{u}$

(ii) For convex mirror forming virtual image

$I = A'B' = +h_1$ (erect image)

$O = AB = +h_1$ (erect object)




Ray diagram for a convex mirror which mostly forms a virtual image

Then, $m = \frac{h_2}{h_1} = -\frac{A'B'}{AB}$, In similar $\Delta$s $A'B'P$ and ABP

$\frac{A'B'}{AB} = \frac{PA'}{PA}$ Then, $m = -\frac{PA'}{PA} = -\frac{+v}{-u}$ i.e $m = -\frac{v}{u}$

It is same as for a concave mirror.

Hence, we conclude that, the linear magnification produced by a mirror is equal to the

ratio of the image distance to the object distance with a minus sign.

**Note:** For a real image u and v, both are negative, hence m is negative.

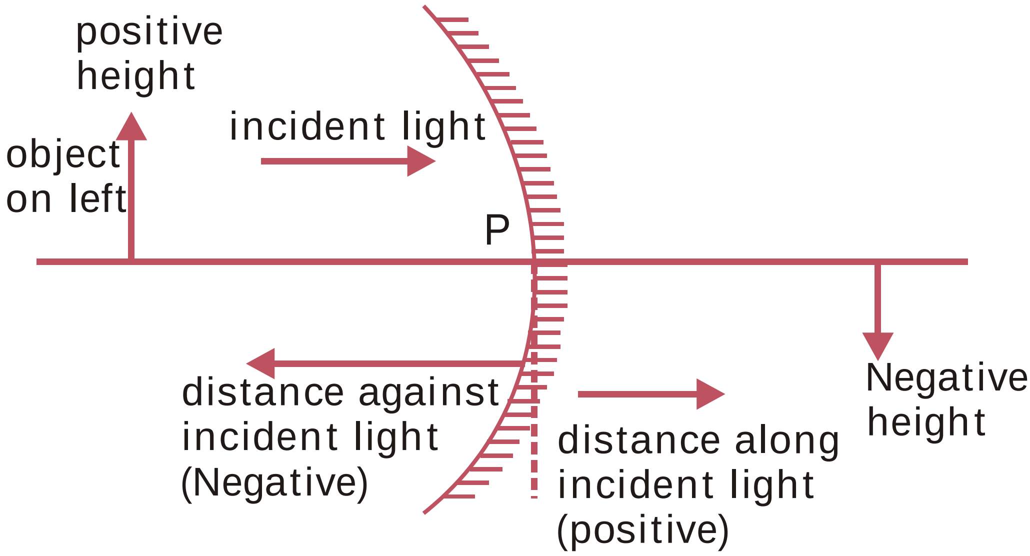
For a virtual image u is negative, v is positive, hence m is positive.

**Areal Magnification :** It is the ratio of the area of the image to the area of the object. It is also called superficial or surface magnification.

Areal magnification = $\frac{area\ of\ image}{area\ of\ object} = \frac{v^2}{u^2}$

**Sign convention for Spherical Mirrors:**

1. All ray diagrams are drawn with the incident light travelling from left to right.
2. All distance are measured from the pole of the mirror.
3. The distances measured in the same direction as that of the incident light are taken as positive.




New cartesian sign covention

4. The distances measured in the direction opposite to the direction of the incident light are taken as negative.

5. Distances measured upward and normal to the axis are taken as positive and those measured perpendicularly downward as negative.

**Note:** (i) On the basis of above sign convention, the focal length as well as radius of curvature of a convex mirror are positive and that of a concave mirror are negative.

(ii) For a convex mirror the image distance v is positive and the object distance u is negative.

(iii) For a concave mirror the value of u is always negative but the value of v is negative when the image formed is real and is positive when the image is virtual.

**or**

Following sign convention is used for measuring various distances in the ray diagrams of spherical mirrors.

(i) All distances are measured from the pole of the mirror.

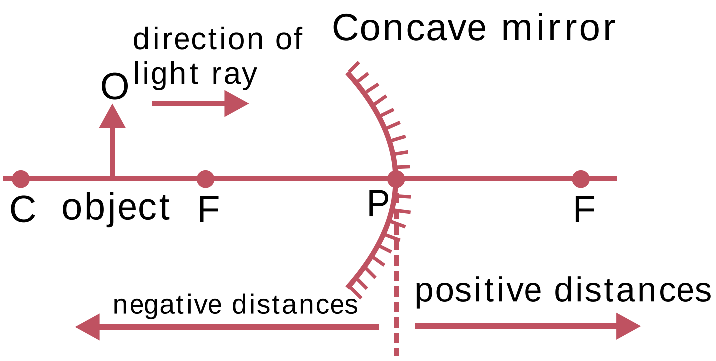
(ii) Distances measured in the direction of the incident ray are positive.

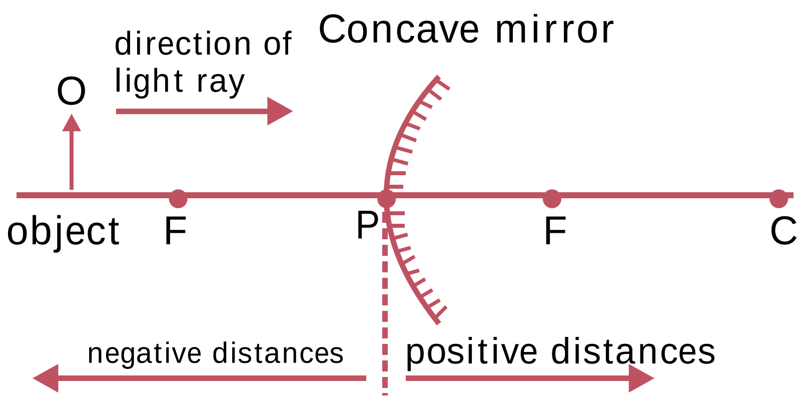
(iii) Distances measured in the direction opposite to that of the incident rays are negative.

(iv) Distances measured above the principal axis are positive i.e. height of an object and of an erect image are positive.

(v) Distance measured below the principal axis are negative, i.e. height of a real inverted image is negative.

This sign convention for a concave and a convex mirror are shown below in the table




**LAUGH & LEARN**

One day sad looking atom, sitting in a hotel, was accosted by another :
" why do you look so sad"?
"I lost an electron"
"Are you sure"
"I am positive"

**Sign convention for spherical mirrors**

**Conclusions from the sign Convention**

| For concave mirror | | For convex mirror | |
|---|---|---|---|
| Distance of the object | u is negative | Distance of the object | u is negative |
| Distance of the real image | v is negative | Distance of the virtual image | v is negative |
| Distance of the virtual image | v is positive | Focal length | f is positive |
| Focal length | f is negative | Radius of curvature | R is negative |
| Radius of curvature | R is negative | Height of the object | O is positive |
| Height of the object | O is positive | Height of the inverted (erect) image | I is positive |
| Height of the inverted (real) image | I is negative | | |
| Height of the erect (virtual) image | I is positive | | |

# OPTICS_WORKSHEET-2

CUQ 1. A spherical mirror whose inner hollow surface is the reflecting surface is called

1) concave mirror 2) convex mirror

3) plane mirror 4) none of these

2. The centre of the speherical mirror is called

1) focus 2) pole 3) centre of curvature 4) none of these

3. Any ray of light travelling parallel to the principal axis of a concave mirror, after reflection passes through

1) pole of mirror 2) focus of mirror

3) centre of curvature of mirror 4) in between pole and focus of mirror

4. Virtual effect and enlarged image will be formed by a concave mirror when object is placed in between

1) infinity and centre of curvature 2) focus and centre of curvature

3) pole and focus 4) at centre of curvature

5. When object is placed between centre of curvature (c) and focus (f) of a concave mirror the nature and site of the image is

1) real, inverted and very small

2) real, inverted, same site of object

3) real, inverted and bigger than the object

4) virtual, erect and enlarged

6. Real, inverted and same site as the object, image will be formed by concave mirror, when the object is at

1) centre of curvature 2) focus

3) between focus and pole 4) beyond centre of curvature

7. All distances in case of spherical mirrors are measured from

1) Focus of mirror 2) Centre of curvature of mirror

3) Principal axis 4) Pole of mirror

8. On the basis of sign convention, the focal length as well as radius of curvature of a convex mirror is

1) Positive 2) Negative

3) Positive or negative 4) None of these

9. The ratio of area of the image to the area of the object is called

1) Magnification 2) Linear magnification

3) Superficial magnification 4) None of these

10. On the basis of sign convention, the focal length as well as radius of curvature of a concave mirror is

1) Positive 2) Negative

3) Both 4) None of these

## JEE MAIN & ADVANCED

**LEVEL-1 Single Correct Choice Type:**

1. The image formed by a convex mirror of real object is larger than the object.

1) When $u < 2f$ 2) When $u > 2f$

3) for all values of u 4) for no value of u

2. The diameter of spherical mirror in which reflection takes place is called

1) radius of curvature 2) centre of curvature

3) linear aperture. 4) focal length.

3. When a convex mirror of focal length 'f' produces an image $\left(\frac{1}{n}\right)$th of the size of the object,then distance of the object from the mirror is

1) $\frac{f}{n}$ 2) $(n+1)f$ 3) $(n-1)f$. 4) $fn$

4. A ray deviates $90^0$ after suffering reflection from a mirror. The angle of incidence is

1) 90° 2) 30° 3) 45° 4) 60°

5. An object is placed at a distance $2f$ from the pole of a convex mirror of focal length $f$. The liear magnification is

1) $\frac{1}{3}$ 2) $\frac{2}{3}$ 3) $\frac{3}{4}$ 4) 1

6. The image of an object placed in front of a concave mirror of focal length 12 cm is formed at a point which is 10 cm more distant from the mirror that the object. The magnification of the image is

1) 1.5 2) 2 3) 2.5 4) 3

**Multi Correct Choice Type:**

7. which of the following forms a virtual and errect image for all positions of a real object

1) plane mirror 2) convex mirror. 3) concave mirror 4) all the above

8. For a convex mirror according to sign convention which of the following is/are positive.

1) focal length 2) object distance
3) image distance 4) radius of curvature

**Statement Type:**

9. Statement I : Any ray of light which travels along centre of curvature of a convex mirror after reflection, from it retraces its path

Statement II : Any ray of light which travels along centre of curvature of a convex mirror after reflection, it will not retrace its path

1) Both Statements are true,
2) Both Statements are false.,
3) Statement - I is true, Statement - II is false.
4) Statement - I is false, Statement - II is true.

**Integer Answer Type:**

10. A 5 cm long object is placed at 30cm infront of a convex mirror of radius of curvature 40 cm. The size of the image is __________ cm.

**Matrix Match Type:**

11. For a concave mirror focal length is 9 cm and radius of curvature is 18 cm. Object distance is u.

| Column - I | Column - II |
|---|---|
| a) $u < 9$ cm | 1) Real image |
| b) $u = 9$ cm | 2) Virtual image |
| c) 9 cm $< u < 18$ cm | 3) inverted image |
| d) $u = 18$ cm | 4) errect image |
| | 5) diminished |

**LEVEL-2 & 3 Single Correct Choice Type:**

12. An object 5 cm tall is placed 1 m from a concave spherical mirror which has a radius of curvature of 20 cm. The size of the image is

1) 0.11 cm 2) 0.50 cm 3) 0.55 cm 4) 0.60 cm

13. At what distance from a convex mirror of focal length 2.5m should a boy stand so that his image has a height equal to half of the original height? The principal axis is perpendicular to the height

1) 5 m 2) 7.5 m 3) 10 m 4) 2.5 m

**Multi Correct Choice Type:**

14. Choose the correct statements

1) convex mirror acts as diverging mirror.

2) image formed by convex mirror is virtual

3) for a concave mirror when object is placed at infinity the image is formed at principal focus.

4) convex mirror acts as converging mirror.

15. A small candle 2.5 cm in size is placed 27 cm in front of a concave mirror of radius of curvature 36 cm.

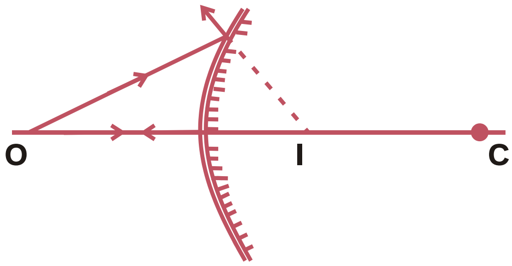
1) the distance from the mirror should a screen be placed in order to receive a sharp image is – 54 cm

2) the nature of image is real inverted w.r.t. object

3) the image formed is 2 times highest the object.

4) the image formed is 3 times highest the object.

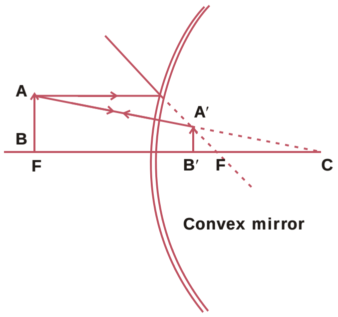
16. A convex mirror has its radius of curvature 30cm. Find the position of the image of an object placed at a distance of 18cm from the mirror




1) $\frac{50}{11}$cm 2) $\frac{60}{11}$cm 3) $\frac{90}{11}$cm 4) 90cm

**Matrix Match Type:**

17. Image formed when the object is placed at the focus(F) of the convex mirror.




| Column - I | Column - II |
|---|---|
| a) image position | 1) virtual and diminished. |
| b) image size | 2) errect and diminished |
| c)image nature | 3) Diminished |
| d) image formation(side) | 4) behind the mirror |
| | 5) at a distance of f/2 from the pole behind the mirror |

**LEVEL-4 & 5** **Single Correct Choice Type:**

18. An object is placed at a distance of 40 cm in front of a concave mirror of focal length 20cm. The image produced is

    1) virtual and inverted 2) real and erect 3) real, inverted and diminished

    4) real, inverted and of the same size as the object.

19. A 1cm object is placed perpendicular to the principal axis of a convex mirror of focal length 7.5 cm. Find its distance from the mirror if the image formed is 0.6 cm in site.

    1) 10 cm 2) 8 cm 3) 6 cm 4) 5 cm

**Integer Answer Type:**

20. An object is placed in front of a convex mirror at a distance of 50cm. A plane mirror is introduced covering lower half of the convex mirror. If the distance between the object and the plane mirror is 30cm, it is found that there is no parallax between between the images formed by the two mirrors. The radius of curvature of the convex mirror ______ cm

**Multi Correct Choice Type:**

21. Which of the following statements is/are incorrect ?

    1) for concave mirror when object is placed at the focus the image is formed at a distance of $\frac{f}{2}$ from the pole.

    2) for concave mirror when object is placed at the focus the nature of image is virtual.

    3) for concave mirror when object is placed at the focus the size of the image is erect and diminised.

    4) for convex mirror when the object is placed at the focus the image is formed at infinity.

# OPTICS

## KEY,HINTS AND SOLUTIONS

## WORKSHEET-1

**CUQ**

1. Ans : 2

   Angle B is the angle of incidence (angle between the incident ray and the normal). Angle C is the angle of reflection (angle between the reflected ray and the normal).

2. Ans: 4

   The angle of incidence is the angle between the incident ray and the normal. As this angle approaches 90 degrees, the reflected ray also approaches a 90 degree angle with the normal; thus, the angle between the incident and reflected ray approach 180 degrees.

3. Ans: 1

   The angle of reflection is 60 degrees. (Note that the angle of incidence is not 30 degrees; it is 60 degrees since the angle of incidence is measured between

   the incident ray and the normal.)

4. Ans: 1

5. Ans: 3

   When you look at your image in a plane mirror, you see an upright image; it is located on the other side of the mirror (and thus is virtual); finally, it has the same dimensions (height, width) as yourself (the object).

6. Ans: 3

   If you stand 3 feet from the mirror, then your image is three feet on the other side of the mirror; this puts your image a total of six feet from you (3 feet to the mirror plus 3 more feet to the image).

7. Ans: 2

   For plane mirrors, the image distance is the same as the object distance ($d_i = -d_o$). The only way to modify the image distance is to modify the object distance.

8. Ans: B

   The portion of mirror required to view the full image of an object is always one-half the height of the object (for plane mirrors only).

9. Ans:4

10. Ans: 2

Consider two mirrors arranged at a 0-degree angle - parallel to each other. There would be an infinite number of images, one located directly behind the other forming a line which seems to extend forever. Suppose the mirrors are adjusted to a 30-degree angle. The mirrors would create 5 images of objects placed between the mirrors. Now suppose that the angle is increased to a 90-degree angle, The mirrors would create 3 images of objects placed between the mirrors.. Finally, suppose the mirrors are adjusted to a 180-degree angle to form a single plane. At such an angle, only one image would be formed.

As the angle between the plane mirrors is increased, the number of images decreases.

## JEE MAIN & ADVANCED

1. Ans: 3

   For a plane mirror, Object distance = Image distance

2. Ans: 1

   (i) When the incident ray is fixed and mirror rotates through angle $20^0$ clock wise then reflected ray rotates clock wise through $40^0$ angle.

   (ii) when mirror (M) in fixed and incident ry rotates through angle $10^0$ clockwise then reflected ray rotates through angle $10^0$ anticlockwise.

   Total angle turned by the reflected ray $= 40^0$ clockwise + $10^0$ anti clockwise $= 30^0$ ( clockwise )

3. Ans: 2

   If the plane mirror is rotated in its own plane, then the direction of reflected ray does not change.

4. Ans: 3; $n = \frac{360}{\theta} - 1$

5. Ans: 3; $D = 360 - 2\theta$ $n = \frac{360}{\theta} - 1$

6. Answer: 1,2,5,7

   1) is true; like any plane mirror image, the image formed by two mirrors oriented at right angles is the same size as the object.

   2) is true; the middle image is the same distance from the crease of the dual mirror system as the image is from the crease. However, image distances are usually measured as the smallest distance measured perpendicularly to the mirror. For middle images, this distance would be measured to an extended mirror line and it would be the same distance as that from the object to that same mirror.

   3) is false; the middle image, unlike the primary images, does not exhibit this feature of left-right reversal.

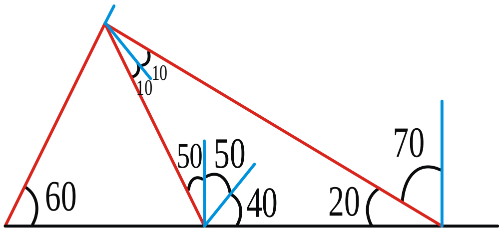
   4) is false; the middle image is a virtual image; it is upright and located behind the mirror (like any virtual image).

   5)is true; just look at the middle image and you will see an upright image.

6)is false; a magnification of -1 means that the image would be inverted (negative M) and the same size as the object (1); the middle image is the same size as the object but it is not inverted.

7) is true; light will reflect twice - once off each mirror - prior to traveling to the viewer's eye.

7. Ans:(1)60 m/s
8. Ans:(2)80 m/s
9. Ans:(3)40 m/s
10. Ans : a - s ; b - s ; c - r ; d - r
11. Ans: 3




12. Ans: 4; $d = 360 - 2i$

13. Ans: 1

Take $\rightarrow$ as + direction. $v_1 - v_m = v_m - v_0$ $v_1 - (-1) = (-1) - 5$

$\therefore$ $v_1 = -7\,m/s$ and direction towards left.

14. Ans:(4)
15. Ans: a-r; b-p; c-q; d-t

16. Ans: 3

. Mirror $L_2$ prouduces image $I_1$ at 1m from it.

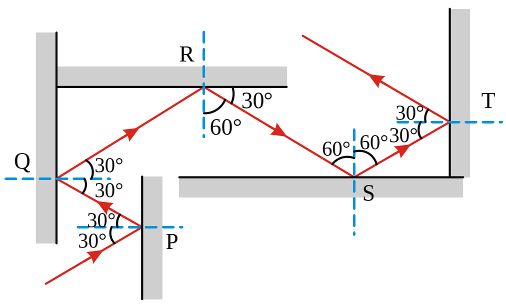
Mirror $L_1$ produces image $I$ at 2m from it.

$I$ is at $(2+2+1)$m from $L_2$ and acts as object for $L_2$ to produce $I_2$

$\therefore$ distance of $I_2$ from the person is 6 m.

17. Ans: 3; $I_1$ is the object for $M_1$ to get $I_3$.

18. Ans:3




19. Ans:2

The incident ray $=(\hat{i}+\hat{j})-\hat{k}$; the reflected ray can be found by reversing the sign of the component along the normal.

20. Ans:2; x-coordinate inverted

21. Ans: (2, 4) Light keeps falling on the same spot.

22. Ans: (a) – s; (b) – q; (c) – p; (d) – q

## WORKSHEET-2

**CUQ**

| | | | | |
|---|---|---|---|---|
| 1) 1 | 2) 2 | 3) 2 | 4) 3 | 5) 3 |
| 6) 1 | 7) 4 | 8) 1 | 9) 3 | 10) 2 |

### JEE MAIN & ADVANCED

| | | | | |
|---|---|---|---|---|
| 1) 4 | 2) 3 | 3) 3 | 4) 3 | 5) 1 |
| 6) 1 | 7) 1,2 | 8) 1,3,4 | 9) 3 | 10) 2 |
| 11) a-2,4;b-1,3;c-1,3;d-1,3 | | 12) 3 | 13) 4 | 14) 1,2,3 |
| 15) 1,2,3 | 16) 3 | 17) a-4,5;b-1,2,3;c-1,2,3;d-4,5 | | |
| 18) 4 | 19) 4 | 20) 25 | 21) 1,2,3,4 | |

### HINTS & SOLUTIONS

12. $m=\frac{v}{u}=\frac{nu}{u}$ ; $\frac{1}{f}=\frac{(n-1)}{nu}\Rightarrow u=\frac{(n-1)}{n}f$

16. $u=-18cm, R=+30cm$

$$\frac{1}{u}+\frac{1}{v}=\frac{2}{30}\Rightarrow\frac{1}{v}=\frac{2}{30}-\frac{1}{-18}\Rightarrow\frac{1}{v}=\frac{6+5}{90}=\frac{11}{90}\Rightarrow v=\left(\frac{90}{11}\right)cm$$ **(or)**

f= 15 cm $u=-18cm, R=+30cm$

$$\frac{1}{15}=\frac{1}{V}-\frac{1}{18}\Rightarrow\frac{1}{V}=\frac{1}{15}+\frac{1}{18}\Rightarrow\frac{1}{V}=\frac{33}{15\times18}\Rightarrow V=\frac{90}{11}\ cm$$

18. $m_A=\frac{I_A}{O_A}; m_B=\frac{I_B}{O_B}$ where $I_A=I_B$ and $O_A=3O_B$

$$\therefore \frac{m_A}{m_B}=\frac{I_A}{I_B}\times\frac{O_B}{O_A}=1\times\frac{1}{3}=\frac{1}{3},$$ But $m_A=\frac{f}{f-u_A}$ and $m_B=\frac{f}{f-u_B}$

$$\therefore \frac{m_A}{m_B}=\frac{f-u_B}{f-u_A},$$ Here $\frac{m_A}{m_B}=\frac{1}{3}; f=-7.5cm; u_A=-30cm$

$$\therefore \frac{1}{3}=\frac{-7.5-ug}{-7.5-(-30)}$$ or $u_B=-15cm$