



# PHYSICS

## Class – IX

### Contents

MOTION

NEWTON'S LAWS OF MOTION

CONSERVATION OF LINEAR MOMENTUM

COLLISIONS

GRAVITATION

FLUID MECHANICS

WORK, POWER & ENERGY

RAY OPTICS

# INDEX

S.NO.	CONTENT DETAILS	PAGE NO.
<b>MOTION</b>		
1.	Synopsis & Worksheet -1	1 - 15
2.	Synopsis & Worksheet -2	15 - 20
3.	Synopsis & Worksheet -3	21 - 27
4.	Synopsis & Worksheet -4	28 - 38
	HINTS AND SOLUTIONS	39 - 43
<b>NEWTON'S LAWS OF MOTION</b>		
5.	Synopsis & Worksheet -1	44 - 52
6.	Synopsis & Worksheet -2	53 - 59
	HINTS AND SOLUTIONS	60 - 60
<b>CONSERVATION OF LINEAR MOMENTUM</b>		
7.	Synopsis & Worksheet -1	61 - 68
8.	Synopsis & Worksheet -2	68 - 71
	HINTS AND SOLUTIONS	72 - 76
<b>COLLISIONS</b>		
9.	Synopsis & Worksheet -1	77- 86
10.	Synopsis & Worksheet -2	86 - 92
	HINTS AND SOLUTIONS	93 - 95
<b>GRAVITATION</b>		
11.	Synopsis & Worksheet -1	96- 104
12.	Synopsis & Worksheet -2	105 - 111
	HINTS AND SOLUTIONS	112 - 112

**FLUID MECHANICS**

13.	Synopsis & Worksheet -1	113- 126
14.	Synopsis & Worksheet -2	127 - 132
	HINTS AND SOLUTIONS	132 - 134

**WORK, POWER & ENERGY**

15.	Synopsis & Worksheet -1	135- 141
16.	Synopsis & Worksheet -2	142 - 147
17.	Synopsis & Worksheet -3	147 - 156
	HINTS AND SOLUTIONS	157 - 157

**RAY OPTICS**

18.	Synopsis & Worksheet -1	158- 174
19.	Synopsis & Worksheet -2	174 - 183
20.	Synopsis & Worksheet -3	183 - 190
21.	Synopsis & Worksheet -4	191 - 203
22.	Synopsis & Worksheet -5	203 - 208
	HINTS AND SOLUTIONS	208 - 217

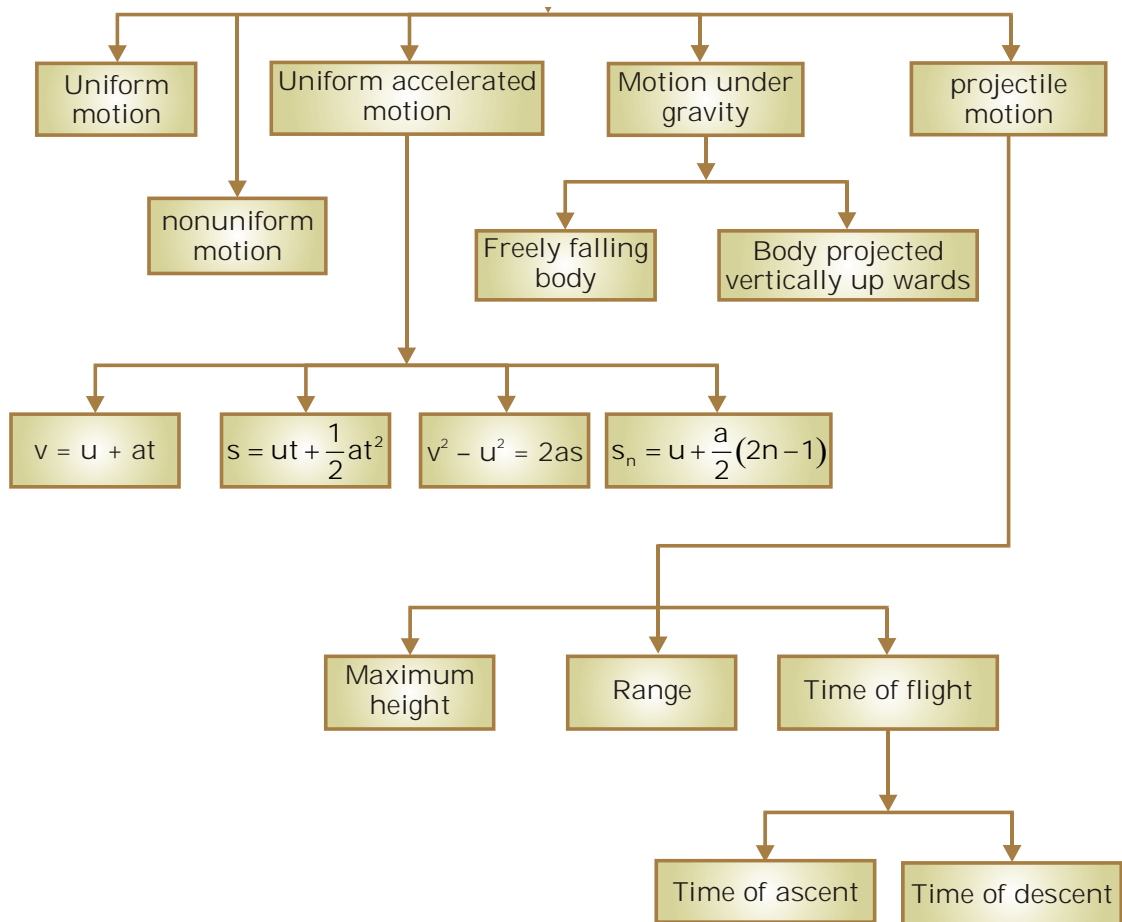




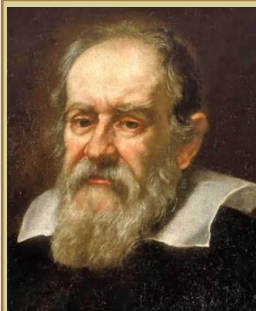
# MOTION

## MEMO GRAPH

### MOTION



## KNOW YOUR SCIENTIST



Galileo Galilei  
(1564 - 1642)

## Galileo Galilei (1564 - 1642)

Galileo Galilei was a key figure in the scientific revolution in Europe about four centuries ago. Galileo proposed the concept of acceleration. From experiments on motion of bodies on inclined planes or falling, freely, he contradicted the Aristotelian notion that a force was required to keep a body in motion and that heavier bodies fall faster than lighter bodies under gravity. He thus arrived at the law of inertia that was the starting point of the subsequent epochal work of Isaac Newton. Newton brought out another masterpiece optics that summarized his work on light and colour.

# MOTION

## MOTION\_SYNOPSIS-1

1. **Mechanics** : The branch of physics which deals with the motion of objects in everyday life is called mechanics.
2. It is divided into , Kinematics, Dynamics and Statics.
  - i) **Kinematics** : Kinematics which is derived from a Greek word kinema meaning motion, is a branch of physics, which deals with the motion of a body without taking into account the cause of motion.
  - ii) **Dynamics** : Dynamics , which is derived from the Greek word dyna meaning power, is a branch of physics which deals with the motion of bodies by taking into account the cause of motion (force).
  - iii) **Statics** : Statics deals with bodies at rest under the effect of different forces.
3. **Point object** :  
An object is said to be a point object if its size is very small as compared to the distance travelled by it in the given time interval.  
**Ex :** i) A bus travelling a distance of 100 km can be considered as a point object. This is because the size of the bus is very small as compared to distance travelled by it.  
ii) A car travelling a distance of 50 km can be considered as a point object. This is because the size of car is very small as compared to the distance travelled by it.
4. **Reference point** : A fixed point or a fixed object with respect to which the given body changes its position is known as reference point.
5. **Rest** : A body is said to be at rest if it does not change its position with respect to the reference point. The objects which remain stationary at a place and do not change their position are said to be at rest.

**Ex :** A chair lying in a room is in the state of rest, because it does not change its position with respect to the surroundings of the room.

7. **Motion :** A body is said to be in motion if it changes its position with respect to the surroundings with the passage of time. All moving things are said to be in motion.

**Ex :** A car is changing its position w.r.t trees, houses etc. is in the state of motion.

8. **Rest and motion are relative terms :** Rest and motion are relative terms. A body can be at rest as well as in motion at the same time. When we say that a body or an object is in motion, then it is essential to see whether the body or object changes its position with respect to other bodies or objects around it or with respect to any fixed point known as reference point. For example, when a bus moves on a road, then the bus as well as the passengers sitting in it change their position with respect to a person standing on the road side. So, the bus and the passengers sitting in it are in motion with respect to the person standing on the road side. However, the passengers sitting in the bus do not change their positions with respect to each other. It means, the passengers sitting in a moving bus are not in motion with respect to each other.

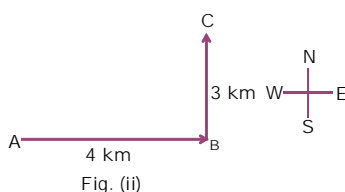
**Ex :** A person sitting in the compartment of a moving train is in the state of rest, with respect to the surroundings of compartment. Yet he is in the state of motion, if he compares himself with surroundings outside the compartment.

**Note :** In order to describe the motion of an object we need to keep in mind three things.

1. The distance of the body from a reference point. This reference point is called the origin of the motion of the body.
2. The direction of motion of the body.
3. The time of motion.

### Terms related to Kinematics :

**Distance :** The distance travelled by a body is actual length of the path covered by a moving body irrespective of the direction in which the body travels.



(i) Suppose a man lives at place A as in figure (ii), and he has to reach another place C, but first he has to meet his friend living at place B.

(ii) Now, the man starts from point A and travels a distance of 4 km to reach B towards East, and then travels another 3 km from B to reach C toward North.

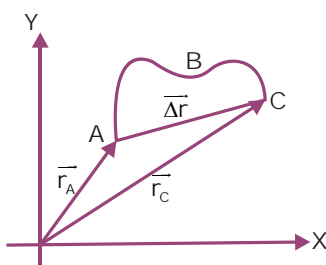
(iii) Thus, the total length of the path i.e.  $AB + BC = 4 \text{ km} + 3 \text{ km} = 7 \text{ km}$  gives the distance travelled by the man and it has no specific direction.

**Displacement** : When a body moves from one position to another, the shortest distance between the initial position and final position.

**Note :**

1. The displacement of an object in a given interval of time can be positive, zero or negative whereas distance travelled is always positive.
2. Distance is a scalar quantity whereas displacement is a vector quantity.

**Displacement (Vector notation)** : We know distance is the actual path length covered by a moving particle or body in a given time interval, while displacement is the change in position vector, i.e., a vector joining initial to final position. If a particle moves from A to C through a path ABC.



Then distance ( $\Delta s$ ) travelled is the actual path length ABC, while the displacement

$$\text{is } \Delta \vec{r} = \vec{r}_C - \vec{r}_A$$

If a particle moves in a straight line without change in direction, the magnitude of displacement is equal to the distance travelled otherwise it is always less than it. Thus  $|\text{displacement}| \leq \text{distance}$ .

1. **Speed :**

The distance covered by a body in unit time is called **Speed**.

2. **Mathematical Expression of Speed** : The speed can be found by dividing the distance covered by the time in which the distance is covered. i.e.,  $\text{Speed} = \frac{\text{distance}}{\text{time}}$

3. **Units of Speed**: C.G.S unit of speed is centimetre per second (cm/s).

S.I. unit of speed is metre per second (m/s).

Bigger unit of speed is km/hr. The relation between km/hr and m/s is

$$1 \text{ kmph} = \frac{1 \text{ km}}{1 \text{ hr}} = \frac{1000 \text{ m}}{3600 \text{ s}} = \frac{5}{18} \text{ m/s} \therefore 1 \text{ km/hr} = \frac{5}{18} \text{ m/s}$$



## 4 Types of Speed

**Uniform Speed :** If a body covers equal distances in equal intervals of time (however small the time intervals may be), then the body is said to have uniform speed or constant speed.

**Note :** If the ratio of distance travelled and time taken by a moving body is constant, i.e.,  $\frac{\text{distance}}{\text{time}} = \text{constant}$  then we say that the body is moving with uniform speed.

**Non-Uniform Speed:**

If a body covers unequal distances in equal intervals of time (however small the time intervals may be), then the body is said to have non-uniform speed or variable speed.

**Note :** If the ratio of distance travelled and time taken by a moving body is not constant, i.e.,  $\frac{\text{distance}}{\text{time}} \neq \text{constant}$  then we say that the body is moving with variable speed.

**AVERAGE SPEED AND INSTANTANEOUS SPEED :**

**Average speed:** When the body is moving with nonuniform speed, then we calculate its average speed by dividing the total distance covered by the total time taken to cover the distance.

$\therefore$  Average speed =  $\frac{\text{Total distance covered}}{\text{Total time taken to cover the distance}}$

$$v_{av} = \frac{s}{t_2 - t_1}$$

**Instantaneous speed:**

Let  $\Delta s$  be the distance travelled in the time interval  $t$  to  $t + \Delta t$ . The average speed in this time interval is  $v_{av} = \frac{\Delta s}{\Delta t}$

The instantaneous speed at a time  $t$  is defined as

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

where  $s$  is the distance travelled in time  $t$ . The average speed is defined for a time interval and the instantaneous speed is defined at a particular instant. Instantaneous speed is also called "speed".



## SCIENTIFIC FACT

### Does the Sun rotate ?

Yes, the sun too rotates about its axis. But unlike the earth, which has rotation period of one day, the sun has a 'differential rotation'.

That is, all parts of the sun do not have the same period of rotation. The period of rotation near its equator is 26.9 days, at sun spot zone (16 degrees north) it is 27.3 days and at the pole it is 13.1 days (synical).

**Introduction to Velocity :** The rate of change of displacement is called velocity.

**Mathematical Expression of Velocity :** Velocity of a body can be found by dividing the displacement with time. i.e.,  $\text{Velocity} = \frac{\text{displacement}}{\text{time taken}}$

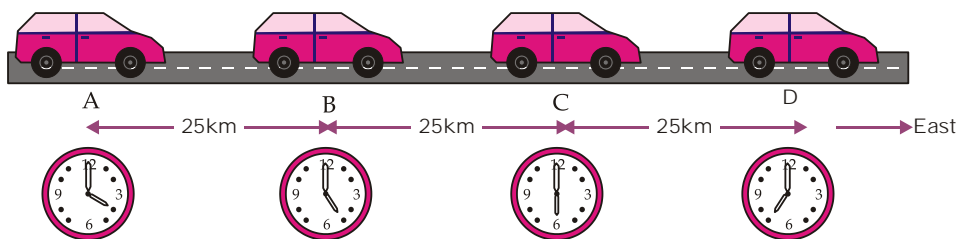
**Units of Velocity :** C.G.S unit of velocity is centimetre per second(cm/s).

S.I. unit of velocity is metre per second (m/s).

**Note:** The units of velocity is the same as the units of speed. However in case of velocity the direction is specified.

### Types of velocity

**Uniform velocity :** Observe a car moving along a straight road towards east.



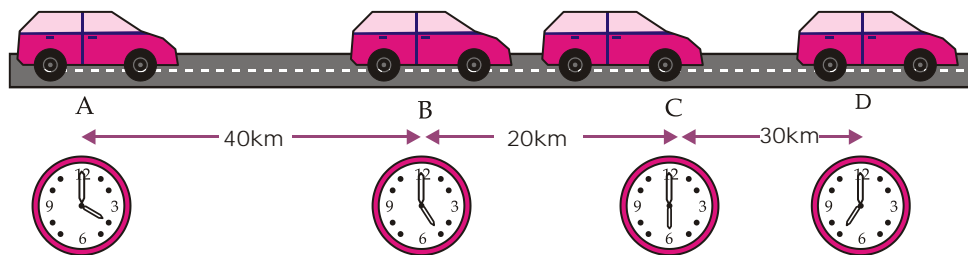
We observe that the car covers equal distances in equal intervals of time in a specified direction.

Here we say that the car is said to be moving with uniform velocity.

**Note:** A body has uniform velocity only if :

- i) If it covers equal displacements in equal intervals of time
- ii) Its direction of motion remains the same.

**Non Uniform velocity :** Observe a car moving on a straight road:



We observe that the car covers unequal displacements in equal intervals of time [or equal displacements in unequal intervals of time (however small these intervals may be)].

Here we say that the car is said to be moving with non uniform velocity or variable velocity.

**Average Velocity :**

When the body moves with non uniform velocity, then we calculate its average velocity by dividing the total displacement by the total time taken.

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time taken}}$$

Mathematically,  $\vec{v}_{av} = \frac{\Delta \vec{S}}{\Delta t}$

**Note:** When a particle moves with different velocities  $\vec{v}_1, \vec{v}_2, \vec{v}_3$  etc., in different time intervals  $\Delta t_1, \Delta t_2, \Delta t_3$  etc. respectively, its average velocity over the total time of motion can be given as

$$v_{av} = \frac{\text{Net displacement vector}}{\text{total time}} = \frac{\Delta \vec{S}}{\Delta t} = \frac{\Delta \vec{S}_1 + \Delta \vec{S}_2 + \Delta \vec{S}_3 + \dots}{\Delta t_1 + \Delta t_2 + \Delta t_3 + \dots}$$

Where the displacement of the particle during time interval  $\Delta t_1, \Delta t_2$  etc. are given as

$$\vec{S}_1 = \vec{v}_1 \Delta t_1, \vec{S}_2 = \vec{v}_2 \Delta t_2 \text{ etc.,} \quad \Rightarrow \quad \vec{v}_{av} = \frac{\vec{v}_1 \Delta t_1 + \vec{v}_2 \Delta t_2 + \dots}{\Delta t_1 + \Delta t_2 + \dots}$$

$$\Rightarrow \quad \vec{v}_{av} = \frac{\sum_{i=1}^{i=n} \vec{v}_i \Delta t_i}{\sum_{i=1}^{i=n} \Delta t_i}$$

**Instantaneous Velocity :**

Instantaneous velocity  $\vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{S}}{\Delta t} \Rightarrow \vec{v} = \frac{\Delta \vec{S}}{\Delta t}$

\* Instantaneous speed is equal to the magnitude of instantaneous velocity.

## ASTONISHING FACT

### Inspite of being 400 times smaller how is the Moon able to mask the Sun ?

The moon is very close to the Earth compared to the sun. By a remarkable coincidence, their sizes and distances are such that for an observer on the Earth, they seem to be of the same size (angular size). If the Moon is at a far off distance, it will not be able to completely cover the sun but cast only a shadow on it.

### Introduction to Acceleration :

The increase in velocity per unit time is called acceleration.

The decrease in velocity per unit time is called deceleration or retardation.

Negative acceleration is called Retardation or Deceleration.

### Mathematical Expression of Acceleration :

$$\therefore \text{Rate of change of velocity} = \frac{\text{Total change in velocity}}{\text{Total time required for change}}$$

But the rate of change of velocity is called acceleration.

$$\text{i.e., Acceleration} = \frac{\text{Change in velocity}}{\text{time taken}}$$

In the above case the acceleration of the car is  $20\text{kmhr}^{-2}$ . It means that for every one hour the increase in velocity is  $20\text{km/hr}$ .

**Units of Acceleration :** C.G.S. unit of acceleration is  $\text{cms}^{-2}$ .

S.I. unit of acceleration is  $\text{ms}^{-2}$ .

### Uniform acceleration :

An object is said to be moving with uniform acceleration if its velocity changes by equal amounts in equal intervals of time.

## MOTION\_WORKSHEET-1

**CUQ** 1. If the distance covered by a particle is zero, what can you say about its displacement

- |                                |                      |
|--------------------------------|----------------------|
| 1) It may (or) may not be zero | 2) It cannot be zero |
| 3) It is negative              | 4) It must be zero   |

2. If the displacement of a particle is zero, distance covered by it
- 1) May (or) may not be zero
  - 2) Must be zero
  - 3) Is negative
  - 4) All are true
3. A man leaves his house for a cycle ride. He comes back to his house after one hour after covering a distance of 2 km. Then the displacement of a man is \_\_\_\_\_ m.
- 1) 2
  - 2) 3
  - 3) 0
  - 4) 1
4. If a body moves in a circular path and reach back to its initial position then
- 1) magnitude of displacement is 0
  - 2) distance is 0
  - 3) both (1) and (2) are correct
  - 4) distance is equal to radius
5. If on a round trip one person travels 6km and arrive back to his starting point, then the distance travelled is \_\_\_\_\_ km.
- 1) 4
  - 2) 6
  - 3) 8
  - 4) 10
6. A body moving at a uniform velocity of 2 m/s will have
- 1) uniform acceleration
  - 2) non uniform acceleration
  - 3) zero acceleration
  - 4) none of these
7. The numerical ratio of displacement to distance is
- 1) Always less than 1.
  - 2) Always greater than 1.
  - 3) Always equal to 1.
  - 4) May be less than 1 or equal to one.
8. The location of a particle is changed. What can we say about the displacement and distance covered by the particle?
- 1) Both cannot be zero
  - 2) One of the two may be zero
  - 3) Both must be zero
  - 4) Both must be equal
9. Consider the motion of the tip of the second hand of a clock. In one minute
- 1) the average speed is zero
  - 2) the distance covered is zero
  - 4) the displacement is zero
  - 3) all of these
10. The numerical value of the ratio of average velocity to average speed is
- 1) always less than one
  - 2) always equal to one
  - 3) always more than one
  - 4) equal to or less than one.



11. If a particle moves in a circle describing equal angles in equal intervals of time, the velocity vector
- 1) remains constant.
  - 2) changes in magnitude.
  - 3) changes in direction.
  - 4) changes both in magnitude and direction.
12. Check up only the correct statement in the following.
- 1) A body has a constant velocity and still it can have a varying speed
  - 2) A body has a constant speed but it can have a varying velocity
  - 3) A body having constant speed cannot have any acceleration.
  - 4) None of these.
13. The acceleration of a body has the direction of
- 1) Distance
  - 2) Velocity
  - 3) Change of velocity
  - 4) None of these
14. The numerical value of the ratio of instantaneous velocity to instantaneous speed is
- 1) Always less than one
  - 2) Always equal to one
  - 3) Always more than one
  - 4) Equal to or less than one
15. The magnitude of displacement is equal to the distance covered in a given interval of time if the particle
- 1) Moves with constant acceleration along the path
  - 2) Moves with constant speed only
  - 3) Moves in a same direction with constant velocity or with variable velocity
  - 4) Moves with constant velocity only
16. The ratio of the average velocity of a train during a journey to the maximum velocity between two stations is
- 1) = 1
  - 2) > 1
  - 3) < 1
  - 4) > or < 1
17. Consider the motion of the tip of the second hand of a clock. In one minute
- 1) the average velocity is zero
  - 2) the distance covered is zero
  - 3) the average speed is zero
  - 4) all of these



18. A body moves with a uniform velocity. Among the following the correct statement is
- 1) Its velocity is zero
  - 2) Its speed is zero
  - 3) Its acceleration is zero
  - 4) Both 1 & 2 are correct
19. Acceleration of a body is zero when
- 1) Velocity of a body increases with time
  - 2) Velocity of a body decreases with time
  - 3) Velocity of a body constant with time
  - 4) both(1) & (2)
20. Acceleration is negative when
- 1) Velocity of a body increases with time
  - 2) Velocity of a body decreases with time
  - 3) Velocity of a body constant with time
  - 4) All of these.
21. Acceleration is positive when
- 1) Velocity of a body increases with time
  - 2) Velocity of a body decreases with time
  - 3) Velocity of a body constant with time
  - 4) All of these.

### JEE MAINS

#### Single Correct Choice Type:

1. A player completes a circular path of radius 'r' in 40s. At the end of 2 minutes 20 seconds, displacement will be
  - 1)  $2r$
  - 2)  $2\pi r$
  - 3)  $7\pi r$
  - 4) Zero
2. A body goes 20 km north and then 10 km due east. The displacement of body from its starting point is
  - 1) 30 km
  - 2) 25.2 km
  - 3) 10 km
  - 4) 22.36 km
3. A man walks 4m towards north then 3m towards east and there he climbs a pole of 12m height.
  - 1) distance covered by man is 13 m
  - 2) total displacement by man is 13 m
  - 3) distance covered by man is 21 m
  - 4) total displacement by man is 19 m
4. A body is moving along the circumference of a circle of radius 'R' and completes half of the revolution. Then, the ratio of its displacement to distance is
  - 1)  $\pi : 2$
  - 2) 2:1
  - 3)  $2 : \pi$
  - 4) 1:2

5. A body completes one round of a circle of radius 'R' in 20 second. The displacement of the body after 45 second is
- 1)  $\frac{R}{\sqrt{2}}$                       2)  $\sqrt{2} R$                       3)  $2\sqrt{R}$                       4) 2R
6. If the distance between the sun and the earth is  $1.5 \times 10^{11}$  m and velocity of light is  $3 \times 10^8$  m/s, then the time taken by a light ray to reach the earth from the sun is
- 1) 500 s                      2) 500 minute                      3) 50 s                      4)  $5 \times 10^3$ s
7. If a body covers first half of its journey with uniform speed  $v_1$  and the second half of the journey with uniform speed  $v_2$ , then the average speed is
- 1)  $v_1 + v_2$                       2)  $\frac{2 v_1 v_2}{v_1 + v_2}$                       3)  $\frac{v_1 v_2}{v_1 + v_2}$                       4)  $v_1 v_2$
8. A body moves with a velocity of 3m/s due east and then turns due north to travel with the same velocity. If the total time of travel is 6s, the acceleration of the body is
- 1)  $\sqrt{3}$  m/s<sup>2</sup> towards north west                      2)  $\frac{1}{\sqrt{2}}$  m/s<sup>2</sup> towards north west
- 3)  $\sqrt{2}$  m/s<sup>2</sup> towards north east                      4) all the above
9. Given the distance between earth and sun is  $1.6 \times 10^8$  km and velocity of light is  $4 \times 10^5$  km/sec. The time taken for sunlight to reach the earth is
- 1) 400 s                      2) 700 s                      3) 500 s                      4) 600 s
10. Rajdhani express moves at a speed of 120 km/h. How long will it take to cover a distance of 15 km?
- 1) 6.4 min                      2) 2.3 min                      3) 7.5 min                      4) 8.3 min
11. A car travels a distance of 250 km from Hyderabad to Guntur in 5 hours, the velocity of the car for this journey is
- 1) 40 kmph                      2) 50 kmph                      3) 60 kmph                      4) 70 kmph
12. A car is moving at a speed of  $15 \text{ ms}^{-1}$ . In how much time will it cover a distance of 1.2 km ?
- 1) 70 s                      2) 80 s                      3) 18 s                      4) 45 s

**Multi Correct Choice Type:**

13. An object may have
- 1) varying speed without having varying velocity
  - 2) varying velocity without having varying speed
  - 3) non zero acceleration without having varying velocity
  - 4) non zero acceleration without having varying speed.
14. In which of the following examples of motion, can the body be considered approximately a point object
- 1) a railway carriage moving without jerks between two stations.
  - 2) a monkey sitting on top of a man cycling smoothly on a circular track
  - 3) a spinning cricket ball that turns sharply on hitting the ground
  - 4) a tumbling beaker that has slipped off the edge of a table
15. Which of the following statements is/ are correct ?
- 1) If the velocity of a body changes, it must have some acceleration.
  - 2) If the speed of a body changes, it must have some acceleration
  - 3) If the body has acceleration, its speed must change
  - 4) If the body has acceleration, its speed may change.
16. Pick out the correct statement from the following.
- 1) displacement is a vector quantity and hence direction is important
  - 2) displacement can be both positive and negative
  - 3) distance is always positive. It never decreases with time
  - 4) distance can be negative

**Statement Type:**

17. Statement I : Acceleration of a body can change its direction without any change in direction of velocity

Statement II: Direction of acceleration is same as that of direction of change in velocity vector

- 1) If Statement I is true, Statement II is true, Statement II is a correct explanation for Statement I
- 2) If Statement I is true, Statement II is true, Statement II is not a correct explanation for Statement I
- 3) If Statement I is true, Statement II is false
- 4) If Statement I is false, Statement II is true

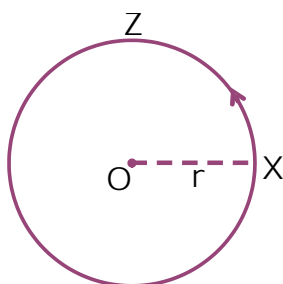
18. Statement I: Magnitude of average velocity is equal to average speed, if velocity is constant.

Statement II : If velocity is constant, then there is no change in the direction.

- 1) If Statement I is true, Statement II is true, Statement II is a correct explanation for Statement I
- 2) If Statement I is true, Statement II is true, Statement II is not a correct explanation for Statement I
- 3) If Statement I is true, Statement II is false
- 4) If Statement I is false, Statement II is true

**Comprehension Type:**

A cyclist moves from a certain point X and moves around a circular path of radius r.

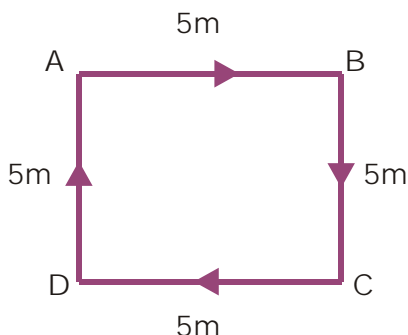


19. If the cyclist completes one revolution around a circular path of radius 'r'. Then, the distance travelled and magnitude of displacement of the cyclist are respectively
  - 1)  $2r$  ,  $\pi r$
  - 2)  $\frac{\pi}{2}$  ,  $2\pi$
  - 3)  $2\pi r$  , zero
  - 4)  $r$  ,  $2r$
20. If the cyclist reaches exactly the other side of the point X (i.e. diametrically opposite point to X). Then, the distance travelled and magnitude of displacement of cyclist are respectively
  - 1)  $\pi r$  ,  $2r$
  - 2)  $\frac{\pi r}{2}$  ,  $2\pi$
  - 3)  $2\pi r$  , zero
  - 4)  $r$  ,  $2r$
21. If the cyclist reaches a point 'Z' as shown in the figure, then the distance travelled and magnitude of displacement of the cyclist are respectively
  - 1)  $\frac{\pi r}{2}$  ,  $2r$
  - 2)  $\frac{\pi r}{4}$  ,  $4r$
  - 3)  $\frac{\pi r}{2}$  ,  $\sqrt{2} r$
  - 4)  $2\pi r$  , zero



**Matrix Match Type:**

22. A boy walks along the square path. ABCD of side 5m. Then match the following.



**Column - I**

- a) Along the path ABC, the distance travelled by a boy is
- b) Along the path CDA, the displacement covered by a boy
- c) Total distance covered by a boy along the path ABCDA
- d) Total displacement covered by a boy along the path ABCDA

**Column - II**

- 1)  $5\sqrt{2}$  m
- 2)  $10\sqrt{2}$  m
- 3) 20 m
- 4) 10 m
- 5) zero

**MOTION\_SYNOPSIS-2**

1. **Recap of equations of motion for a body moving with uniform acceleration are:**

a)  $v = u + at$       b)  $s = ut + \frac{1}{2}at^2$       c)  $v^2 - u^2 = 2as$       d)  $s_n = u + \frac{a}{2}(2n - 1)$

Where  $u$  = initial velocity,  $v$  = final velocity,  $a$  = acceleration,  $t$  = total time,  $s$  = displacement and  $s_n$  = displacement in  $n^{\text{th}}$  second.

2. **Motion due to gravity :** Motion due to gravity can be studied as:

- a) Freely falling body      b) Body projected vertically upwards
- c) Projectile motion.(will be discussed later)

**Freely falling body:** Whenever a body is falling freely ( no initial force is applied ) then  $u = 0$  . The equations for a freely falling body are:

a)  $v=gt$       b)  $s(\text{or})h = \frac{1}{2}gt^2$       c)  $v^2 = 2gh \Rightarrow v = \sqrt{2gh}$       d)  $s_n = \frac{g}{2}[2n - 1]$



## MOTION\_WORKSHEET-2

- CUQ** 1. A boy is Running towards east with a velocity of 10m/s. he suddenly turns towards north. Then at that place acceleration due to gravity is
1. Increases
  2. Decreases
  3. Remains same
  4. Zero
2. A 1000kg and 10kg objects are dropped from the same height, then the ratio of acceleration of fall of two bodies is
1. 1:1
  2. 100:1
  3. 1: 100
  4. 10: 1
3. A huge rock and stone are dropped from same height, then
- 1 Both reaches the ground at the same time
  2. Huge rock take more time than a stone
  3. Stone takes more time than a huge rock
  4. They reach the ground with different velocity
4. The direction of acceleration due to gravity is always
1. Towards the centre of earth
  2. Along the tangent of earth
  3. Along the chord of earth
  4. Away from the surface of earth
5. The dimensional formula for 'g' is
1.  $(M^0L^2T^{-2})$
  2.  $(M^0LT^{-2})$
  3.  $(M^0L^0T^2)$
  4.  $(MLT^{-2})$
6. If a body is falling freely under the acceleration due to gravity, the final velocity of the body when it reaches the ground is directly praportional to
1. Time taken by the body
  2. Square of time taken by the body
  3. Cube of time taken by the body
  4. Independent on time
7. If a body is falling freely then the acceleration due to gravity is taken as \_\_\_\_\_.
- 1) equal
  - 2) negative
  - 3) positive
  - 4) infinity
8. For a freely falling body, initial velocity is \_\_\_\_\_.
- 1) zero
  - 2) infinite
  - 3) non zero
  - 4) one
9. Two stones of different masses are dropped simultaneously from the top of a building. Then which is correct among these?
- 1) larger stone hits the ground earlier.
  - 2) smaller stone hits the ground earlier.
  - 3) velocity of the stones are independent on the mass
  - 4)none of these.

10. A body, freely falling under gravity will have uniform
- 1) speed                      2) velocity                      3) momentum                      4) acceleration
11. Two bodies of different masses are dropped simultaneously from the top of a tower. If air resistance is proportional to the mass of the body, then,
- 1) the heavier body reaches the ground earlier.  
 2) the lighter body reaches the ground earlier.  
 3) both the bodies reach the ground simultaneously.  
 4) cannot be decided.
12. If two balls of same density but of different masses are dropped from a height of 100 m, then
- 1) both will come together on the earth  
 2) both will come late on the earth  
 3) first will come first and second after that  
 4) second will come first and first after that
13. The distances moved by a freely falling body (starting from rest) during 1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup>,.....n<sup>th</sup> second of its motion are proportional to
- 1) even numbers                                      2) odd numbers  
 3) all integral numbers                                      4) squares of integral numbers
14. A body falls freely from a tower of height 'h'. The velocity with which it strikes the ground is [g = acceleration due to gravity]
- 1)  $\sqrt{2gh}$                       2)  $\frac{\sqrt{2g}}{h}$                       3)  $\frac{\sqrt{g}}{h}$                       4)  $\frac{h}{\sqrt{g}}$
15. Two bodies of different masses are dropped from heights of 16m and 25m respectively. The ratio of the time taken by them is
- 1) 25 : 16                      2) 5 : 4                      3) 4 : 5                      4) 16 : 25

### JEE MAINS

#### Single Correct Choice Type:

1. If a body is dropped freely from a height of 20m, then velocity of the body on reaching to the ground is [g = 10m/s<sup>2</sup>]
1. 20m/s                      2. 100m/s                      3. 150m/s                      4. 200m/s

2. If a body is dropped freely from a height of 20m, then the time taken by the body to reach the ground is
1. 2 sec                      2.  $\sqrt{2}$  sec                      3.  $\sqrt{10}$  sec                      4. 10 sec
3. If a body is dropped freely, then what is velocity of the body after '2' sec  
[ $g = 10\text{m/s}^2$ ]
1. 10m/s                      2. 20 m/s                      3. 15 m/s                      4. 200 m/s
4. A stone is dropped from the top of the tower and reaches the ground in 3s. Then the height of the tower is ( $g = 9.8\text{m/s}^2$ )
- 1) 18.6m                      2) 39.2m                      3) 44.1m                      4) 98m
5. The average velocity of a freely falling body is numerically equal to half of the acceleration due to gravity. The velocity of the body as it reaches the ground is
- 1)  $g$                       2)  $\frac{g}{2}$                       3)  $\frac{g}{\sqrt{2}}$                       4)  $\sqrt{2}g$
6. A boy standing at the top of a tower of 20 m height drops a stone Assuming  $g = 10\text{ms}^{-2}$ , the velocity with which it hits the ground is [AIPMT 2011]
- 1)  $20\text{ms}^{-1}$                       2)  $40\text{ms}^{-1}$                       3)  $5\text{ms}^{-1}$                       4)  $10\text{ms}^{-1}$
7. The height from which a body is released is numerically equal to the velocity acquired finally. Then that height is equal to.
1.  $g$                       2.  $2g$                       3.  $4g$                       4.  $8g$
8. A freely falling body describes a distance  $x$  in the first 2 seconds and a distance  $y$  in the next 2 seconds . then
1.  $y=x$                       2.  $y = 2x$                       3.  $x = 2y$                       4.  $y = 3x$
9. A body is dropped from height 'h' at time  $t = 0$ , reaches the ground at time ' $t_0$ '. The time taken by which it would have reached a height 'h/2' is
1.  $t_0/2$                       2.  $2t_0$                       3.  $t_0/\sqrt{2}$                       4.  $\sqrt{2}t_0$
10. A ball is dropped freely while another is thrown vertically downward with an initial velocity 'v' from the same point simultaneously. After 't' second they are separated by a distance of
- 1)  $\frac{vt}{2}$                       2)  $\frac{1}{2}gt^2$                       3)  $vt$                       4)  $vt + \frac{1}{2}gt^2$

**Multi Correct Choice Type:**

11. The distance travelled by a freely falling body is directly proportional to
- 1) the mass of the body
  - 2) the acceleration due to gravity.
  - 3) the square of the time of fall
  - 4) the time of fall
12. For a freely falling body
1. initial velocity is zero
  2. Acceleration is positive
  3. On reaching the ground its velocity is  $\sqrt{2gh}$
  4. Time in which it reaches the ground is  $\frac{2u}{g}$
13. A body is falling under gravity, then the distance covered
- 1) in 1st second of its motion is 4.9m
  - 2) in 2nd second of its motion is 14.7m
  - 3) in 3rd second of its motion is 24.5m
  - 4) in 4th second of its motion is 34.3 m

**Statement Type:**

14. *Statement I* : A body falling freely moves with constant velocity.

*Statement II*: The body falls freely, when acceleration of the body is equal to acceleration due to gravity.

- 1) Both Statements are true, Statement II is the correct explanation of Statement I.
- 2) Both Statements are true, Statement II is not correct explanation of Statement I.
- 3) Statement I is true, Statement II is false.
- 4) Statement I is false, Statement II is true.

**Comprehension Type:**

When a body is dropped freely it accelerates.

15. A stone released with zero velocity from the top of a tower reaches the ground in 4 second, the height of the tower is about  $[g = 10\text{ms}^{-2}]$
- 1) 20 m
  - 2) 40 m
  - 3) 80 m
  - 4) 160 m



16. Take the uniform acceleration near the surface of earth to be  $9.8\text{ms}^{-2}$  for a free fall. What is the velocity of a body at the end of two second of free fall, if the initial velocity is zero? ( $g = 9.8\text{m/s}^2$ )
- 1)  $9.8\text{ms}^{-1}$                       2)  $19.6\text{ ms}^{-1}$                       3)  $29.4\text{ ms}^{-1}$                       4)  $4.9\text{ ms}^{-1}$
17. How far a stone shall free fall in 1 second released from rest? ( $g = 9.8\text{m/s}^2$ )
- 1)  $4.9\text{m}$                               2)  $9.8\text{m}$                               3)  $19.6\text{m}$                               4)  $29.4\text{ m}$

**Matrix Match Type:**

18. For a freely falling body, match the following

**Column -I**

- a) Velocity of the body on reaching the ground is
- b) Distance travelled by the body is in 't' sec is
- c) velocity of the body after a time 't' sec is
- d) time (t) taken by a body  $t_0$  reach the ground is

**Column-II**

- 1)  $\sqrt{\frac{2h}{g}}$
- 2)  $gt$
- 3)  $\frac{1}{2}gt^2$
- 4)  $\sqrt{2gh}$
- 5)  $2g$

19. For a freely falling body, match the following

**Column -I**

- a)  $V = gt$
- b)  $S = \frac{1}{2}gt^2$
- c)  $V = \sqrt{2gh}$
- d)  $S_3 = \frac{5g}{2}$

**Column-II**

- p) Distance travelled by the body is 3rd sec.
- q) velocity of body on reaching the ground
- r) Distance travelled by the body in 't' sec.
- s) velocity of body after time 't' sec



## MOTION\_SYNOPSIS-3

**Body projected vertically upwards:** Whenever a body is projected vertically upwards, then  $a = -g$ . The equations of motion for a body projected vertically upwards are:

$$\text{a) } v = u - gt \quad \text{b) } s \text{ or } h = ut - \frac{1}{2}gt^2 \quad \text{c) } v^2 - u^2 = -2gh \quad \text{d) } s_n = u - \frac{g}{2}(2n - 1)$$

Whenever a body is projected vertically upwards: velocity becomes zero at the highest point.

**i) Maximum height :** The vertical distance travelled by a body before its velocity becomes zero is called maximum height reached by the body.

If a body is projected vertically upwards with a velocity 'u', the maximum height (h) reached by the body is  $h = \frac{u^2}{2g}$ . Where g = acceleration due to gravity.

**ii) Time of ascent :** Time taken by the body to reach the highest point is called time of ascent ( $t_a$ ) and is given by  $t_a = \frac{u}{g}$

**iii) Time of descent :** Time taken by the body to travel from the maximum height to the ground is called time of descent ( $t_d$ ) is given by  $t_d = \frac{u}{g}$ .

In the absence of air resistance, time of ascent is equal to time of descent.

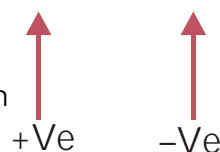
**iv) Time of flight :** The total time spent by the body in air is called time of flight (t) is given by  $t = t_a + t_d = \frac{u}{g} + \frac{u}{g} = \frac{2u}{g}$ .

**Note:** If 'u' is the velocity with which a body is projected vertically upwards and 'v' is the velocity with which the body reaches the point of projection, then  $v = u$ .

Vertical projection of an object from a tower:  $h = -ut + \frac{1}{2}gt^2$  or  $h = \frac{1}{2}gt^2 - ut$

**\*3. Sign convention:** Normally we take vertically upward direction positive (and downward negative) and horizontally rightwards positive (or leftwards negative)

(a) Sign convention for motion in vertical direction



(b) Sign convention for motion in horizontal direction  $\xrightarrow{+Ve}$   $\xleftarrow{-Ve}$

**Velocity on reaching the ground:**

When a body is dropped from a height  $h$ , its initial velocity is zero. Let the final velocity on reaching the ground be  $v$ . For a freely falling body,

$$v^2 - u^2 = 2gh, \text{ but } u = 0 \text{ therefore, } v^2 - 0 = 2gh \text{ or } v = \sqrt{2gh} \text{ ——— (10)}$$

Further, in case of a body thrown upwards, we have for initial velocity.

$$u = \sqrt{2gh} \text{ ——— (11)}$$

Comparing equation (10) and (11), we conclude that the velocity of the body falling from height  $h$ , on reaching the ground is equal to the velocity with which it is projected vertically upwards to reach the same height  $h$ .

The upwards velocity at any point in its flight is the same as its downward velocity at that point.

**Body projected vertically up from the top of a tower**

1. If a body is projected vertically up from the top of a tower of height ' $h$ ' with velocity ' $u$ '.

a) Displacement after time  $t$  is  $S = ut - \frac{1}{2}gt^2$

b) If it reaches the ground after time  $t$ , then  $-h = ut - \frac{1}{2}gt^2$

c) Its velocity on reaching the ground is  $\sqrt{u^2 + 2gh}$

d) Its maximum height above the ground is  $\left\{ h + \left( \frac{u^2}{2g} \right) \right\}$

**MOTION\_WORKSHEET-3**

**CUQ** 1. When a body projected vertically upwards its velocity is at the highest point is \_\_\_\_\_.

- 1) non-zero                      2) infinite                      3) zero                      4) one

2. The Time of flight is equal to the sum of \_\_\_\_\_.

- 1) time of ascent and velocity                      2) time of ascent and time of descent  
 3) time of descent and velocity                      4) time of ascent and maximum height



11. To reach the same height on the moon as on the earth, a body must be projected up with
- 1) Higher velocity on the moon.
  - 2) Lower velocity on the moon.
  - 3) Same velocity on the moon and earth.
  - 4) It depends on the mass of the body.
12.  $B_1, B_2$  and  $B_3$  are three balloons ascending with velocities  $v, 2v$  and  $3v$  respectively. If a bomb is dropped from each when they are at the same height, if  $T_1, T_2$  and  $T_3$  are the times taken by the bombs while reaching the ground. Then
- 1)  $T_1 < T_2 < T_3$
  - 2)  $T_1 = T_2 = T_3$
  - 3)  $T_1 > T_2 > T_3$
  - 4)  $T_2 > T_1 > T_3$
13.  $B_1, B_2$  and  $B_3$  are three balloons ascending with velocities  $v, 2v$  and  $3v$ , respectively. If a bomb is dropped from each when they are at the same height, then
- 1) bomb from  $B_1$  reaches ground first
  - 2) bomb from  $B_2$  reaches ground first
  - 3) bomb from  $B_3$  reaches ground first
  - 4) they reach the ground simultaneously
14. A person standing near the edge of the top of a building throws two balls A and B. The ball A is thrown vertically upward and B is thrown vertically downward with the same speed, The ball A hits the ground with a speed  $V_A$  and the ball B hits the ground with a speed  $V_B$ . then
- 1)  $V_A < V_B$
  - 2)  $V_A < V_B$
  - 3)  $V_A = V_B$
  - 4) the relation between  $V_A$  and  $V_B$  depends on height of the building above the ground
15. The time taken by a vertically projected body before reaching the ground is
- 1) directly proportional to initial velocity.
  - 2) directly proportional to square of initial velocity.
  - 3) inversely proportional to square of initial velocity.
  - 4) inversely proportional to initial velocity.



## JEE MAINS

## Single Correct Choice Type:

- A stone is projected vertically upwards with a velocity 10 m/s. Find the maximum height reached by the body. (take  $g = 10\text{m/s}^2$ .)  
1) 5m                      2) 10m                      3) 15m                      4) 20m
- A body is thrown vertically upwards and rises to a maximum height of 5m. The velocity with which the body was thrown upwards is (take  $g = 10\text{m/s}^2$ )  
1) 15 m/s                      2) 10 m/s                      3) 20 m/s                      4) 40 m/s
- A body is projected vertically upwards with a velocity of 9.8 m/s. The total time for which the body will remain in the air is \_\_\_\_\_ (take  $g = 9.8\text{m/s}^2$ )  
1) 2 sec                      2) 3 sec                      3) 1 sec                      4) 0.5 sec
- A body is projected vertically upwards with a velocity 19.6 m/s, its time of ascent is = \_\_\_\_ ( $g = 9.8\text{m/s}^2$ )  
1) 3 sec                      2) 4 sec                      3) 2 sec                      4) 1 sec
- A body projected vertically up reaches a maximum height of 9.8m. Its time of flight is  
1) 4s                      2) 2s                      3)  $\sqrt{2}\text{s}$                       4)  $2\sqrt{2}\text{s}$
- A pebble is thrown vertically upwards with a speed of  $20\text{ms}^{-1}$ . How high will it be after 2seconds? (Take  $g = 10\text{ms}^{-2}$ )  
1) 20m                      2) 30m                      3) 40m                      4) 60m
- A ball thrown vertically upwards with an initial velocity of 1.4 m/s returns in 2 s. The total displacement of the ball is  
1) 22.4 cm                      2) zero                      3) 44.8 m                      4) 33.6m
- A stone thrown vertically up from the ground reaches a maximum height of 50m in 10s. Time taken by the stone to reach the ground from maximum height is  
1) 5s                      2) 10s                      3) 20s                      4) 25s
- A body is projected vertically up with a speed of 20m/s. The distance travelled by it is 3 sec is  
1. 5 m                      2 15 m                      3. 20 m                      4 25 m
- A body is projected vertically up and total distance travelled by it is S. Then total time of its flight will be

1.  $\sqrt{\frac{s}{g}}$

2.  $\sqrt{\frac{2s}{g}}$

3.  $\sqrt{\frac{4s}{g}}$

4.  $\sqrt{\frac{8s}{g}}$

11. A body is projected vertically up from the top of a tower with 19.6 m/s takes 5sec to reach the ground. Then height of the tower is
1. 44.1 m                      2. 24.5 m                      3. 39.2 m                      4. 19.6 m
12. From the top of a tower a body is projectd vertically up with a velocity of 4.9 m/s. The time after which it will pass through the point of projectopm is
1. 1sec                      2. 2sec                      3. 4sec                      4. 6sec

**Multi Correct Choice Type:**

13. Pickup the correct statement.
- 1) A body is projected vertically upwards at maximum height  $v = 0$  and  $a \neq 0$   
 2) A body is projected vertically upwards at maximum height  $v \neq 0$  and  $a \neq 0$   
 3) A body is projected vertically upwards other than maximum height along its path  $v \neq 0$  and  $a \neq 0$ .  
 4) A body is projected vertically upwards other than maximum height along its path  $v = 0$  and  $a = 0$ .
14. A particle is projected vertically upward with velocity  $u$  from a point A, when it returns to point of projection
- 1) Its average speed is  $u/2$                       2) Its average velocity is zero  
 3) Its displacement is zero                      4) Its average speed is  $u$

**Statement Type:**

15. *Statement I* : A body is projected vertically upwards then its velocity gradually decreases.
- Statement II* : A body is projected vertically upward then its acceleration gradually increases.
- 1) Both Statements are true, Statement II is the correct explanation of Statement I.  
 2) Both Statements are true, Statement II is not correct explanation of Statement I.  
 3) Statement I is true, Statement II is false.  
 4) Statement I is false, Statement II is true.
16. *Statement-I*: Two balls of different masses are thrown vertically upward with the same speed. They will pass through their point of projection in the downward direction with the same speed.
- Statement-II*: The maximum height and downward velocity attained at the point of projection are dependent on the mass of the ball.
- 1) Both Statements are true, Statement II is the correct explanation of Statement I.  
 2) Both Statements are true, Statement II is not correct explanation of Statement I.  
 3) Statement I is true, Statement II is false.  
 4) Statement I is false, Statement II is true.

**Comprehension Type:**

Whenever a body is projected vertically upwards, its velocity decreases.

17. A body is projected vertically upwards with a velocity 40m/s. The maximum height reached by the body. [ $g = 10\text{m/s}^2$ ]
- 1) 80m                      2) 20m                      3) 30m                      4) 40m
18. A body is thrown vertically upwards and rises to a maximum height of 10m. The velocity with which the body was thrown upwards is ( $g = 9.8\text{m/s}^2$ )
- 1) 10m/s                      2) 14m/s                      3) 20m/s                      4) 18m/s
19. A ball is thrown vertically upwards. It returns 6seconds later. The greatest height reached by ball is ( $g = 10\text{ms}^{-2}$ )
- 1) 90m                      2) 70m                      3) 20m                      4) 45m

**Matrix Match Type:**

20. If the body projected vertically upwards, Then match the following:

Column -I

Column-II

- |                                       |                                    |
|---------------------------------------|------------------------------------|
| a) Maximum height reached by the body | 1) $\frac{2u^2}{g}$                |
| b) time of ascent                     | 2) $\frac{u}{g}$                   |
| c) time of descent                    | 3) $\frac{2u}{g}$                  |
| d) time of flight                     | 4) $\frac{u^2}{2g}$                |
|                                       | 5) depends on the initial velocity |

## KINEMATICS\_SINOPSIS-4

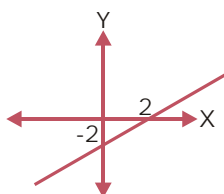
## 1. Graphs

i) A linear equation between  $x$  and  $y$  represents a straight line between  $x$  and  $y$ , e.g.  $y = 4x - 2$ ,  $y = 5x + 3$ ,  $3x = y - 2$  equations represent straight line on  $x$ - $y$  graph.

To draw the straight line find  $x$ -intercept and  $y$ -intercept. Joining these two intercepts will give the graph of straight line.

For example to draw the graph of  $y = 4x - 2$ . Find  $x$ -intercept and  $y$ -intercept. To find  $x$ -intercept put  $y = 0$  in the given equation i.e.,  $4x - 2 = 0 \Rightarrow 4x = 2 \Rightarrow x = \frac{1}{2}$  and to find  $y$ -intercept put  $x = 0$  in the given equation i.e.,  $y = 4(0) - 2 \Rightarrow y = -2$

Join these two intercepts to get the required graph of straight line as shown.

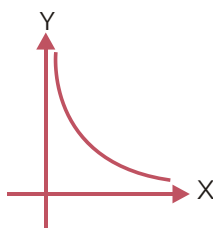


ii)  $x \propto y$  or  $y = kx$  represents a straight line passing through origin.

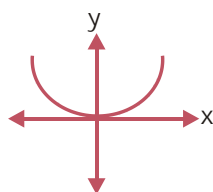
iii) The graph of  $y = k$  (constant) is a straight line parallel to  $x$ -axis.

iv) The graph of  $x = k$  (constant) is a straight line parallel to  $y$ -axis.

v)  $x \propto \frac{1}{y}$  represents a rectangular hyperbola in  $x$ - $y$  graph. Shape of rectangular hyperbola is as shown in figure



vi) A quadratic equation in  $x$  and  $y$  represents a parabola in  $x$ - $y$  graph, e.g.,  $y = 3x^2 + 2$ ,  $y^2 = 4x$ ,  $x^2 = y - 2$  equations represent parabola in  $x$ - $y$  graph. The shape of parabola of form  $y = kx^2$  where  $k$  is a constant is as shown in the figure.





2. If  $z = \frac{dy}{dx}$  or  $\frac{y}{x}$ , then the value of  $z$  at any point on  $x$ - $y$  graph can be obtained by the slope of the graph at that point.

For example, for one dimensional motion

a) slope of displacement–time graph gives velocity (as  $v = \frac{ds}{dt}$ ).

b) slope of velocity–time graph gives acceleration (as  $a = \frac{dv}{dt}$ ).

3. If  $z = yx$  or  $y(dx)$  or  $x(dy)$ , then the value of  $z$  between  $x_1$  and  $x_2$  or between  $y_1$  and  $y_2$  can be obtained by the area of graph between  $x_1$  and  $x_2$  or  $y_1$  and  $y_2$ .

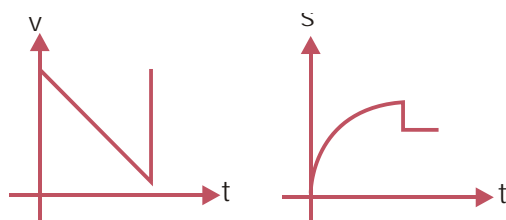
For example, for one dimensional motion

a) area under velocity–time graph gives displacement (as  $ds = v dt$ ).

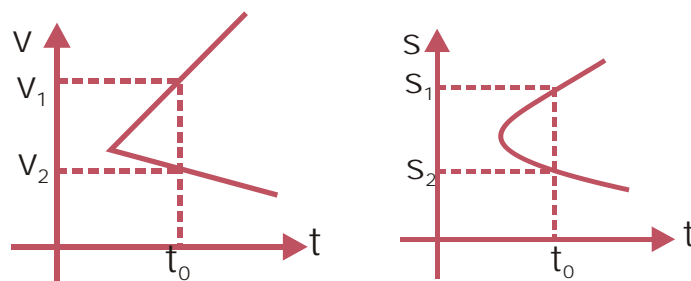
b) area under acceleration–time graph gives change in velocity (as  $dv = a dt$ ).

**Note:** Go through these points after completing the work sheet.

1 Slopes of  $v$ - $t$  or  $s$ - $t$  graphs can never be infinite at any point, because infinite slope of  $v$ - $t$  graph means infinite acceleration. Similarly, infinite slope of  $s$ - $t$  graph means infinite velocity. Hence, the following graphs are not possible:



2. At one time, two values of velocity or displacement are not possible. Hence, the following graphs are not acceptable:

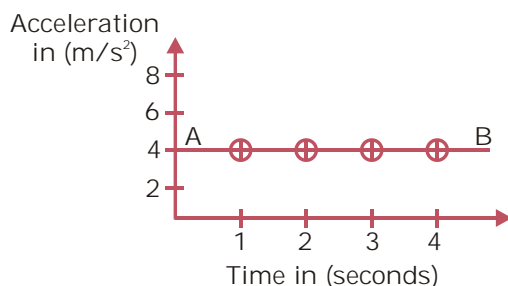


3. Different values of displacements in  $s$ - $t$  graph corresponding to given  $v$ - $t$  graph can be calculated just by calculating areas under  $v$ - $t$  graph. There is no need of using equations like  $v = u + at$  etc.

## KINEMATICS\_WORKSHEET-4

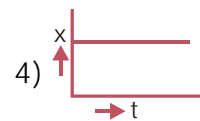
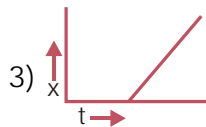
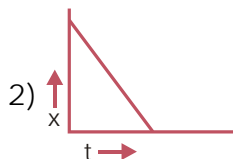
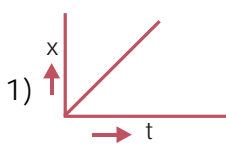
**CUQ** 1. If displacement - time graph is parallel to time axis, then body is \_\_\_\_

- 1) Uniform velocity
  - 2) Stationary
  - 3) Variable acceleration
  - 4) Variable velocity
2. If the velocity - time graph is parallel to time axis, then body is moving with
    - 1) Uniform velocity
    - 2) Stationary
    - 3) Variable velocity
    - 4) Variable acceleration
  3. If displacement-time graph is a curve, the body is moving with a \_\_\_\_
    - 1) Uniform velocity
    - 2) Stationary
    - 3) Variable acceleration
    - 4) Variable velocity
  4. If the velocity-time graph is a curve, then body has
    - 1) Variable acceleration
    - 2) Stationary
    - 3) Variable velocity
    - 4) both (1) and (3)
  5. If the velocity-time graph is a curve, then area under the curve represents \_\_\_\_
    - 1) velocity
    - 2) displacement
    - 3) acceleration
    - 4) all of these
  6. If displacement - time graph is a straight line, then body is moving with
    - 1) Uniform Velocity
    - 2) Stationary
    - 3) Variable Velocity
    - 4) Variable acceleration
  7. If Velocity - time graph is a straight line but moving away from velocity - time axis, then body is
    - 1) moving with variable velocity
    - 2) moving with uniform velocity
    - 3) moving with stationary
    - 4) displacement
  8. If acceleration - time graph, parallel to time axis from figure

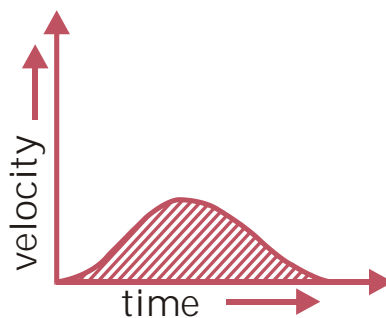


- 1) body is moving with a uniform velocity
- 2) Acceleration change
- 3) body is moving with a uniform acceleration
- 4) all of these

9. Which of the following cannot be the distance (x)- time (t) graph ?



10. The figure given here shows the velocity-time graph of a one-dimensional motion. Which of the following characteristics of the particle is represented by the shaded area ?



- 1) Velocity                      2) Acceleration                      3) Displacement                      4) Speed

11. Choose the correct statement :

- 1) The area of displacement - time graph gives velocity.
- 2) The slope of velocity - time graph gives acceleration.
- 3) The slope of displacement - time graph gives acceleration.
- 4) The area of velocity - time graph gives average velocity.

12. Velocity-time graph of a body thrown vertically up is

- 1) a straight line    2) a parabola
- 3) a hyperbola    4) circle

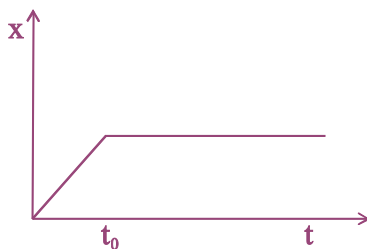
13. Velocity - displacement graph of a freely falling body is

- 1) Straight line passing through the origin
- 2) Straight line intersecting 'x' and 'y' axes
- 3) Parabola    4) Hyperbola

14. Displacement - time graph of a body projected vertically up is

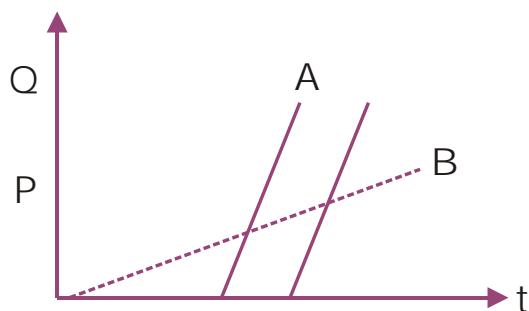
- 1) a straight line    2) a parabola                      3) a hyperbola                      4) a circle

15. If the distance travelled by a particle and corresponding time be laid off along y and x axes respectively, then the correct statement of the following is
- 1) the curve may lie in fourth quadrant
  - 2) the curve lies in first quadrant
  - 3) the curve exhibits peaks corresponding to maxima
  - 4) the curve may drop as time passes
16. In relation to a velocity - time graph
- 1) the curve can be a circle
  - 2) the area under the curve and above the time axis between any two instants gives the average acceleration
  - 3) the slope at any instant gives the rate of change of acceleration at that instant
  - 4) the area under the curve and above the time axis gives the displacement
17. The displacement - time graph of a particle moving with respect to a fixed point is a straight line
- 1) the object is stationary with zero velocity
  - 2) the acceleration of the object is zero
  - 3) body moves with uniform velocity
  - 4) All the above
18. For a uniform motion
- 1) the velocity - time graph is a straight line parallel to time axis
  - 2) the position - time graph is a parabola
  - 3) the acceleration - time graph is a straight line inclined with time axis
  - 4) the position - time graph is a straight line
19. Figure shows the displacement- time graph of a particle moving on the x-axis





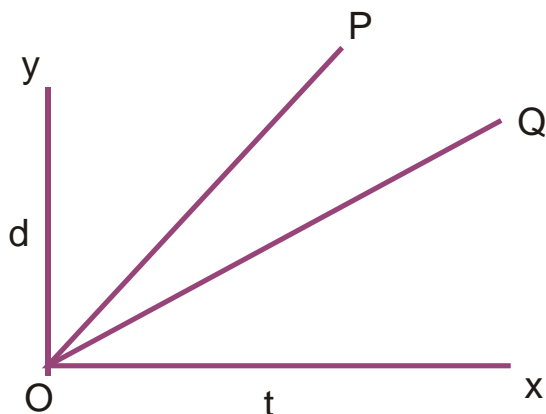
- 1) the particle is continuously going in positive X direction
  - 2) the particle is at rest
  - 3) the velocity increases up to a time  $t_0$  and then becomes constant.
  - 4) the particle moves at constant velocity up to a time  $t_0$  and then stops.
20. The position time (x-t) graphs for two children A and B returning from their school to their homes P and Q respectively are shown in figure. Choose the correct entries in the below.



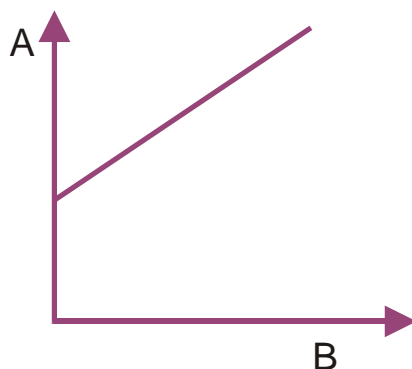
- a) A lives closer to the school than B
  - b) B starts from the school earlier than A
  - c) A walks faster than B
  - d) A and B reach home at the same time
  - e) B over takes A on the road once
- 1) a,d,e                      2) b,c                      3) c                      4) a,b,c
21. Shape of displacement - time graph for a body projected vertically up.
- 1) Curve with constant slope
  - 2) Curve with positive slope
  - 3) Curve with decreasing slope
  - 4) Curve with increasing slope



4. If velocity - time graph has positive slope, then body has
- 1) Negative acceleration
  - 2) Positive acceleration
  - 3) Zero acceleration
  - 4) Both 1 and 2
5. The displacement - time graphs of two bodies A and B are OP and OQ respectively. If  $\angle POX$  is  $60^\circ$  and  $\angle QOX$  is  $45^\circ$ , the ratio of the velocity of A to that of B is

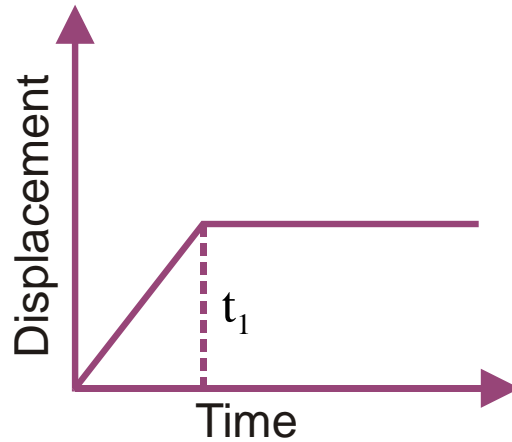


- 1)  $\sqrt{3} : \sqrt{2}$
  - 2)  $\sqrt{3} : 1$
  - 3)  $1 : \sqrt{3}$
  - 4) 3:1
6. The variation of quantity A with quantity B, plotted in Fig. describes the motion of a particle in a straight line.
- a) Quantity B may represent time.
  - b) Quantity A is velocity if motion is uniform.
  - c) Quantity A is displacement if motion is uniform.
  - d) Quantity A is velocity if motion is uniformly accelerated.

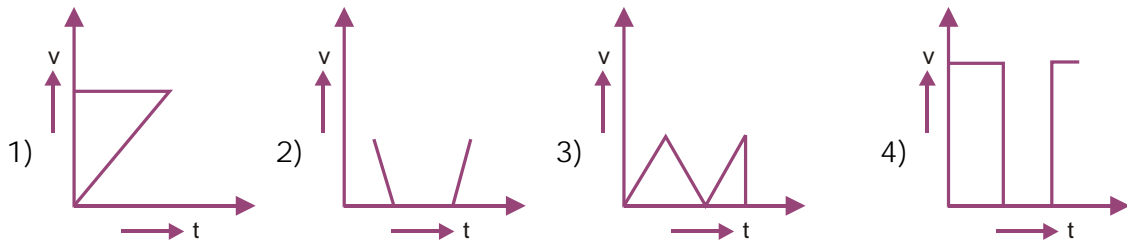


- 1) a,c,d
- 2) b,c,d
- 3) a,b
- 4) c,d

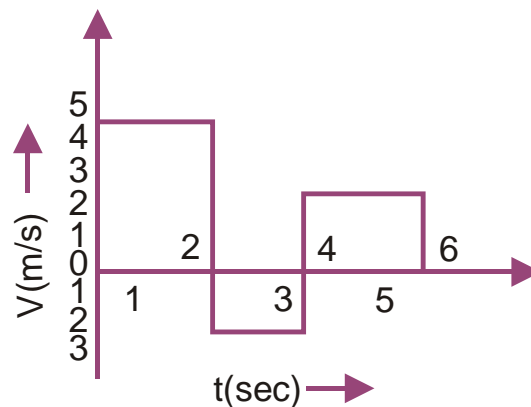
7. The x-t graph shown in figure represents



1. constant velocity
  2. velocity of the body is continuously changing.
  3. instantaneous velocity
  4. the body travels with constant speed upto time  $t_1$  and then stops
8. Which of the following velocity time graphs is NOT possible



9. The velocity-time graph of a body moving in a straight line is shown in the figure. The displacement and distance travelled by the body in 6 sec are respectively.



1. 8m,16m
2. 16m,8m
3. 16m,16m
4. 8m,8m

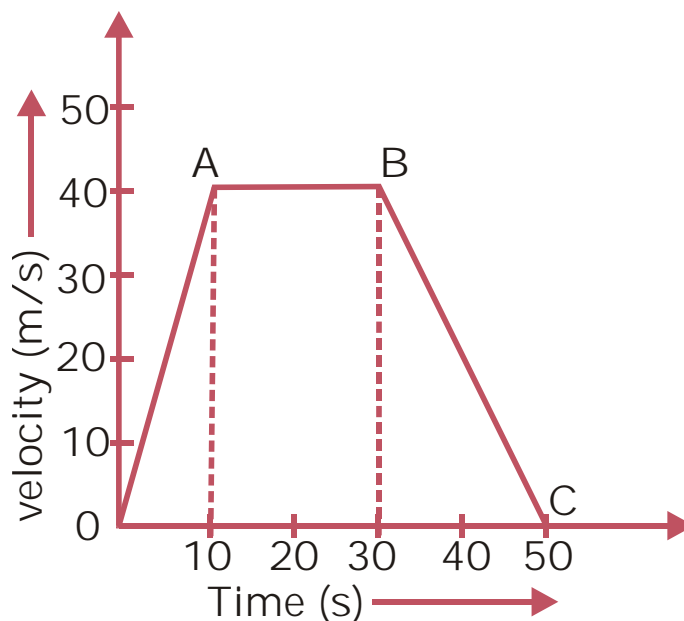


10. If velocity - time graph is parallel to time axis
- 1) Both will be moving with uniform velocity
  - 2) Body has zero acceleration
  - 3) Its displacement can be found by finding the area of the graph.
  - 4) Slope of graph will be zero

**Statement Type:**

11. *Statement I* : The slope of displacement - time graph gives velocity.  
*Statement II* : The area covered under velocity - time graph gives displacement.
- 1) Both Statements are true, Statement II is the correct explanation of Statement I.
  - 2) Both Statements are true, Statement II is not correct explanation of Statement I.
  - 3) Statement I is true, Statement II is false.
  - 4) Statement I is false, Statement II is true.

**Comprehension Type:**



The velocity - time graph of a car given above and solve the given questions.

12. what is the acceleration of car from O to A ?
- 1)  $2 \text{ m/s}^2$
  - 2)  $4 \text{ m/s}^2$
  - 3)  $6 \text{ m/s}^2$
  - 4)  $0.5 \text{ m/s}^2$

13. what is the acceleration of car from A to B ?

- 1)  $1\text{m/s}^2$       2)  $4\text{m/s}^2$       3)  $2\text{m/s}^2$       4)  $0\text{m/s}^2$

14. what is the retardation of car from B to C ?

- 1)  $1\text{m/s}^2$       2)  $2\text{m/s}^2$       3)  $3\text{m/s}^2$       4)  $4\text{m/s}^2$

**Matrix Match Type:**

15. **Column-I**

a) Uniform velocity

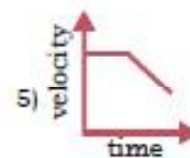
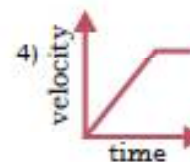
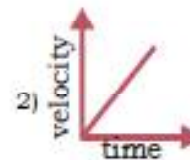
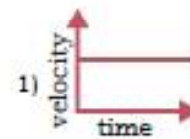
b) Uniform acceleration of the body from rest

c) Uniform acceleration of the body (initially not at rest)

d) Body starting with uniform

acceleration that becomes zero

**Column-II**



MOTION\_WORKSHEET-1\_KEY

- CUQ** 1. 4                    2. 1                    3. 3                    4. 1                    5. 2  
 6. 3                    7. 4                    8. 1                    9. 3                    10. 4  
 11. 3                    12. 2                    13. 3                    14. 2                    15. 3  
 16. 3                    17. 1                    18. 3                    19. 3                    20. 2                    21. 1

JEE MAINS

1. 1                    2. 4                    3. 2                    4. 3                    5. 2  
 6. 1                    7. 2                    8. 2                    9. 1                    10. 3  
 11. 2                    12. 2                    13. 2,4                    14. 1,2                    15. 1,2,4  
 16. 1,2,3                    17. 2                    18. 1                    19. 3                    20. 1  
 21. 3                    22. a-4;b-1;c-3;d-5

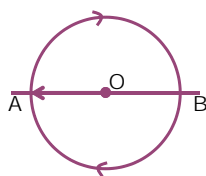
HINTS AND SOLUTIONS:

1. A player takes 40s to complete one revolution of a circular path.  
 Here 2 minutes 20s = 140s

Let n is the number of revolution he made  $n = \frac{140}{40} = \frac{7}{2} = 3\frac{1}{2}$

that means he made three and half revolutions in 140seconds. So, he reached the opposite of the starting point in a circle of radius 'r'

So, the displacement is  $r + r = 2r$ .



2. Displacement =  $\sqrt{x^2 + y^2} = \sqrt{10^2 + 20^2} = \sqrt{500} = 22.36 \text{ km}$   
 4.  $2R:\pi R$  (or)  $2:\pi$ ;                    5.  $S = \sqrt{R^2 + R^2} = \sqrt{2}R$                     6.  $S = vt$   
 7. Average velocity =  $\frac{\text{total displacement}}{\text{total time}}$                     8.  $a = \frac{\text{change in velocity}}{\text{time}}$   
 10. Average speed, =  $\frac{\text{total distance travelled}}{\text{total time taken}} = \frac{S_1 + S_2 + S_3}{t_1 + t_2 + t_3}$

Let 's' the total distance travelled and let  $t_1$  time taken for 1<sup>st</sup> one third distance, with velocity of  $1\text{ms}^{-1}$   $\therefore t_1 = \frac{s/3}{1} = s/3$  ----- (1)

Let  $t_2$  be the time taken for 2nd one third distance with a velocity of  $2\text{ms}^{-1}$   
 $\therefore t_2 = \frac{s/3}{2} = s/6$  ----- (2)

Let  $t_3$  be the time taken for next one third distance with a velocity of  $3\text{ms}^{-1}$   
 $\therefore t_3 = \frac{s/3}{3} = s/9$  ----- (3)

$$\therefore V_{ave} = \frac{s}{t_1 + t_2 + t_3} = \frac{s}{\frac{s}{3} + \frac{s}{6} + \frac{s}{9}} = \frac{s}{s \left[ \frac{1}{3} + \frac{1}{6} + \frac{1}{9} \right]} = \frac{1}{11/18} = \frac{18}{11} = 1)64 \text{ ms}^{-1}$$

11. I Average Velocity =  $\frac{\text{Total Displacement}}{\text{Total time taken}} = \frac{V_1t + V_2t}{t + t} = \frac{V_1 + V_2}{2}$

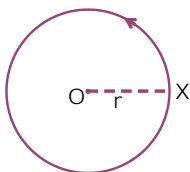
II. Average Velocity =  $\frac{\text{Total Displacement}}{\text{Total time taken}} = \frac{s + s}{\frac{s}{V_1} + \frac{s}{V_2}} = \frac{2V_1V_2}{V_1 + V_2}$

12. Distance = Speed  $\times$  time  $\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{1.2\text{km}}{15\text{ms}^{-1}} = \frac{1200\text{m}}{15\text{ms}^{-1}} = 80\text{s}$

15.  $a = \frac{dv}{dt}$ , If velocity changes, definitely there will be acceleration. If speed changes, then velocity also changes, so definitely there will be acceleration. Acceleration may be due to change in the direction of velocity only and not magnitude.

If body has acceleration, its speed may changes if acceleration is due to change in magnitude of velocity.

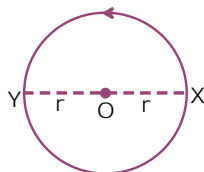
19. Let the cyclist start from point X. radius of circle OX = r (given)



Distance covered by cyclist = Circumference of circle (once) =  $2\pi r$   
 Initial position of cyclist = X. After one revolution, final position = X  
 Since initial position and final position are same displacement is zero.

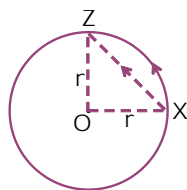


20. Distance covered = Half of the circumference of circle =  $\frac{1}{2} \times 2\pi r = \pi r$



Initial position = X, final position = Y Magnitude of displacement XY = ?  
From figure XY = XO + OY = r + r = 2r.

21. Distance covered by cyclist



= Quadrant of circle =  $\frac{1}{4} \times$  Circumference of circle =  $\frac{1}{4} \times 2\pi r = \frac{\pi r}{2}$

Initial position = X Final position = Z; Magnitude of displacement = XZ = ?  
Consider right triangle OXZ, we have from pythagorus theorem,

$$XZ = \sqrt{(OX)^2 + (OZ)^2} = \sqrt{(r)^2 + (r)^2} = \sqrt{2r^2} = \sqrt{2}r$$

### MOTION\_WORKSHEET-2\_KEY

**CUO:** 1. 3 2. 1 3. 1 4. 1 5. 2 6. 1 7. 3 8. 1  
9. 3 10. 4 11. 3 12. 1 13. 2 14.1

### JEE MAINS

1. 1 2. 1 3.2 4.3 5. 1 6. 1 7. 2 8. 4  
9. 3 10. 3 11. 2,3 12. 3 13.1,2,3,4 14.4 15. 3  
16. 2 17. 1 18. a-2,4;b-3;c-2;d-1 19. a-4;b-3;c-2;d-1

### HINTS AND SOLUTIONS:

9. The acceleration of a freely falling body does not depend upon the mass of the body.

10. Using  $v^2 - u^2 = 2as$ , we get  $v^2 = 2gh \Rightarrow v = \sqrt{2gh}$

14.  $h = \frac{1}{2}gt^2 = \frac{1}{2} \times 9.8 \times 3 \times 3 = 44.1 \text{ m}$

16. Given,  $g = 10\text{ms}^{-1}$  and  $h=20 \text{ m}$ ; We have  $v = \sqrt{2gh}$

## MOTION\_WORKSHEET-3\_KEY

1. 3      2. 2      3. 1      4. 2      5. 2      6. 2      7. 4  
 8. 1      9. 3      10. 2      11. 2      12. 1      13. 1      14. 3

## JEE MAINS

1. 1      2. 2      3. 1      4. 3      5. 4      6. 1      7. 2  
 8. 2      9. 4      10. 3      11. 2      12. 1      13. 1,3      14. 1,2,3  
 15. 3      16. 3      17. 1      18. 2      19. 4      20. a-4,5; b-2,5; c-2,5; d-3,5

## HINTS AND SOLUTIONS

6.  $u = 20\text{ms}^{-1}$ ,  $g = 10\text{ms}^{-2}$ ,  $t = 2\text{s}$ ,  $h = ?$

$$h = ut - \frac{1}{2}gt^2 = 20 \times 2 - \frac{1}{2} \times 10 \times (2)^2 = 40 - 20 = 20\text{m}.$$

7. displacement = shortest distance between initial point and final point

8. time of ascent = time of descent      17.  $h_{\text{max}} \propto u^2$ ;  $\frac{h}{h'} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

14. For vertically projected body, if it returns to the starting point, displacement and average velocity become zero. As acceleration is constant, average speed during upward or downward motion is  $(u+0)/2 = u/2$ . The same will be the average speed for the whole motion.

17.  $\therefore$  velocity after 5s =  $0 + 2 \times 5 = 10\text{m/s}$

18. height upto this point =  $\frac{1}{2}at^2 = \frac{1}{2} \times 2 \times (5)^2 = 25\text{m}$

20. further height travelled by the body after 5s. =  $\frac{0^2 - 10^2}{-2g} = 5\text{m}$

$\therefore$  Total height =  $25 + 5 = 30\text{m}$

Total time = time taken from A to B + time taken from B to max ht (C) and then to the ground.

$t = 5 + t'$ . How to get  $t'$ ?  $s = +25\text{m}$ ,  $u = 10\text{m/s}$ ,  $t = t' = ?$ ,  $g = 10\text{m/s}^2$ .

$$25 = 10t' - \frac{1}{2} \times 10 \times t'^2 \Rightarrow 25 = 10t' - 5t'^2 \Rightarrow 5 = 2t' - t'^2$$

On solving this we get,  $t' = 1 + \sqrt{6}\text{s}$   $\therefore$  total time  $t = 5 + 1 + \sqrt{6} = 6 + \sqrt{6}\text{s}$

## MOTION\_WORKSHEET-4\_KEY

1. 2    2. 1    3. 4    4. 4    5. 2    6. 1    7. 1    8. 3  
 9. 2    10. 3    11. 2    12. 1    13. 3    14. 2    15. 2    16. 4  
 17. 4    18. 1    19. 4    20. 1    21. 3

## JEE MAINS

1. 1    2. 1    3. 1    4. 2    5. 2    6. 1    7. 4    8. 1  
 9. 1    10. 1,2,3,4    11. 2    12. 2    13. 4    14. 2  
 15. a-1;b-2;c-3;d-4

## HINTS AND SOLUTIONS

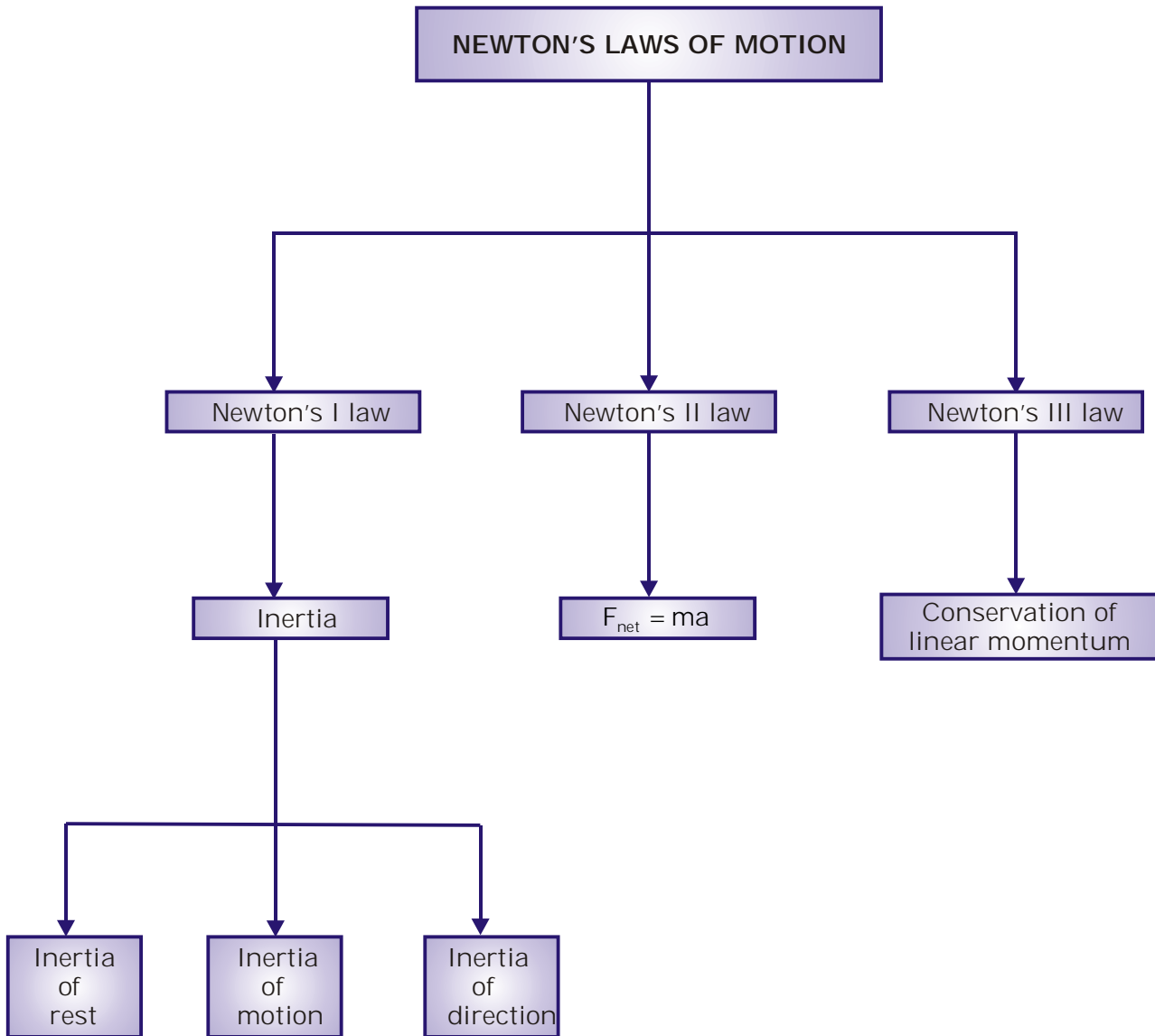
9. Area of  $v-t$  graph gives distance

10. The slope of velocity - time graph gives acceleration.  $\text{acceleration} = \frac{40}{10} = 4\text{m/s}^2$

11. Car moves with uniform velocity from A to B. So, the acceleration becomes zero.

12. Retardation =  $\frac{40}{20} = 2\text{m/s}^2$

MEMO GRAPH





## KNOW YOUR SCIENTIST

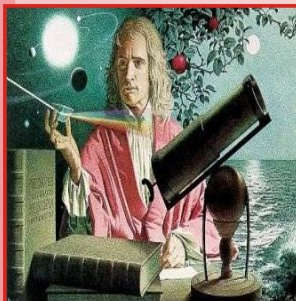


**Galileo Galilei**  
(1564 - 1642)

### Galileo Galilei (1564 - 1642)

Galileo Galilei was a key figure in the scientific revolution in Europe about four centuries ago. Galileo proposed the concept of acceleration. From experiments on motion of bodies on inclined planes or falling, freely, he contradicted the Aristotelian notion that a force was required to keep a body in motion and that heavier bodies fall faster than lighter bodies under gravity. He thus arrived at the law of inertia that was the starting point of the subsequent epochal work of Isaac Newton. Newton brought out another masterpiece optics that summarized his work on light and colour.

### Issac Netwon (1643 - 1727)



In 1684, encouraged by his friend Edmund Halley, Netwon embarked on wring what was to be The principia Mathematica was one of the greatest scientific works ever published. He enunciated all three laws of motion and the universal law of gravitation, which explained all the three Kepler's laws of planetary motion. The book was packed with a host of path - breaking achievements: basic principles of fluid mechanics, mathematics of wave motion, calculation of masses of the earth, the sun and other planets, explanation of the precession of equinoxes, the tides, etc. Newton brought out another masterpiece Opticks that summarized his work on light and colour.

## NEWTON'S LAWS OF MOTION

### SYNOPSIS-1

Isaac Newton (a 17th century scientist) put forth a variety of laws that explain why objects move (or don't move) as they do. These three laws have become known as Newton's three laws of motion.

#### Inertia :

- \* It is the inability of a body to change its state of rest or of uniform motion or its direction by itself.
- \* Mass is a measure of inertia in translatory motion
- \* Heavier the mass, larger the inertia & vice-versa.

**Types of inertia:** There are three types of inertia. (i) Inertia of rest (ii) Inertia of motion and (iii) Inertia of direction.

- \* **Inertia of rest:** It is the inability of a body to change its state of rest by itself .

**Ex:**When a bus is at rest and starts suddenly moving forward the passengers inside it will fall back.

**Inertia of motion:** It is the inability of a body to change its state of uniform motion by itself.

**Ex:** Passengers in a moving bus fall forward, when brakes are applied suddenly.

**Inertia of direction:** It is the inability of a body to change its direction of motion by itself .

**Ex:**When a bus takes a turn, passengers in it experience an outward force.

- \* A person sitting in a moving train, throws a coin vertically upwards, then
  - i) it falls behind him, if the train is accelerating
  - ii) it falls in front of him, if the train is retarding
  - iii) it falls into the hand of the person, if the train is moving with uniform velocity.
  - iv) It falls into the hand of the person if the train is at rest

#### Newton's First Law ( law of Inertia)

- \* Every body continues to be in its state of rest (or) uniform motion in a straight line unless it is acted upon by a net external force to change its state
- \* It defines inertia, force and mechanical equilibrium.
- \* If the net external force on an object is zero , then acceleration of object is zero.

#### Linear momentum :

- \* Linear momentum is the product of the mass of a body and its velocity.  $\vec{p} = m\vec{v}$
- \* Linear momentum is a vector. It has the same direction as the direction of velocity of the body. SI unit:  $\text{kg m s}^{-1}$  , CGS unit:  $\text{g cm s}^{-1}$
- \* Dimensional Formula :  $[\text{MLT}^{-1}]$

**Change in momentum of a body in different cases :**

- \* Consider a body of mass  $m$  moving with velocity  $\vec{v}_i$  and momentum  $\vec{P}_i$ . Due to a collision (or) due to the action of a force on it suppose its velocity changes to  $\vec{v}_f$  and momentum changes to  $\vec{P}_f$  in a small time interval  $\Delta t$ .

Change in momentum of body =  $\Delta\vec{P} = \vec{P}_f - \vec{P}_i$

Where  $P_i$  = initial momentum  $P_f$  = final momentum  $\Delta\vec{P} = m\vec{v}_f - m\vec{v}_i$

$|\Delta\vec{P}| = |\vec{P}_f - \vec{P}_i| = \sqrt{P_f^2 + P_i^2 - 2P_fP_i \cos\theta}$  where  $\theta$  = angle between  $\vec{P}_f$  and  $\vec{P}_i$

**Newton's second law:**

- \* The rate of change of momentum of a body is directly proportional to the resultant (or) net external force acting on the body and takes place along the direction of force.

\* 
$$\vec{F} = \frac{m\vec{v} - m\vec{u}}{t} = \frac{m(\vec{v} - \vec{u})}{t}$$

- \* Force is a vector and the acceleration produced in the body is in the direction of net force,
- \* SI unit : newton (N). CGS unit : dyne.
- \* One newton =  $10^5$  dyne.
- \* Dimensional Formulae =  $[MLT^{-2}]$

**Gravitational units of force:** Kilogram weight (kg wt) and gram weight (g wt);  
 1 kg.wt = 9.8 N, 1 gm.wt = 980 dyne.

**Newton's third law:**

- \* For every action there is always an equal and opposite reaction
- \* Action and reaction do not act on the same body and they act on different bodies at same instant of time
- \* Action and reaction, known as pair of forces, are equal in magnitude and opposite in directions acting on different bodies in interaction. So they never cancel each other.

**Examples:**

- \* When we walk on a road we push the road backwards and road applies equal (in magnitude) and opposite force on us, so that we can move forward.
- \* When we swim on water we push water backward and water applies equal (in magnitude) and opposite force on us, so that we can move forward.



## WORKSHEET-1

- CUQ** 1. The behaviour of a body under zero resultant force is given by
- 1) First law of motion
  - 2) Second law of motion
  - 3) Third law of motion
  - 4) Law of gravitation
2. You move forward when your car suddenly comes to a halt and you are thrown backward when your car rapidly accelerates. Which law of Newtons is involved in these?
- 1) third law
  - 2) second law
  - 3) first law
  - 4) law of gravitation
3. You are thrown out side when your car suddenly takes a turn. Which law of Newton is involved in this?
- 1) third law
  - 2) second law
  - 3) first law
  - 4) law of gravitation
4. An object is thrown vertically upward with some velocity. If gravity is turned off at the instant the object reaches the maximum height, what happens?
- 1) the object continues to move in a straight line
  - 2) the object will be at rest
  - 3) the object falls back with uniform velocity
  - 4) the object falls back with uniform acceleration
5. Which of the following is the most significant law of motion given by Newton?
- 1) First law of motion
  - 2) Second law of motion
  - 3) Third law of motion
  - 4) Zeroth law of motion
6. The quantity of motion of a body is best represented by
- 1) its mass
  - 2) its speed
  - 3) its velocity
  - 4) its linear momentum
7. A certain particle undergoes erratic motion. At every point in its motion, the direction of the particle's momentum is always
- 1) the same as the direction of its velocity
  - 2) the same as the direction of its acceleration
  - 3) the same as the direction of its net force
  - 4) the same as the direction of its kinetic energy
8. To keep a particle moving with constant velocity on a frictionless surface, an external force
- 1) should act continuously
  - 2) should be a variable force
  - 3) not necessary
  - 4) should act opposite to the direction of motion
9. If action force acting on a body is gravitational in nature, then reaction force
- 1) may be a contact force
  - 2) may be gravitational force
  - 3) may be a gravitational or contact force
  - 4) may be a force of any origin



10. Action and reaction can never balance out because
- 1) they are equal but not opposite always
  - 2) they are unequal in magnitude even though opposite in direction
  - 3) though they are equal in magnitude and opposite in direction they act on different bodies
  - 4) they are unequal in magnitudes
11. You hold a rubber ball in your hand. The Newton's third law companion force to the force of gravity on the ball is the force exerted by the
- 1) ball on the earth      2) ball on the hand
  - 3) hand on the ball      4) earth on the ball

### JEE MAINS

#### Single Correct Choice Type:

1. A constant force ( $F$ ) is applied on a stationary particle of mass ' $m$ '. The velocity attained by the particle in a certain displacement will be proportional to
- 1)  $m$
  - 2)  $1/m$
  - 3)  $\sqrt{m}$
  - 4)  $\frac{1}{\sqrt{m}}$
2. A constant force ( $F$ ) is applied on a stationary particle of mass ' $m$ '. The velocity attained by the particle in a certain interval of time will be proportional to
- 1)  $m$
  - 2)  $1/m$
  - 3)  $\sqrt{m}$
  - 4)  $\frac{1}{\sqrt{m}}$
3. A force produces an acceleration of  $a_1$  in a body and the same force produces an acceleration of  $a_2$  in another body. If the two bodies are combined and the same force is applied on the combination, the acceleration produced in it is
- 1)  $a_1 + a_2$
  - 2)  $\frac{a_1 + a_2}{a_1 a_2}$
  - 3)  $\frac{a_1 a_2}{a_1 + a_2}$
  - 4)  $\sqrt{a_1 a_2}$
4.  $n$  balls each of mass  $m$  impinge elastically in each second on a surface with velocity  $u$ . The average force experienced by the surface will be
- 1)  $mnu$
  - 2)  $2mnu$
  - 3)  $4mnu$
  - 4)  $mnu/2$
5. A ball reaches a racket at 60 m/s along + X direction, and leaves the racket in the opposite direction with the same speed. Assuming that the mass of the ball as 50gm and the contact time is 0.02 second, the force exerted by the racket on the ball is
- 1) 300 N along + X direction
  - 2) 300 N along - X direction
  - 3) 3,00,000 N along + X direction
  - 4) 3,00,000 N along - X direction

6. 'P' and 'Q' horizontally push in the same direction a 1200 kg crate. 'P' pushes with a force of 500 newton and 'Q' pushes with a force of 300 newton. If a frictional force provides 200 newton of resistance, what is the acceleration of the crate?
- 1)  $1.3 \text{ m/s}^2$     2)  $1.0 \text{ m/s}^2$     3)  $0.75 \text{ m/s}^2$     4)  $0.5 \text{ m/s}^2$
7. A ball of mass ' $m$ ' moves normal to a wall with a velocity ' $u$ ' and rebounds with a velocity ' $v$ '. The change in momentum of the ball during the rebounding is
- 1)  $m(u+v)$  towards the wall    2)  $m(u-v)$  towards the wall
- 3)  $m(u+v)$  away from the wall    4)  $m(u-v)$  away from the wall.
8. If a force of 250N acts on a body, the momentum required is  $125 \text{ kgms}^{-1}$ . The period for which the force acts on the body is
- 1)  $0.1 \text{ s}$     2)  $0.3 \text{ s}$     3)  $0.5 \text{ s}$     4)  $0.2 \text{ s}$
9. A machine gun fires a bullet of mass 40g with a velocity  $1200 \text{ ms}^{-1}$ . The man holding it can exert a maximum force of 144 N on the gun. How many bullets can he fire per second at the most?
- 1) One    2) Three    3) Two    4) Four
10. A truck of mass 500kg is moving with constant speed  $10 \text{ ms}^{-1}$ . If sand is dropped into the truck at the constant rate  $10 \text{ kg/min}$ , the force required to maintain the motion with constant velocity is
- 1)  $\frac{3}{2} \text{ N}$     2)  $\frac{5}{4} \text{ N}$     3)  $\frac{7}{5} \text{ N}$     4)  $\frac{5}{3} \text{ N}$
11. A 5000 kg rocket is set for vertical firing. The exhaust speed is  $800 \text{ ms}^{-1}$ . To give an upward acceleration of  $20 \text{ ms}^{-2}$ , the amount of gas ejected per second to supply the needed thrust is ( $g = 10 \text{ ms}^{-2}$ )
- 1)  $127.5 \text{ kg s}^{-1}$     2)  $137.5 \text{ kg s}^{-1}$     3)  $187.5 \text{ kg s}^{-1}$     4)  $185.5 \text{ kg s}^{-1}$
12. A 0.2 kg object at rest is subjected to a force  $(0.3\hat{i} - 0.4\hat{j}) \text{ N}$ . What is its velocity vector after 6 sec
- 1)  $(9\hat{i} - 12\hat{j})$     2)  $(8\hat{i} - 16\hat{j})$     3)  $(12\hat{i} - 9\hat{j})$     4)  $(16\hat{i} - 8\hat{j})$

### Multi Correct Choice Type:

13. Force effects
- 1) A stationary object into motion    2) A moving body to stop
- 3) Direction of moving body    4) Dimensions of the body



19. S I: If two objects of different masses have same momentum, the lighter body posses greater velocity.  
S II: For all bodies momentum always remains same if force applied is same. For a given time
- 1) Both I & II True
  - 2) Both I & II False
  - 3) Statement I is True & Statement II is False
  - 4) Statement I is False & Statement II is True

**Multi Correct Choice Type:**

20. Choose the correct statements:
- 1) Both action and reaction are forces
  - 2) Action and rection act simultaneously but act on differnt bodies
  - 3) Action and reaction cannot cancel each other
  - 4) Action and reaction forces occurs in pairs only
21. Choose the correct statements:
- 1)  $1\text{kg-wt} = 9.8 \text{ kg m /s}^2$
  - 2)  $1\text{g-wt} = 980 \text{ g m /s}^2$
  - 3)  $1\text{kg-wt} = 980 \text{ g cm /s}^2$
  - 4)  $1\text{g-wt} = 980 \text{ dyne}$

**Comprehension Type:**

Inertia of a body may be defined as the tendency of a body to oppose any change in its state of rest or uniform motion.

22. Two bodies 'A' and 'B' of same masses are moving with velocities  $V$  and  $2V$  respectively. Then the ratio of their inertia is
- 1) 1 : 1
  - 2) 2 : 1
  - 3) 3 : 1
  - 4) 4 : 1
23. Which of the following has the largest inertia ?
- 1) A pin
  - 2) A pen
  - 3) Your physics book
  - 4) Your loaded school bags
24. An athlete runs some distance before taking a long jump, because
- 1) It helps him to gain energy
  - 2) It helps to apply large force
  - 3) It gives himself large amount of inertia
  - 4) None of these



## SYNOPSIS-2

### Law of conservation of momentum :

According to this law, the total momentum of a system remains constant if no net external force acts on the system.

That is, momentum of a system,  $\vec{p} = \text{constant}$ , if net external force acting on it is zero (i.e.  $\vec{F}_{\text{external}} = 0$ ).

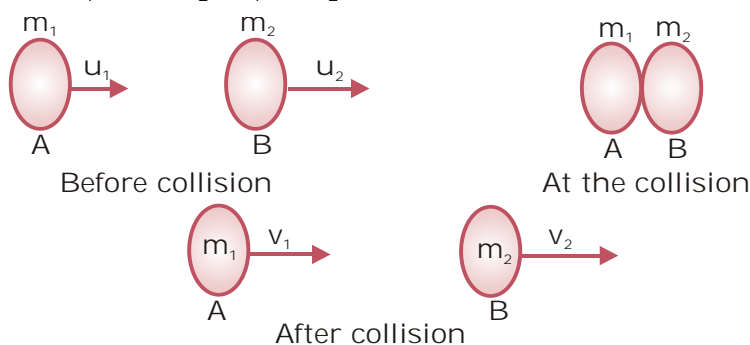
### Conservation of momentum from third law of motion :

If a number of bodies collide with one another the total momentum of the bodies just before collision is equal to the total momentum just after collision.

**Example :** Let a moving ball collides with another stationary ball lying on a ground. Observe, what happens after collision? The moving ball will slow down i.e., its velocity decreases after colliding with the stationary ball. On the other hand, the stationary ball begins to move i.e. its velocity increases after collision. We know, momentum of a body = mass of the body  $\times$  velocity of the body (i.e.  $\vec{p} = m\vec{v}$ ). Therefore, the momentum of moving ball decreases after collision and the momentum of the stationary ball increases after collision. Thus, we find that when two balls collide with each other, then moving ball loses momentum and the stationary ball gains momentum. The loss of momentum of one ball is equal to the gain of momentum of other ball. However, the total momentum of these colliding balls before and after the collision remains the same. This is the law of conservation of momentum.

### Derivation :

Let us consider two marbles A and B having masses  $m_1$  and  $m_2$  moving with initial velocities,  $u_1$  and  $u_2$  ( $u_1 > u_2$ ) in the same direction.



After collision the two bodies move with velocities  $v_1$  and  $v_2$  in the same direction say as shown in figure.

Momentum of body A before collision =  $m_1 u_1$

Momentum of body B before collision =  $m_2 u_2$

$\therefore$  Total momentum of body A and B before collision =  $m_1 u_1 + m_2 u_2$

### Similarly :

momentum of body A after collision =  $m_1 v_1$

momentum of body B after collision =  $m_2 v_2$

$\therefore$  Total momentum of body A and B after collision =  $m_1 v_1 + m_2 v_2$

Suppose the two marbles collide with each other for 't' seconds.

The momentum of marble A before and after collision is  $m_1u_1$  and  $m_1v_1$  respectively.

⇒ Change of momentum of body A during the collision =  $m_1v_1 - m_1u_1$

∴ The rate of change of momentum during the collision of body A will be

$$= \frac{\text{change of momentum}}{\text{Time taken}} = \frac{m_1v_1 - m_1u_1}{t} = \frac{m_1(v_1 - u_1)}{t}$$

Similarly, the rate of change of momentum of marble B will be

$$\frac{m_2v_2 - m_2u_2}{t} = \frac{m_2(v_2 - u_2)}{t}$$

If the force exerted by marble A on B is  $F_1$  and that by B on A is  $F_2$ , then according to Newton's 2nd law of motion,

$$F_1 = \frac{m_1(v_1 - u_1)}{t} \quad \text{_____ (1)} \qquad F_2 = \frac{m_2(v_2 - u_2)}{t} \quad \text{_____ (2)}$$

According to Newton's 3rd law of motion, the force exerted by marble A on B and marble B on marble A are equal and opposite to each other.

$$\therefore F_1 = -F_2$$

$$\Rightarrow \frac{m_1(v_1 - u_1)}{t} = -\frac{m_2(v_2 - u_2)}{t} \Rightarrow m_1v_1 - m_1u_1 = -m_2v_2 + m_2u_2$$

$$\Rightarrow m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2 \text{ or, } m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

i.e. total momentum before collision = total momentum after collision.

**Note :** If the two bodies stick together after collision, then they move with common

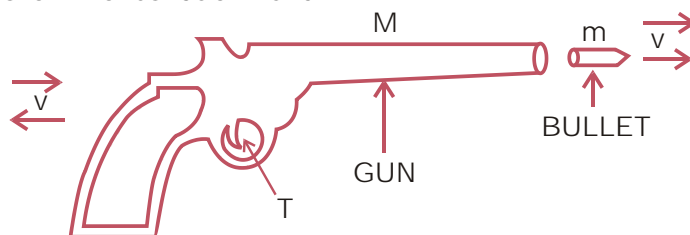
$$\text{velocity } v \text{ (say) then } m_1u_1 + m_2u_2 = m_1v + m_2v \Rightarrow v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}$$

**Examples to illustrate the law of conservation of momentum :**

1. **Recoil of a gun :** We have already explained the recoil of a gun with the help of Newton's third law of motion. However, the recoil of a gun can also be explained with the help of the law of conservation of momentum.

The bullet inside the gun and the gun itself forms a system. Thus, the system is (gun + bullet). Before firing, the gun and the bullet are at rest, therefore, momentum of the system before firing is zero.

When the bullet is fired, it leaves the gun in the forward direction with certain momentum. Since no external force acts on the system, so the momentum of the system (gun + bullet) must be zero after firing. This is possible only if the gun moves backward with a momentum equal to the momentum of the bullet. That is why gun recoils or moves backward.



Gun and bullet

The velocity with which the gun moves backward after firing a bullet is known as recoil velocity.

**Calculation of Recoil Velocity of a gun :**

Let, mass of the bullet = m;

velocity of the bullet after firing =  $\vec{v}$ ; mass of the gun = M

Recoil velocity of the gun after firing =  $\vec{V}$

Since the system is at rest before firing. So momentum of the system (gun + bullet) before firing = 0

Total momentum of the system (gun + bullet) after firing

= momentum of gun + momentum of bullet =  $M\vec{V} + m\vec{v}$

According to the law of conservation of momentum,

$$M\vec{V} + m\vec{v} = 0 \Rightarrow M\vec{V} = -m\vec{v} \text{ or } \vec{V} = -\left(\frac{m}{M}\right)\vec{v}$$

Negative sign shows that the direction of the velocity of the gun after firing is opposite to the direction of the velocity of the bullet.

Action and reaction being equal and opposite and acting simultaneously for same duration, have equal and opposite impulses. They produce equal and opposite changes of momentum in the pair of bodies involved. It keeps the total momentum of the two body system constant (conserved)

**Impulsive Force :** A large force which acts for a small interval of time is called impulsive force.

**Impulse :** Impulse of a force is defined as the change in momentum produced by the given force and it is equal to the product of force and the time for which it acts. According to Newton's 2<sup>nd</sup> law of motion

$$\vec{F} = m\vec{a} = m \left( \frac{\vec{v} - \vec{u}}{t} \right) = \frac{m\vec{v} - m\vec{u}}{t} \Rightarrow \vec{F}t = m\vec{v} - m\vec{u}$$

Impulsive force = change in momentum.

For constant force,  $J = F \times t$ ,

The area bounded by the force-time graph measures Impulse.

**Note :** Impulse is a vector quantity, whose direction is same as that of force.

**Unit :** S. I. unit of impulse = N s or kg m/s

C.G.S unit of impulse = dyne second or g cm/s.

**Application of Impulse :**

- a) shock absorbers are used in vehicles to reduce the magnitude of impulsive force.
- b) A cricketer lowers his hands, while catching the ball to reduce the impulsive force.

**Elastic & Inelastic Collisions :** During collisions, there will be an exchange of momentum between the bodies. However the kinetic energy of the bodies may remain constant or change. Accordingly, the collisions are classified into two kinds. They are

- 1) Elastic Collisions and
- 2) Inelastic collisions.

**Elastic Collisions :** The collisions in which both momentum and kinetic energy are conserved are known as elastic collisions. The collision between nuclei and fundamental particles are elastic collisions.

**Inelastic Collisions :** The collisions in which kinetic energy is not conserved but law of conservation of momentum holds good are known as inelastic collisions. When two bodies stick together after collision, the collision is said to be completely inelastic. The collision between a bullet and its target when the bullet remains embedded in the target is an example of perfectly inelastic collision. During these collisions the loss of kinetic energy appears in the form of heat or as excitation energy as in the case of atomic collisions.

**ACTIVITY :-**

Take a big rubber balloon and inflate it fully. Tie its neck using a thread. Also using adhesive tape, fix a straw on the surface of this balloon.

Pass a thread through the straw and hold one end of the thread in your hand or fix it on the wall.

Ask your friend to hold the other end of the thread or fix it on a wall at some distance.

Now remove the thread tied on the neck of balloon. Let the air escape from the mouth of the balloon.

Observe the direction in which the straw moves.

## WORK SHEET-2

- CUQ** 1. Mutual forces between two bodies are always
- |                                    |                       |
|------------------------------------|-----------------------|
| 1) Unequal and opposite            | 2) Equal and opposite |
| 3) Equal and in the same direction | 4) Unequal            |
2. The recoil of a gun can be explained with the help of the
- |                       |                        |
|-----------------------|------------------------|
| 1) Newton's First Law | 2) Newton's Second Law |
| 3) Newton's Third Law | 4) None of these       |
3. According to Newton's Second Law of motion impulsive force is equal to
- |               |               |                          |                          |
|---------------|---------------|--------------------------|--------------------------|
| 1) $m\vec{v}$ | 2) $m\vec{u}$ | 3) $m\vec{v} - m\vec{u}$ | 4) $m\vec{v} + m\vec{u}$ |
|---------------|---------------|--------------------------|--------------------------|
4. The momentum of moving ball \_\_\_\_\_ after collision
- |         |              |              |         |
|---------|--------------|--------------|---------|
| 1) Zero | 2) Decreases | 3) Increases | 4) None |
|---------|--------------|--------------|---------|
5. The momentum of the stationary ball \_\_\_\_\_ after collision.
- |         |              |              |         |
|---------|--------------|--------------|---------|
| 1) Zero | 2) Decreases | 3) Increases | 4) None |
|---------|--------------|--------------|---------|
6. When a body is travelling at constant velocity, the net force on it is \_\_\_\_
- |          |          |      |             |
|----------|----------|------|-------------|
| 1) $< 1$ | 2) $> 1$ | 3) 0 | 4) $\infty$ |
|----------|----------|------|-------------|
7. The momentum and energy are conserved in this collision.
- |                      |                     |
|----------------------|---------------------|
| 1) Inelastic         | 2) Elastic          |
| 3) Perfectly elastic | 4) In any collision |
8. The momentum only conserved but not energy in this collision
- |                        |                     |
|------------------------|---------------------|
| 1) Inelastic           | 2) Elastic          |
| 3) Perfectly inelastic | 4) In any collision |



9. After collision they stick together in this collision
- 1) Perfectly inelastic
  - 2) Perfectly elastic
  - 3) Elastic
  - 4) Inelastic

### JEE MAINS

#### Single Correct Choice Type:

1. The momentum of the system is conserved
  - 1) Always
  - 2) Never
  - 3) Only in absence of external force
  - 4) Only when an external force acts
2. Conservation of linear momentum is a direct consequence of
  - 1) Newton's first law of motion
  - 2) Newton's second law of motion
  - 3) Newton's third law of motion
  - 4) None of these
3. When two bodies of masses  $m_1$  and  $m_2$  moving with velocities  $u_1$  and  $u_2$  in the same direction collide with each other and  $v_1$  and  $v_2$  are their velocities after collision in the same direction, then
  - 1)  $m_1v_1 + m_2v_2 = m_2u_2 - m_1u_1$
  - 2)  $m_1v_1 + m_2v_2 = m_1u_1 - m_2u_2$
  - 3)  $m_2u_2 + m_2u_1 = m_2v_1 + m_1v_2$
  - 4)  $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
4. Which of the following is not a vector quantity?
  - 1) mass
  - 2) Impulse
  - 3) Momentum
  - 4) Force
5. Choose the correct answer
  - 1). S.I Unit of impluse is N-S
  - 2) C.G.S Unit of Impluse in N-S
  - 3). Impluse is a scalar quantity
  - 4) Impluse has no direction
6. A bullet of mass 100g is fired from a gun of mass 20 kg with a velocity of  $100\text{ms}^{-1}$ . Then the velocity of recoil of the gun is
  - 1)  $-1.5\text{ m/s}$
  - 2)  $-0.5\text{ m/s}$
  - 3)  $-2.5\text{ m/s}$
  - 4)  $3.5\text{ m/s}$
7. Statement I: Collision between two particles is not necessarily associated with physical contact between them.  
Statement II: Only in physical contact momentum transfer takes place.
  - 1) Both I & II True
  - 2) Both I & II False
  - 3) Statement I is True & Statement II is False
  - 4) Statement I is False & Statement II is True
8. The car A of mass 1500 kg travelling at 25 m/s collides with another car B of mass 1000 kg travelling at 15 m/s in the same direction. After collision the velocity of car A becomes 20 m/s. The velocity of car B after the collision is
  - 1) 12.2 m/s
  - 2) 11.5 m/s
  - 3) 22.5 m/s
  - 4) 5.22 m/s
9. A body of mass 2Kg moving with uniform velocity of  $40\text{ms}^{-1}$  collides with another body at rest. If the two bodies move together with a velocity of  $25\text{ms}^{-1}$ , mass of the other body is
  - 1) 0.6 Kg
  - 2) 0.9 Kg
  - 3) 1.2 Kg
  - 4) 1.5Kg

10. A body of mass 6Kg travelling with a velocity of  $10\text{ms}^{-1}$  collides elastically with a body of mass 4Kg travelling at a speed of  $5\text{ms}^{-1}$  in opposite direction and comes to rest. Then velocity of the second body is
- 1) 0                                      2)  $6\text{ms}^{-1}$                                       3)  $8\text{ms}^{-1}$                                       4)  $10\text{ms}^{-1}$
11. A truck weighing 2500 kg and moving with a velocity  $28\text{ ms}^{-1}$  collides with a stationary car weighing 300 kg. The truck and the car move together after the impact. Then their common velocity is
- 1) 25 m/s                                      2) 20 m/s                                      3) 15 m/s                                      4) 10 m/s
12. A body of mass 20Kg moving with a velocity of  $20\text{ms}^{-1}$  collides with another body of mass 40Kg moving in the same direction with a velocity of  $10\text{ms}^{-1}$ . After collision, if they move together, their common velocity is
- 1)  $\frac{10}{3}\text{ms}^{-1}$                                       2)  $\frac{20}{3}\text{ms}^{-1}$                                       3)  $\frac{40}{3}\text{ms}^{-1}$                                       4)  $\frac{50}{3}\text{ms}^{-1}$
13. When two bodies of masses  $m_1$  and  $m_2$  moving with velocities  $u_1$  and  $u_2$  in the opposite direction collide with each other and move together after collision in the same direction with a common velocity  $v$ , then
- 1)  $v = \frac{m_1u_1 - m_2u_2}{m_1 + m_2}$                                       2)  $m_2u_1 - m_1u_2 = (m_1 + m_2)v$
- 3)  $m_1u_1 - m_2u_2 = (m_1 - m_2)v$                                       4)  $m_1u_1 + m_2u_2 = m_1v - m_2v$
14. A sphere of mass 25Kg moving with a velocity of  $40\text{ms}^{-1}$  collides with another sphere of mass 15Kg which is at rest. After collision if they move with the same velocity, that velocity is equal to
- 1)  $25\text{ms}^{-1}$                                       2)  $40\text{ms}^{-1}$                                       3)  $15\text{ms}^{-1}$                                       4)  $12\text{ms}^{-1}$
15. Two spheres of masses 2Kg and 3Kg travelling in opposite direction with velocities  $8\text{ms}^{-1}$  and  $6\text{ms}^{-1}$  collide. If the collision is perfectly inelastic, then final velocity is
- 1) 0.1m/s                                      2) 0.2m/s                                      3) 0.3m/s                                      4) 0.4m/s

**Reasoning Type:**

16. *Statement I* : A large force which acts for a small interval of time is called impulsive force.
- Statement II* : The total momentum of a system remains constant if no net external force acts on the system.
- 1) Both Statements are true, Statement II is the correct explanation of Statement I.
- 2) Both Statements are true, Statement II is not correct explanation of Statement I.
- 3) Statement I is true, Statement II is false.
- 4) Statement I is false, Statement II is true.



## KEY & SOLUTIONS

### NEWTON'S LAWS OF MOTION\_

#### WORK SHEET-1

**CUQ:** 1) 1      2) 3      3) 3      4) 2      5) 2      6) 4      7) 1      8) 3  
           9) 2      10) 3      11) 1

**JEE MAINS :** 7) 3      8) 3      9) 2      10) 4      11) 3      12) 1      13) 1,2,3,4  
           14) 1      15) 2,4      16) 1,2,3,4      17) 1,2,3      18) a-5,b-3,c-4, d-1,2  
           19) 1      20) 1,2,3,4      21) 1,3,4      22) 1      23) 4      24) 3

#### WORK SHEET-2

**CUQ:** 1. 2      2. 3      3. 3      4. 2      5. 3  
           6. 3      7. 2      8. 1      9. 1

**JEE MAINS :**

1. 4      2. 3      3. 4      4. 1      5. 1      6. 2  
 7. 4      8. 3      9. 3      10. 4      11. 1      12. 3  
 13. 1      14. 2      15. 1      16. 1      17. 3  
 18. a-1,2; b-1,3; c-4; d-5      19. 3      20. 2      21. 2      22. 1

#### HINTS & SOLUTIONS

6.  $m_1v_1 = m_2v_2$        $m_1 = 100\text{g} = 0.1\text{kg}$ ,  $m_2 = 20\text{kg}$   
 $v_1 = 100\text{m/s}$ ;       $0.1 \times 100 = -20v_2$ ;       $v_2 = 0.5\text{m/s}$

8.  $m_A = 1500\text{kg}$        $m_B = 1000\text{kg}$   
 $u_1 = 25\text{m/s}$        $u_2 = 15\text{m/s}$   
 $v_1 = 20\text{m/s}$        $v_2 = ?$

According to conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$1500 \times 25 + 1000 \times 15 = 1500 \times 20 + 1000 \times v_2$$

$$v_2 = 22.5\text{m/s}$$

9.  $m_1u_1 + m_2u_2 = (m_1 + m_2)v$ ;      10.  $m_1u_1 - m_2u_2 = m_1v_1 + m_2v_2$

11.  $m_1u_1 + m_2u_2 = (m_1 + m_2)v$

\*\*\*\*\*

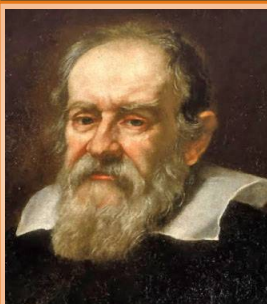




## CONTENTS

Conservation of linear momentum  
Impulse

## KNOW YOUR SCIENTIST

Galileo Galilei  
(1564 - 1642)

Galileo Galilei (1564 - 1642)

Galileo Galilei was a key figure in the scientific revolution in Europe about four centuries ago. Galileo proposed the concept of acceleration. From experiments on motion of bodies on inclined planes or falling, freely, he contradicted the Aristotelian notion that a force was required to keep a body in motion and that heavier bodies fall faster than lighter bodies under gravity. He thus arrived at the law of inertia that was the starting point of the subsequent epochal work of Isaac Newton. Newton brought out another masterpiece optics that summarized his work on light and colour.

## CONSERVATION OF LINEAR MOMENTUM

### SYNOPSIS-1

**Linear momentum**

1. The product of mass and the velocity of a particle is defined as its linear momentum ( $\vec{P}$ ). So,  $\vec{P} = m\vec{v}$

The magnitude of linear momentum may be written as  $P = mv$

S.I. unit of momentum is  $\text{kgm/s}$

C.G.S unit of momentum is  $\text{gcm/s}$

**Relation between momentum and kinetic energy**

$$\text{Thus, } P = \sqrt{2Km} \quad \text{or} \quad K = \frac{P^2}{2m}$$

Here,  $K$  is the kinetic energy of the particle. In accordance with Newton's second law,

$$\vec{F} = m\vec{a} = m \frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{P}}{dt} \quad \text{Thus, } \vec{F} = \frac{d\vec{P}}{dt}$$

In case the external force applied to a particle (or a body) be zero, we have

$$\vec{F} = \frac{d\vec{P}}{dt} = 0 \quad \text{or} \quad \vec{p} = \text{constant}$$

Showing that in the absence of an external force, the linear momentum of a particle (or the body) remains constant. This is called the law of conservation of linear momentum. The law may be extended to a system of particles or to the centre of mass of a system of particles. For example, for a system of particles it takes the form.

If net force (or the vector sum of all the forces) on a system of particles is zero, the vector sum of linear momentum of all the particles remain conserved, or

$$\text{If } \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots + \vec{F}_n = 0 \quad \text{Then, } \vec{P}_1 + \vec{P}_2 + \vec{P}_3 + \dots + \vec{P}_n = \text{constant}$$

The same is the case for the centre of mass of a system of particles, i.e., if

$$\vec{F}_{\text{COM}} = 0, \vec{P}_{\text{COM}} = \text{constant}.$$

Thus, the law of conservation of linear momentum can be applied to a single particle, to a system of particles or even to the centre of mass of the particles.

The law of conservation of linear momentum enables us to solve a number of problems which can not be solved by a straight application of the relation  $\vec{F} = m\vec{a}$ .

- i. When no external force acts upon a system, then the vector sum of momenta of all bodies of that system remains constant.

For any collision conservation of momentum holds good.

$$\vec{F} = \frac{d\vec{P}}{dt} \text{ and when } \vec{F} = 0, \vec{P} \text{ is constant}$$

- ii. Mathematical treatment of Newton's third law gives law of conservation of linear momentum.
- iii. Examples for applications of law of conservation of linear momentum are (a) collisions (b) recoil of gun when bullet is fired (c) explosion of bomb shell.
- iv. If a bullet of mass  $m$  is fired from a gun of mass  $M$  with velocity  $u$ , recoil

$$\text{velocity of the gun is } V = \frac{mu}{M}$$

Here total momentum of the system before and after firing is zero.

Here relative velocity of bullet with respect to gun is  $u_{\text{rel}} = (u + v)$

$$\text{Here } \frac{u}{v} = \frac{M}{m}$$

If  $K_R$  and  $K_b$  are the kinetic energies of gun and bullet,  $\frac{K_g}{K_b} = \frac{m}{M}$

When a bullet is fired from a gun, both bullet and gun will have the same momentum in magnitude.

- v. If a stationary bomb shell explodes into a number of fragments, the vector sum of the momenta of all fragments adds up to zero.

$$\sum m_n V_n = 0$$

If a stationary shell explodes into two fragments of masses  $m_1$  and  $m_2$ , they move in opposite directions so that total momentum is zero.

$$\Rightarrow m_1V_1 + m_2V_2 = 0$$

$$\Rightarrow m_1V_1 = -m_2V_2 \quad (- \text{ve sign indicates the opposite direction of motion})$$

- vi. A stationary shell of mass M exploded into two fragments. A fragment of mass m flies off with velocity u. Then the other fragment flies off with velocity V given by the following relation

$$M(0) = mu + (M - m)V$$

$$V = \frac{-mu}{(M - m)}$$

(Negative sign indicates that the two fragments fly off in opposite directions)

- vii. A shell of mass M moving with velocity  $\vec{V}$  explodes into two fragments with the masses  $m_1$  and  $m_2$  which fly off with velocities  $\vec{V}_1$  and  $\vec{V}_2$ . Then from conservation of momentum,

$$v = \frac{\sqrt{(m_1V_1)^2 + (m_2V_2)^2}}{(m_1 + m_2)} \quad \text{where } M = (m_1 + m_2)$$

If the shell explodes into a number of fragments which move with different velocities,

$$M\vec{V} = m_1\vec{V}_1 + m_2\vec{V}_2 + \dots + m_n\vec{V}_n$$

$$\text{here } M = (m_1 + m_2 + \dots + m_n)$$

- viii. Two bodies of masses  $m_1$  and  $m_2$  are moving with velocities  $\vec{V}_1$  and  $\vec{V}_2$ . They collide and after collision they merge and the compound mass moved with a velocity  $\vec{V}$ .

$$\text{If } \vec{V}_1 \text{ and } \vec{V}_2 \text{ are perpendicular to each other, then } V = \frac{\sqrt{(m_1V_1)^2 + (m_2V_2)^2}}{(m_1 + m_2)}$$

- ix. In uniform circular motion, momentum is not conserved but K.E is conserved.  
 x. When a stationary bomb shell explodes final K.E is greater than the initial K.E.  
 xi. When a body is projected vertically up from the ground, momentum of the body



earth system is conserved.

## WORKSHEET-1

- CUQ** 1. Momentum is which quantity?
- 1) Scalar                      2) Vector                      3) Tensor                      4) All of these
2. The formulae for linear momentum is,  $\vec{p} =$
- 1)  $m\vec{V}$                       2)  $\frac{m}{\vec{V}}$                       3)  $m + \vec{V}$                       4)  $m - \vec{V}$
3. Units of momentum is
- 1) kg ms                      2) kg m/s                      3) kg m s<sup>2</sup>                      4) kgm/s<sup>2</sup>
4. Dimensional formula of momentum is
- 1) [MLT<sup>-2</sup>]                      2) [ML<sup>2</sup>T<sup>-2</sup>]                      3) [M<sup>2</sup>LT]                      4) [MLT<sup>-1</sup>]
5. The relation between linear momentum( $\vec{P}$ ) and kinetic energy(K.E) is
- 1)  $\frac{P^2}{m}$                       2)  $\frac{P^2}{2m}$                       3)  $\frac{P^2}{3m}$                       4)  $\frac{P^2}{4m}$
6. If the external force applied to a particle(or a body) be zero, then momentum( $\vec{P}$ ) is
- 1) Zero                      2) constant                      3) variable                      4) All of these
7. The change in momentum per unit time of a body represents
- 1) impulse                      2) force                      3) kinetic energy                      4) resultant force
8. The momentum of the system is conserved
- 1) always                      2) never  
3) only in absence of external force                      4) only when an external force acts
9. Law of conservation of linear momentum is
- 1) I law                      2) II law                      3) III law                      4) None of these
10. A shell explodes and many pieces fly off in different directions. Which of the following is conserved?
- 1) kinetic energy                      2) momentum  
3) neither momentum nor KE                      4) momentum and KE

## JEE MAIN &amp; ADVANCED

**LEVEL-1** Single Correct Choice Type:

- A cricket ball of mass 100g is moving with velocity 25m/s. Then the momentum of the ball is  
1) 7.5kg m/s      2) 19.5kg m/s      3) 2.5kg m/s      4) 27.5kg m/s
- A bullet of mass 20 g is horizontally fired with a velocity 150 ms<sup>-1</sup> from a pistol of mass 2 kg. What is the recoil velocity of the pistol?  
1) 1 ms<sup>-1</sup>      2) -1.5 ms<sup>-1</sup>      3) 2 ms<sup>-1</sup>      4) -3.5 ms<sup>-1</sup>
- A bullet of mass 5 g is fired at a velocity 900 ms<sup>-1</sup> from a rifle of mass 2.5 kg. What is the recoil velocity of the rifle ?  
1) -1.8 m/s      2) 1.8 m/s      3) 2.8 m/s      4) 3.8 m/s
- A shell of mass 40 kg moving with a velocity of 80 ms<sup>-1</sup> explodes and bursts into two fragments of masses 32 Kg and 8Kg. If the latter comes to rest, velocity of the former is  
1) 10 ms<sup>-1</sup>      2) 0.1 ms<sup>-1</sup>      3) 1000 ms<sup>-1</sup>      4) 100 ms<sup>-1</sup>
- A bullet of mass a and velocity b is fired into a large block of mass c. The final velocity of the system is  
1)  $\frac{cb}{(a+b)}$       2)  $\frac{ab}{a+c}$       3)  $\frac{(a+b)a}{c}$       4)  $\frac{(a+c)b}{a}$
- One projectile moving with velocity v in space, gets burst into 2 parts of masses in the ratio 1 : 3. The smaller part becomes stationary. What is the velocity of the other part?  
1)  $\frac{2v}{3}$       2)  $\frac{4v}{3}$       3)  $\frac{5v}{3}$       4) 3v
- A particle of mass 'm' moving with velocity v strikes a stationary particle of mass '2m' and sticks to it, the speed of system will be  
1) v/3      2) 2v      3) v/2      4) 3
- A space craft of mass 2000 kg moving with a velocity of 600m/s suddenly explodes into two pieces. One piece of mass 500 kg is left stationary. The velocity of the other part must be  
1) 600 m/s      2) 800 m/s      3) 1500 m/s      4) 1000 m/s

**Statement Type:**

- Statement I** : In uniform circular motion, momentum is not conserved but kinetic energy is conserved  
**Statement II** : When a stationary bomb shell explodes final kinetic energy is greater than the initial kinetic energy.  
1) Both Statements are true, Statement II is the correct explanation of Statement I.  
2) Both Statements are true, Statement II is not correct explanation of Statement I.  
3) Statement I is true, Statement II is false.  
4) Statement I is false, Statement II is true.

**Integer Type:**

10. A 50 kg man standing on the surface of negligible friction kicks forward a 0.1 kg stone so that it acquires a speed of  $10 \text{ ms}^{-1}$ . Then the velocity acquired by the man is \_\_\_\_\_ cm/s

**LEVEL-2 & 3**

11. A man and a cart move towards each other. The man weighs 64 kg and the cart 32kg. The velocity of the man is 5.4 km/hr and that of the cart is 1.8 km/hr. When the man approaches the cart, he jumps on to it. The velocity of the cart carrying the man will be  
 1) 3 km/hr                      2) 30 km/hr                      3) 1.8 km/hr                      4) zero
12. A man and a cart move towards each other. The man weighs 64 kg and the cart 36kg. The velocity of the man is 2 km/hr and that of the cart is 1 km/hr. When the man approaches the cart, he jumps on to it. The velocity of the cart carrying the man will be  
 1) 3 km/hr                      2) 0.92 km/hr                      3) 1.8 km/hr                      4) zero
13. The hero of a stunt film fires 50 g bullets from a machine gun, each at a speed of 1.0 km/s. If he fires 20 bullets in 4 seconds, what average force does he exert against the machine gun during this period ?  
 1) 150N                      2) 250N                      3) 350 N                      4) 450N
14. A railway truck of mass  $10^4 \text{ Kg}$  travelling at  $1 \text{ ms}^{-1}$  collides with another of double its mass moving in the opposite direction with velocity of  $0.2 \text{ ms}^{-1}$ . If the trucks couple automatically on collision, their common velocity in  $\text{ms}^{-1}$ .  
 1) 0.2                      2) 0.4                      3) 0.6                      4) 0.1
15. When U-238 nucleus originally at rest decays by emitting an  $\alpha$  particle with a speed of  $V \text{ ms}^{-1}$ , the recoil speed of the residual nucleus is (in  $\text{ms}^{-1}$ )  
 1)  $\frac{V}{4}$                       2)  $-\frac{4V}{234}$                       3)  $\frac{4V}{238}$                       4)  $-\frac{4V}{238}$

**Multi correct answer Type:**

16. A 15.0 kg explosive moving vertically upward at speed 10.0 m/s suddenly explodes into three fragments of masses 2.0 kg, 3.0kg and 10.0 kg. Immediately after the explosion, the 2.0 kg mass is moving upward at a speed of 20.0 m/s and the 3.0 kg mass is discovered to be moving horizontally at a speed of 5.0 m/s. If  $\vec{v}$  is the velocity of 10 kg then  
 1) Momentum of the system after the explosion is  $(15)\hat{i} + (40)\hat{j} + (10)\vec{v}$   
 2) the velocity of the 10.0 kg piece is  $(-1.5)\hat{i} + (11.0)\hat{j}$   
 3) Momentum of the system after the explosion is  $(-1.5)\hat{i} + (11.0)\hat{j}$   
 4) the magnitude of speed of the 10.0 kg piece is 11.1m/s

**LEVEL-4 & 5** 17. A bomb of mass 6 kg initially at rest explodes in to three identical fragments. One of the fragments moves with a velocity of  $10\sqrt{3}\hat{i} \text{ m/s}$ , another fragment moves with a velocity of  $10\hat{j} \text{ m/s}$ , then the third fragment moves with a velocity of magnitude.

- 1) 30 m/s                      2) 20 m/s                      3) 15 m/s                      4) 5 m/s

18. A man of mass  $m$  is standing on a platform of mass  $M$  kept on smooth ice. If the man starts moving on the platform with a speed  $v$  relative to the platform, with what velocity relative to the ice does the platform recoil ?

- 1)  $\frac{mv}{M+m}$                       2)  $\frac{mv}{M-m}$                       3)  $\frac{mv}{M+2m}$                       4)  $\frac{2mv}{M+m}$

### STAR FACT

#### Why are engines fitted at the rear end in some vehicles?

Rear engines reduce fatigue of the driver. He is also free from noise and vibration from the engine. Moreover, the weight on the front portion of the vehicle is reduced and so the torque needed to steer the vehicle would be less. Also, the hot air from the engine does not enter inside the cabin. However, the cost of manufacturing a rear engine vehicle is higher compared to front engine vehicle.

### SYNOPSIS-2

#### IMPULSE :

Consider a constant force  $\vec{F}$  which acts for a time  $t$  on a body of mass  $m$ , thus, changing its velocity from  $\vec{u}$  to  $\vec{v}$ . Because the force is constant, the body will travel with constant acceleration  $\vec{a}$  where

$$\vec{F} = m\vec{a} \quad \text{and} \quad \vec{a}t = \vec{v} - \vec{u}$$

$$\text{hence,} \quad \frac{\vec{F}}{m}t = \vec{v} - \vec{u} \quad \text{or} \quad \vec{F}t = m\vec{v} - m\vec{u}$$

The product of constant force  $\vec{F}$  and the time  $t$  for which it acts is called the impulse ( $\vec{J}$ ) of the force and this is equal to the change in linear momentum which it produces. Thus,

**Impulse momentum theorem:**



$$\text{Impulse}(\vec{J}) = \vec{F} t = \Delta\vec{P} = \vec{P}_f - \vec{P}_i$$

**Instantaneous Impulse :** There are many occasions when a force acts for such a short time that the effect is instantaneous, e.g., a bat striking a ball. In such cases, although the magnitude of the force and the time for which it acts may each be unknown but the value of their product (i.e., impulse) can be known by measuring the initial and final momenta. Thus, we can write

$$\vec{J} = \int \vec{F} dt = \Delta\vec{P} = \vec{P}_f - \vec{P}_i$$

Regarding the impulse it is important to note that impulse applied to an object in a given time interval can also be calculated from the area under force – time (F – t) graph in the same time interval.

### NOBEL LAUREATE IN PHYSICS - 1902



Lorentz  
(1853 - 1928)

LORENTZ (1853 - 1928)

Leiden University, Leiden, the Netherlands  
"in recognition of the extraordinary services he rendered by his research into the influence of magnetism upon radiation phenomena"

### WORKSHEET-2

**CUQ** 1. A large force is acting on a body in short interval of time is called

- 1) Impulse                      2) Momentum                      3) Energy                      4) Pressure

2. Impulse = \_\_\_\_\_ × \_\_\_\_\_.

- 1) Force, time                      2) Acceleration, Force  
3) Mass, acceleration                      4) force, Velocity

3. Impulse, in the case of constant force( $\vec{J}$ ) =

- 1)  $\vec{F}t$                       2)  $\frac{\vec{F}}{t}$                       3)  $\frac{t}{\vec{F}}$                       4)  $\vec{F} + t$

4. Formula for instantaneous impulse(Incase of variable force)

- 1)  $\vec{J} = \int \vec{F} dt$                       2)  $\vec{J} = \vec{F} dt$                       3)  $\vec{J} = \frac{d\vec{F}}{dt}$                       4)  $\vec{J} = \frac{\vec{F}}{t}$

5. Impulse is a \_\_\_\_\_ quantity.
  - 1) Vector
  - 2) Scalar
  - 3) Tensor
  - 4) All of these
6. Units of impulse is
  - 1) Ns
  - 2) Kg ms
  - 3) kg m<sup>-1</sup>s
  - 4) Ns<sup>-1</sup>
7. Impulse is equal to
  - 1) momentum
  - 2) change in momentum
  - 3) rate of change of momentum
  - 4) rate of change of force
8. Area under force-time graph and time axis gives
  - 1) Linear momentum
  - 2) Displacement
  - 3) Work done
  - 4) Impulse
9. A cricketer drags his hands backward to catch the ball, because
  - 1) by increasing the time in which he stops the ball, reduces force on his hands
  - 2) by increasing distance, he allows ball to travel a greater distance for more runs
  - 3) it is an art to catch the ball
  - 4) it is rule in cricket
10. Shock absorbers are used in vehicles because
  - 1) It reduces the impulse force by increased by impulsive time
  - 2) It increases the impulsive force by reducing impulsive time
  - 3) It reduces the impulsive force by reducing impulsive time
  - 4) It increases the impulsive force by increasing impulsive time

### JEE MAIN & ADVANCED

#### LEVEL-1 Single Correct Choice Type:

1. A force of 50 N acts on a body for 10 s. What will be the change in momentum?
  - 1) 200 Ns
  - 2) 400 Ns
  - 3) 500 Ns
  - 4) 1000 Ns
2. A force of 100N acts on a body in 10s, what will be the change in momentum?
  - 1) 200Ns
  - 2) 400Ns
  - 3) 500Ns
  - 4) 1000Ns
3. A train of mass  $m$  is moving with velocity  $\vec{v}_1$ , It is given an impulse such that the velocity becomes  $\vec{v}_2$ . Then magnitude of impulse is equal to
  - 1)  $m(\vec{v}_2 - \vec{v}_1)$
  - 2)  $m(\vec{v}_1 - \vec{v}_2)$
  - 3)  $m + (\vec{v}_2 - \vec{v}_1)$
  - 4)  $0.5m(\vec{v}_2 - \vec{v}_1)$
4. A 1 kg ball drops vertically onto the floor with a speed of 25 m/s and rebounds with a speed of 10 m/s. What is the impulse acting on the ball in
  - 1) 35N-s
  - 2) 40 N-s
  - 3) 56 N-s
  - 4) 70 N-s
5. A cricket ball of mass 500 g is moving with speed of 36 km h<sup>-1</sup>. It is reflected back with the same speed. What is the impulse applied on it ?
  - 1) 10 kg m s<sup>-1</sup>
  - 2) 20 kg m s<sup>-1</sup>
  - 3) 30 kg m s<sup>-1</sup>
  - 4) 40 kg m s<sup>-1</sup>
6. A 2kg ball drops vertically on to the floor with a speed of 25m/s and rebounds with the speed of 10m/s. What is the impulse acting on the ball is
  - 1) 35Ns
  - 2) 40Ns
  - 3) 56Ns
  - 4) 70Ns

7. A cricket ball of mass 1000g moving with the speed of 18kmph it is reflected back with the same speed. Then impulse applied on it is  
 1) 10 kgm/s                      2) 20Ns                      3) 30 Ns                      4) 40 Ns
8. When a ball of mass 5 kg hits a bat with a velocity 3 m/s, in positive direction and it moves back with a velocity = 4 m/s, find the impulse in SI units.  
 1) 25 N-s                      2) 35 N-s                      3) 45 N-s                      4) 55 N-s

**Matrix Match Type**

- |  |   |
|--|---|
| <p>9. <b>Column-I</b></p> <p>a) the change in momentum is equal to</p> <p>b) the product of the force and time interval which the force acts is equal to</p> <p>c) unit of impulse</p> <p>d) dimensional formula of impulse is</p> | <p><b>Column-II</b></p> <p>1) Impulse</p> <p>2) dyne second</p> <p>3) kg m s<sup>-1</sup></p> <p>4) MLT<sup>-1</sup></p> <p>5) ML<sup>-1</sup>T</p> |
|--|---|

**Multi Correct Choice Type:**

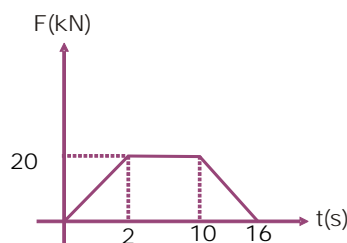
10. A force of 16 N acts on a ball of mass 80g for one micro-second. Then  
 1) the acceleration is 200 ms<sup>-2</sup>                      2) the impulse is 16 × 10<sup>-6</sup> N s  
 3) the acceleration is 100 ms<sup>-2</sup>                      4) the impulse is 12 × 10<sup>-6</sup> N s

**LEVEL-2 & 3**

11. Calculate the impulse necessary to stop a 1500kg car travelling at 90km h<sup>-1</sup>.  
 1) -375 N s                      2) -3700 N s                      3) -3500 N s                      4) -37500 N s
12. Calculate the impulse necessary to stop the car of mass 1000kg travelling at 72kmph.  
 1) 10000 Ns                      2) 20000 Ns                      3) 30000 Ns                      4) 40000 Ns

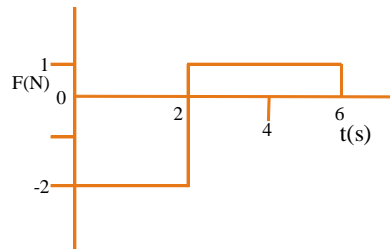
**Integer Answer Type:**

13. The thrust (F) versus time graph of a rocket is shown in figure.



The mass of the rocket is 1200 kg. The velocity of rocket, 16 seconds after starting from rest is \_\_\_\_\_ m/s. Neglect gravity.

14. A force time graph for the motion of a body is as shown in figure. Change in linear momentum between 0 and 6s is



1. zero                      2. 8 Ns                      3. 4 Ns                      4. 2 Ns
15. An impulse 15N-s given to a body changes its velocity from 15m/s to 25m/s. The increase in the kinetic energy of the body is given by  
 1)100J                      2) 200J                      3) 300J                      4) 50J
16. An impulse 5N-s given to a body changes its velocity from 4m/s to 6m/s. The increase in the kinetic energy of the body is given by  
 1)50J                      2) 10J                      3) 25J                      4) 5J
17. A body of mass 3 kg moves with initial velocity  $\vec{u} = \hat{i} + 4\hat{j}$  m/s and it's final velocity  $\vec{v} = 2\hat{i} + 3\hat{j}$  m/s then

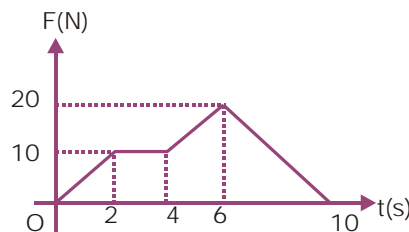
- 1) Impulse is equal to  $3(\hat{i} - \hat{j})$ Ns                      2) Magnitude of impulse is  $3\sqrt{2}$  Ns  
 3) Impulse is equal to  $2(\hat{i} - \hat{j})$ Ns                      4) Magnitude of impulse is  $2\sqrt{2}$  Ns

- LEVEL-4 & 5** 18. A truck of mass  $2 \times 10^3$  kg travelling at 4 m/s is brought to rest in 2 s when it strikes a wall. What force ( assume constant ) is exerted by the wall ?

- 1)  $2 \times 10^3$ N                      2)  $4 \times 10^3$ N                      3)  $5 \times 10^3$ N                      4)  $6 \times 10^3$ N
19. A bullet of mass  $10^{-3}$  kg strikes an obstacle and moves at  $60^\circ$  to its original direction. If its speed also changes from 20 m/s to 10 m/s. Find the magnitude of impulse acting on the bullet.

- 1)  $\sqrt{3} \times 10^2$ N-s                      2)  $\sqrt{3} \times 10^{-3}$ N-s                      3)  $\sqrt{3} \times 10^{-2}$ N-s                      4)  $\sqrt{30} \times 10^{-2}$ N-s

20. A particle of mass 2 kg is initially at rest. A force starts acting on it in one direction whose magnitude changes with time. The force time graph is shown in figure.



Then the velocity of the particle at the end of 10 s is  $\_\_\_ \times 10$ m/s .



**CONSERVATION OF LINEAR MOMENTUM****WORK SHEET-1\_KEY**

**CUQ:** 1) 2    2) 1    3) 2    4) 4    5) 2    6) 2    7) 4    8) 3  
 9) 3    10) 2

**JEE MAINS AND ADVANCED:**

1) 3    2) 2    3) 1    4) 4    5) 2    6) 2    7) 1    8) 2  
 9) 2    10) 2    11) 1    12) 2    13) 2    14) 1    15) 2  
 16) 1,3,4    17) 2    18) 1

**HINTS & SOLUTIONS**

2. Apply conservation of momentum
3. Here the system is bullet + rifle. Total initial momentum of the system = 0. Let  $v_1$  and  $v_2$  be the final velocities of the bullet and rifle respectively. Total final momentum of the system =  $m_1v_1 + m_2v_2$ . Therefore from the principle of conservation of linear momentum,
- $$m_1v_1 + m_2v_2 = 0 \Rightarrow v_2 = -\frac{m_1}{m_2}v_1$$

$$\Rightarrow v_2 = \frac{-5 \times 10^{-3} \times 900}{2.5} = -1.8 \text{ m/s}$$

The negative sign indicates that they are moving in opposite direction.

4. using conservation of momentum
- $$40 \times 80 = 32V + 8 \times 0 \Rightarrow 40 \times 80 = 32V \Rightarrow V = 100 \text{ m/s}$$
6. Initial momentum of the system =  $mv$ . Let  $v^l$  be the velocity of bigger part after burst.

$$\text{Final momentum of the system} = \left(\frac{3}{4}m\right)v^l + \left(\frac{m}{4}\right)0 = \frac{3}{4}mv^l$$

$$\text{According to law of conservation of momentum } mv = \left(\frac{3}{4}m\right)v^l \therefore v^l = \frac{4v}{3}$$

8.  $M(0) = m_1v_1 + m_2v_2$
12.  $m_1u_1 + m_2u_2 = (m_1 + m_2)v$

13. The momentum of each bullet =  $(0.050 \text{ kg}) (1000 \text{ m/s}) = 50 \text{ kg-m/s}$ .  
The gun is imparted this much of momentum by each bullet fired.

Thus, the rate of change of momentum of the gun =  $\frac{(50 \text{ kg - m/s}) \times 20}{4 \text{ s}} = 250 \text{ N}$

14. From conservation of momentum

$$(10^4 \times 1) + \{2 \times 10^4 (-0.2)\} = (10^4 + 2 \times 10^4) V$$

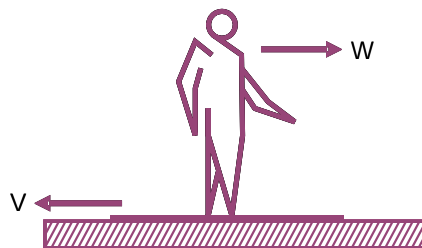
$$V = \frac{0.6 \times 10^4}{3 \times 10^4} = 0.2 \text{ ms}^{-1}$$

15. From conservation of momentum

$$(238 \times 0) = (234) V^1 + 4V \Rightarrow V^1 = -\frac{4V}{234}$$

17.  $\vec{P}_3 = -(\vec{P}_1 + \vec{P}_2)$

18. Consider the situation shown in figure . Suppose the man moves at a speed  $w$  towards right and the platform recoils at a speed  $V$  towards left, both relative to the ice. Hence, the speed of the man relative to the platform is  $V + w$ . By the question,  
 $V + w = v$ , or  $w = v - V$  - - - - - (1)



Taking the platform and the man to be the system, there is no external horizontal force on the system. The linear momentum of the system remains constant. Initially, both the man and the platform were at rest. Thus,  $0 = MV - mw$

or,  $MV = m(v - V)$  [ Using (1)] or,  $V = \frac{mv}{M+m}$

**WORKSHEET-2 KEY**

- CUQ:** 1) 1    2) 1    3) 1    4) 1    5) 1    6) 1    7) 2    8) 4  
 9) 1    10) 1

**JEE MAINS AND ADVANCED:**

- 1) 3    2) 4    3) 1    4) 1    5) 1    6) 4    7) 1    8) 2  
 9) a-1,b-1,c-2,3;d-4    10) 1,2    11) 4    12) 2    13) 200    14) 1  
 15) 3    16) 3    17)1,2    18)2    19) 3    20)5

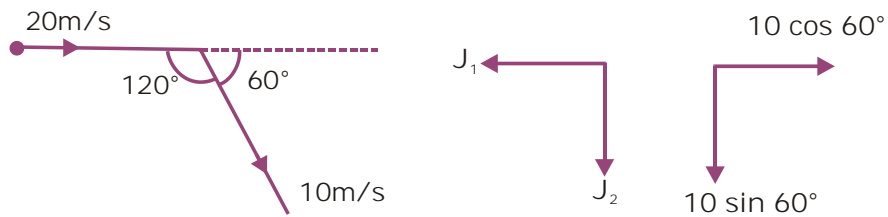
**HINTS & SOLUTIONS**

2.  $I = F \times t = 100 \times 10 = 1000\text{Ns}$   
 3. Sol : Impulse = Change in momentum =  $m\vec{v}_2 - m\vec{v}_1$   
 5. Sol : Impulse = Change in momentum =  $[0.5 \times 10 - 0.5 \times -10] \text{ kg m s}^{-1}$   
 $= 0.5 [10 + 10] \text{ kg ms}^{-1} = 10 \text{ kg m s}^{-1}$   
 6.  $\vec{I} = m(\vec{v} - \vec{u})$   
 7.  $I = 2mu$   
 8. Impulse = change in linear momentum =  $P_f - P_i = mv - mu = -5 \times 4 - 5 \times 3 = -35 \text{ N-s}$   
 $\therefore$  Magnitude of impulse in SI units = 35  
 9. Conceptual  
 11. Sol : Impulse = Change in momentum =  $Mv - Mu = M(v - u)$   
 $= 1500(0 - 25) \text{ Ns} = -37500 \text{ N s}$  [ $\therefore 90 \text{ km h}^{-1} = 25 \text{ m s}^{-1}$ ]  
 18. Using impulse = change in linear momentum.



We have  $F.t = mv_f - mv_i = m(v_f - v_i)$   
 or  $F(2) = 2 \times 10^3 [0 - (-4)]$  or  $2F = 8 \times 10^3$  or  $F = 4 \times 10^3 \text{ N}$

19. Mass of the bullet  $m = 10^{-3} \text{ kg}$



Consider components parallel to  $J_1$ .

$$J_1 = 10^{-3} [-10 \cos 60^\circ - (-20)] \quad \text{or} \quad J_1 = 15 \times 10^{-3} \text{ N-s}$$

Similarly, parallel to  $J_2$ , we have  $J_2 = 10^{-3} [10 \sin 60^\circ - 0] = 5\sqrt{3} \times 10^{-3} \text{ N-s}$

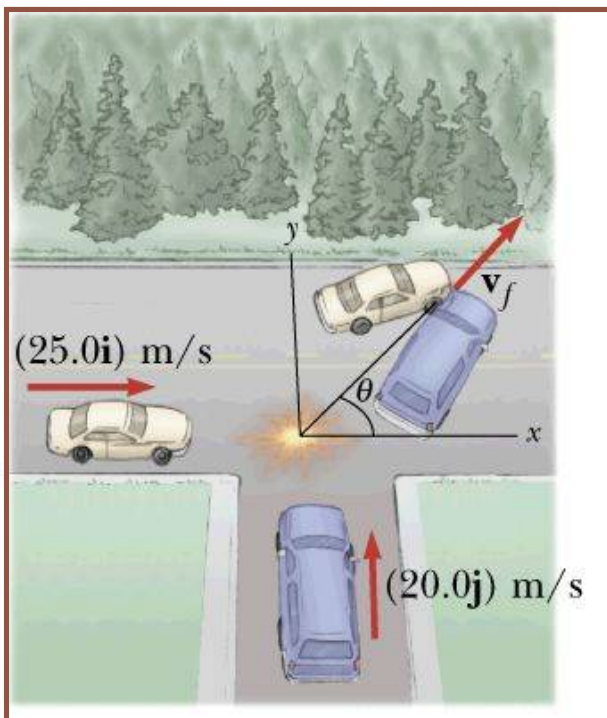
The magnitude of resultant impulse is given by

$$J = \sqrt{J_1^2 + J_2^2} = 10^{-3} \sqrt{(15)^2 + (5\sqrt{3})^2} \quad \text{or} \quad J = \sqrt{3} \times 10^{-2} \text{ N-s}$$

\*\*\*\*\*



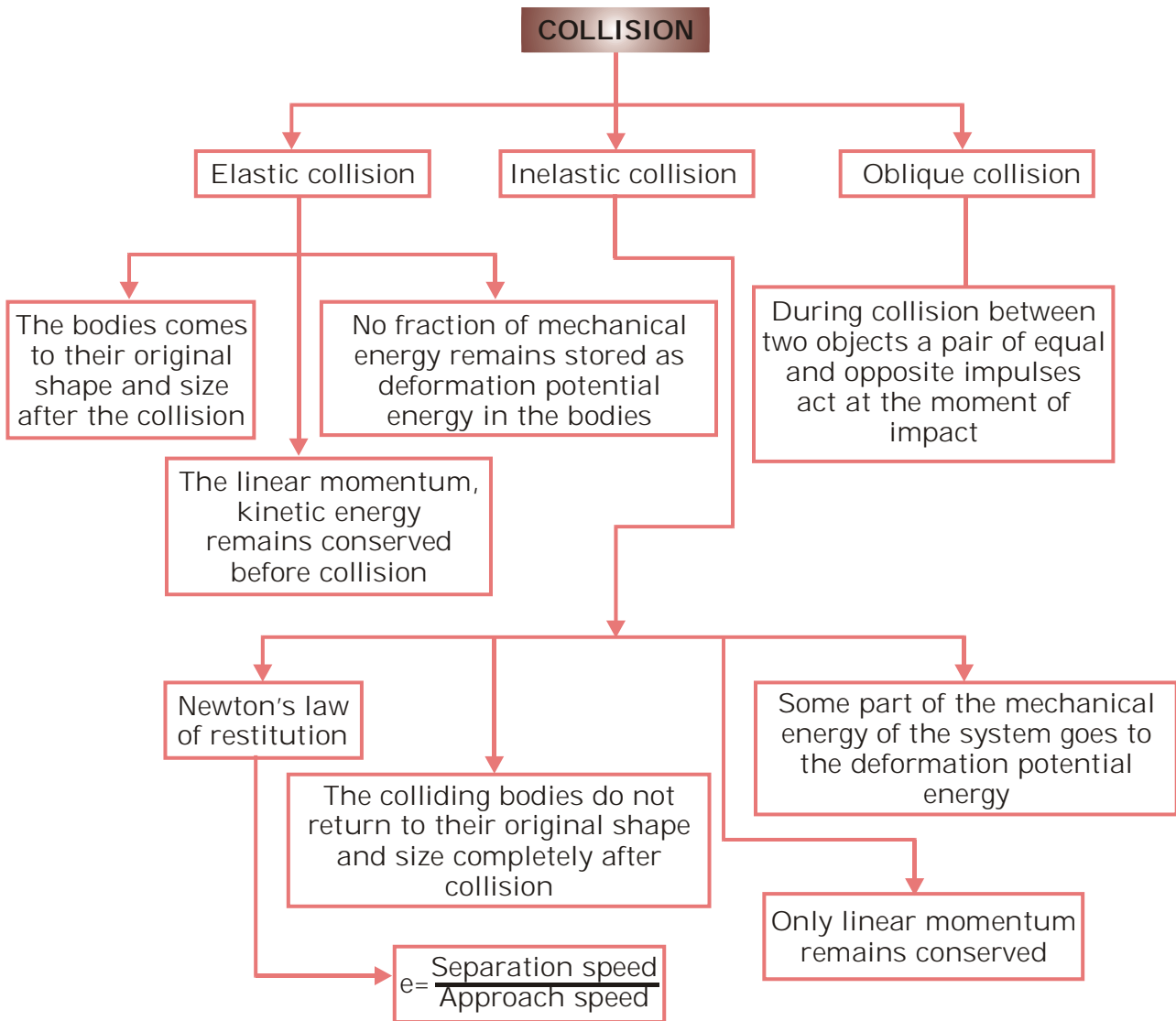
# COLLISSIONS



## CONTENTS

- Introduction to Collisions
- Introduction to Elastic Collision
- Introduction to Inelastic Collision
- Special cases of Head on Elastic Collision
- Special cases of Head on Inelastic collision
- Newton's Law of Restitution

# MEMO GRAPH

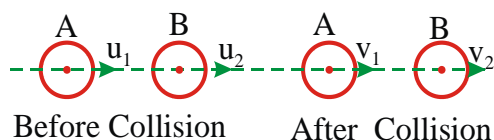


## COLLISSIONS SYNOPSIS-1

### Collisions

- The strong interaction among bodies involving exchange of momentum in a short interval of time is called collision.
- During collision bodies may or may not come into physical contact.
- In the collision of  $\alpha$  particle with nucleus, due to coulombic repulsive forces  $\alpha$  particle is scattered away without any physical contact.
- Based on the direction of motion of colliding bodies, collisions are classified into
  - (i) Head on or one dimensional collision
  - (ii) oblique collision

### Head on (or) one dimensional collision



- It is the collision in which the velocities of the colliding bodies are confined to same straight line before and after collision.

### Oblique Collision:

- It is the collision in which the velocities of the colliding bodies are not confined to same straight line before and after collision.
- Oblique collision may be two dimensional or three dimensional.
- When a particle hits elastically and obliquely another stationary particle of same mass, then they always move perpendicular to each other after collision.

**Types of Collision:** Based on conservation of kinetic energy collisions are classified into

- (i) Elastic Collision    (ii) Inelastic collision

**Elastic Collision:** It is the collision in which both momentum and kinetic energy are conserved. Forces involved during collision are conservative in nature

- Ex.1.** Collision between atomic particles.  
 2. Collision between two smooth billiard balls.  
 3. Collision of  $\alpha$  particle with nucleus.

**Inelastic collision:** It is the collision in which momentum is conserved but not kinetic energy. Some or all the forces involved during collision are non conservative.

**Ex:** Collision between two vehicles.

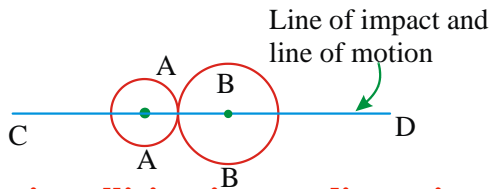
### Perfectly inelastic collision:

- It is the collision in which the colliding bodies stick together and move as a single body after collision.
- In perfectly inelastic collision the momentum remains conserved but the loss of kinetic energy is maximum.

**Ex:** A bullet is fired into a wooden block and remains embedded in it.

**Line of impact:** The line passing through the common normal to the surfaces in contact during impact is called line of impact. The force during collision acts along this line on both bodies.

**Ex 1:** Two balls A and B are approaching each other such that their centres are moving along line CD.



**Elastic collision in one dimension:**

When two particles of masses  $m_1$  and  $m_2$  moving along the line joining their centers with velocities  $u_1$  and  $u_2$  ( $u_1 > u_2$ ) before collision.  $v_1$  and  $v_2$  are velocities after collision



From conservation of linear momentum

$$m_1 (\vec{u}_1 - \vec{v}_1) = m_2 (\vec{v}_2 - \vec{u}_2)$$

From Law of conservation of K.E

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$\therefore \vec{u}_1 - \vec{u}_2 = \vec{v}_2 - \vec{v}_1$$

i.e Relative velocity of approach before collision = Relative velocity of separation after collision

➤ **Velocities after collision are**

$$\vec{v}_1 = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) \vec{u}_1 + \left( \frac{2m_2}{m_1 + m_2} \right) \vec{u}_2$$

$$\vec{v}_2 = \left( \frac{2m_1}{m_1 + m_2} \right) \vec{u}_1 + \left( \frac{m_2 - m_1}{m_1 + m_2} \right) \vec{u}_2$$

**Special cases:**

➤ If colliding particles have equal masses

i.e  $m_1 = m_2 = m$  ;  $\vec{v}_1 = \vec{u}_2$  ,  $\vec{v}_2 = \vec{u}_1$

➤ If two bodies are of equal masses and the second body is at rest i.e.,  $m_1 = m_2 = m$  and  $\vec{u}_2 = \vec{0}$  then

$$\vec{v}_1 = \vec{0} \quad ; \quad \vec{v}_2 = \vec{u}_1$$

➤ A lighter particle collides with heavier particle which is at rest  $m_1 \lll m_2$  ,  $\vec{u}_2 = \vec{0}$

$$\vec{v}_1 = -\vec{u}_1 \quad , \quad \vec{v}_2 = \vec{0}$$

➤ A heavier body collides with lighter body at rest

$m_1 \ggg m_2$  ,  $\vec{u}_2 = \vec{0}$  ;

$$\vec{v}_1 = \vec{u}_1 \quad , \quad \vec{v}_2 = 2\vec{u}_1$$



**Applications:**

➤ A body of mass  $m_1$  moving with a velocity  $v_1$  collides elastically with a stationary mass  $m_2$

1) Velocity of first body after collision  $\vec{v}_1 = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) \vec{u}_1$

2) velocity of second body after collision

$$\vec{v}_2 = \left( \frac{2m_1}{m_1 + m_2} \right) \vec{u}_1$$

3) KE of first body after collision (or) KE retained by first body  $K.E_1 = \frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 \left( \frac{m_1 - m_2}{m_1 + m_2} \right)^2 u_1^2$

$$K.E_{ret} = \frac{1}{2} m_1 u_1^2 \left[ \frac{m_1 - m_2}{m_1 + m_2} \right]^2 = KE_i \left[ \frac{m_1 - m_2}{m_1 + m_2} \right]^2$$

4) fraction of KE retained by 1<sup>st</sup> body

$$\frac{K.E_{ret}}{K.E_i} = \left[ \frac{m_1 - m_2}{m_1 + m_2} \right]^2$$

5) K.E. of second body after collision (or) KE transferred to the second body

$$K.E_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_2 \left( \frac{2m_1}{m_1 + m_2} \right)^2 u_1^2$$

$$KE_2 = \left( \frac{4m_1 m_2}{(m_1 + m_2)^2} \right) \left( \frac{1}{2} m_1 u_1^2 \right)$$

$$KE_{tra} = \left( \frac{4m_1 m_2}{(m_1 + m_2)^2} \right) K.E_i$$

6) Fraction of KE transferred from 1<sup>st</sup> body to second body (or) Fraction of KE lost by 1<sup>st</sup> body is

$$\frac{KE_{tra}}{KE_i} = \frac{4m_1 m_2}{(m_1 + m_2)^2}$$

7) Fraction of momentum retained by  $m_1$

$$\frac{P_1}{P_i} = \frac{m_1 v_1}{m_1 u_1} = \frac{m_1 - m_2}{m_1 + m_2}$$

8) Fraction of momentum transferred from 1<sup>st</sup> body to second body

$$\frac{P_2}{P_i} = \frac{P_i - P_1}{P_i} = 1 - \frac{P_1}{P_i} = 1 - \left( \frac{m_1 - m_2}{m_1 + m_2} \right) = \frac{2m_2}{m_1 + m_2}$$

**COLLISIONS WORKSHEET-1**

**CUQ**

1. In any collision, which of the following is conserved.
  - 1) Linear momentum of the system
  - 2) Kinetic energy of the system
  - 3) Both 1 and 2
  - 4) None of these

2. Which physical quantity remains constant in an elastic collision?
  - 1) Linear momentum only
  - 2) Kinetic energy only
  - 3) Both Linear momentum and kinetic energy
  - 4) force
3. The quantities remaining constant in a collision are
  - 1) momentum, kinetic energy and temperature
  - 2) momentum and kinetic energy but not temperature
  - 3) momentum and temperature but not kinetic energy
  - 4) momentum, but neither kinetic energy nor temperature.
4. If both the bodies come to their original shape and size after the collision is said to be
  - 1) elastic
  - 2) inelastic
  - 3) oblique
  - 4) none of these
5. When two bodies of equal mass moving towards each other collide elastically with same velocity in magnitude, after collision
  - 1) they stop
  - 2) one body stops and the other moves
  - 3) they move away with the same velocity in magnitude
  - 4) they move with a common velocity
6. A heavier body moving with certain velocity collides head on elastically a lighter body at rest. Then which of the following is true?
  - 1) The heavier body moves with the same velocity along the same direction
  - 2) The lighter body moves with twice that of the initial velocity of the heavier body
  - 3) Both 1 and 2
  - 4) None of these
7. In inelastic collision of two bodies, which of the following do not change after the collision?
  - 1) total kinetic energy
  - 2) total linear momentum
  - 3) total angular momentum
  - 4) none of these
8. In an elastic collision
  - 1) the initial kinetic energy is equal to the final kinetic energy
  - 2) the final kinetic energy is less than the initial kinetic energy.
  - 3) the kinetic energy remains constant
  - 4) the kinetic energy first increases then decreases.
9. A body of mass  $M_1$  collides elastically with another body of mass  $M_2$  at rest. There is maximum transfer of energy when:
  - 1)  $M_1 > M_2$
  - 2)  $M_1 < M_2$
  - 3)  $M_1 = M_2$
  - 4) same for all values of  $M_1$  and  $M_2$

10. Internal forces can change:
- 1) the linear momentum but not the kinetic energy.
  - 2) the kinetic energy but not the linear momentum.
  - 3) linear momentum as well as kinetic energy.
  - 4) neither the linear momentum nor the kinetic energy.

### JEE MAIN & ADVANCED

#### LEVEL-1 Single Correct Choice Type:

1. A smooth sphere of mass  $M$  moving with velocity  $u$  directly collides elastically with another sphere of mass  $m$  at rest. After collision their final velocities are  $V$  and  $v$  respectively. The value of  $v$  is\_\_\_\_\_.

  - 1)  $\left(\frac{2M}{m+M}\right)u$
  - 2)  $\left(\frac{2m}{m+M}\right)u$
  - 3)  $\left(\frac{M}{m+M}\right)u$
  - 4)  $\left(\frac{M}{2m+M}\right)u$

2. A body of mass 4kg moves with a velocity of 10m/s collide elastically with a body of mass 8kg is at rest, then the velocity of 8kg body is

  - 1) 4 m/s
  - 2) 6.67 m/s
  - 3) 10 m/s
  - 4) 12 m/s

3. A 6 kg mass travelling at  $2.5 \text{ ms}^{-1}$  collides head on with a stationary 4 kg mass. After the collision the 6 kg mass travels in its original direction with a speed of  $1 \text{ ms}^{-1}$ . The final velocity of 4 kg mass is

  - 1) 1 m/s
  - 2)  $2.25 \text{ ms}^{-1}$
  - 3)  $2 \text{ ms}^{-1}$
  - 4)  $0 \text{ ms}^{-1}$

4. A block of mass 1 kg moving with a speed of  $4 \text{ ms}^{-1}$ , collides with another block of mass 2 kg which is at rest. The lighter block comes to rest after collision. The speed of the heavy body after collision is

  - 1)  $2 \text{ ms}^{-1}$
  - 2)  $1 \text{ m s}^{-1}$
  - 3)  $1.5 \text{ ms}^{-1}$
  - 4)  $0.5 \text{ ms}^{-1}$

5. A ball of 4 kg mass and a speed of  $3 \text{ ms}^{-1}$  has a head on elastic collision with a 6 kg mass initially at rest. The speeds of both the bodies after collision are respectively

  - 1)  $0.6 \text{ ms}^{-1}, 2.4 \text{ ms}^{-1}$
  - 2)  $-0.6 \text{ ms}^{-1}, -2.4 \text{ ms}^{-1}$
  - 3)  $-0.6 \text{ ms}^{-1}, 2.4 \text{ ms}^{-1}$
  - 4)  $-0.6 \text{ ms}^{-1}, -2.4 \text{ ms}^{-1}$

6. A marble going at a speed of  $2 \text{ ms}^{-1}$  hits another marble of equal mass at rest. If the collision is perfectly elastic. The velocity of the first after collision is

  - 1) 4
  - 2) 0
  - 3) 2
  - 4) 3

**Matrix Match Type**

7. A particle of mass  $m$ , kinetic energy  $K$  and momentum  $P$  collides head on elastically with another particle of mass  $2m$  at rest. Match the following (after collision)

**Column-I**

- a) momentum of first particle  
b) momentum of second particle  
c) kinetic energy of first particle  
d) kinetic energy of second particle

**Column-II**

- 1)  $\frac{4}{3}P$   
2)  $\frac{8K}{9}$   
3)  $-\frac{P}{3}$   
4)  $\frac{K}{9}$   
5)  $1.33 P$

**Multi correct answer type:**

8. A block moving horizontally on a smooth surface with a speed of 20 m/s bursts into two equal parts continuing in the same direction. If one of the parts moves at 30 m/s, then
- 1) The speed with which the second part move is 10m/s.
  - 2) The speed with which the second part move is 5m/s.
  - 3) The fractional change in the kinetic energy is  $1/4$
  - 4) The fractional change in the kinetic energy is  $1/2$
9. Two bodies of masses  $m_1$  and  $m_2$  collide each other elastically with velocities  $V_1, V_2$  respectively ( $V_1 > V_2$ ). Then final velocities becomes  $V_1', V_2'$  after collision if  $m_1 \gg m_2$  and  $V_2 = 0$ , then final velocity of balls are
- 1)  $2V_2, V_2$
  - 2)  $3V_2, 0$
  - 3)  $V_2, 3V_1$
  - 4)  $V_1, 2V_1$
10. Two balls bearings of same masses collide each other with initial velocities  $V_1, V_2$  and final velocitites  $V_1', V_2'$  respectively then  $V_1', V_2'$  values are
- 1)  $0, 2V_1$
  - 2)  $2V_2, 0$
  - 3)  $2V_2, 3V_1$
  - 4)  $V_2, V_1$

**LEVEL-2 & 3**

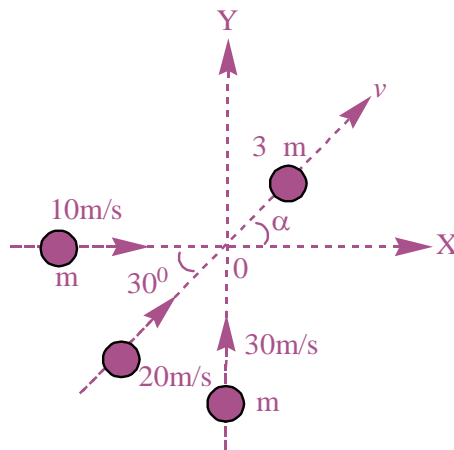
11. Two balls of masses  $m_1$  and  $m_2$  ( $m_2 \gg m_1$ ) collide with each other with initial velocities  $V_1, V_2$  ( $\approx 0$ ). Then the final velocities of balls are
- 1)  $0, -V_1$
  - 2)  $V_1, -2V_2$
  - 3)  $-V_1, 0$
  - 4)  $V_1, 3V_2$
12. A body of mass 10 kg moving with a velocity of  $5 \text{ ms}^{-1}$  hits a body of 1 gm at rest. The velocity of the second body after collision assuming it to be perfectly elastic is
- 1)  $10 \text{ ms}^{-1}$
  - 2)  $5 \text{ ms}^{-1}$
  - 3)  $15 \text{ ms}^{-1}$
  - 4)  $0.10 \text{ ms}^{-1}$



13. A body of mass 5kg moving with a speed of  $3\text{ms}^{-1}$  collides head on with a body of mass 3kg moving in the opposite direction at a speed of  $2\text{ms}^{-1}$ . The first body stops after the collision. The final velocity of the second body is
- 1)  $3\text{ms}^{-1}$                       2)  $5\text{ms}^{-1}$                       3)  $-9\text{ms}^{-1}$                       4)  $30\text{ms}^{-1}$
14. Two masses  $m_A$  and  $m_B$  moving with velocities  $v_A$  and  $v_B$  in opposite directions collide elastically. After that the masses  $m_A$  and  $m_B$  move with velocity  $v_B$  and  $v_A$  respectively. Then the ratio  $\frac{m_A}{m_B}$  is \_\_\_\_\_.
- 1) 1:1                      2) 1:2                      3) 2:1                      4) 1:3

**COMPREHENSION TYPE :**

Three particles of equal masses travelling with velocities of  $10\text{m/s}$ ,  $20\text{m/s}$  and  $30\text{m/s}$ , respectively, along x-axis, at an angle of  $30^\circ$  to the direction of positive x-axis and y-axis (as shown in Fig.) collide simultaneously and get stucked to each other



15. The combined particle will move with velocity
- 1)  $\frac{10}{3}\sqrt{20+2\sqrt{3}}\text{ m/s}$                       2)  $\frac{10}{3}\sqrt{20-2\sqrt{3}}\text{ m/s}$
- 3)  $\frac{10}{3}\sqrt{(5-\sqrt{3})}\text{ m/s}$                       4)  $\frac{10}{3}\sqrt{(5+\sqrt{3})}\text{ m/s}$
16. Combined particle will move at angle  $\alpha$  with x-axis where
- 1)  $\alpha = \tan^{-1}\left[2(\sqrt{3}+1)\right]$                       2)  $\alpha = \tan^{-1}\left[2(\sqrt{3}-1)\right]$
- 3)  $\alpha = \tan^{-1}\left[(\sqrt{3}-2)\right]$                       4)  $\alpha = \tan^{-1}\left[2(\sqrt{3}+2)\right]$

**LEVEL-4 & 5** 17. A particle of mass 'm' moving with a velocity  $(3\hat{i} + 2\hat{j})m/s$  collides with stationary mass 'M' and finally 'm' moves with a velocity  $(-2\hat{i} + \hat{j})m/s$  if  $\frac{m}{M} = \frac{1}{13}$  the velocity of the M after collision is?

- 1)  $(5\hat{i} + \hat{j})m/s$       2)  $(5\hat{i} - \hat{j})m/s$       3)  $\left(\frac{5\hat{i}}{13} - \frac{\hat{j}}{13}\right)m/s$       4)  $\left(\frac{5\hat{i}}{13} + \frac{\hat{j}}{13}\right)m/s$

**Multi correct answer type:**

18. A particle (A) of mass  $m_1$  elastically collides with another stationary particle (B) of mass  $m_2$ . then:

- 1)  $\frac{m_1}{m_2} = \frac{1}{2}$  and the particles fly a part in the opposite direction with equal velocities.  
 2)  $\frac{m_1}{m_2} = \frac{1}{3}$  and the particles fly apart in the opposite direction with equal velocities.  
 3)  $\frac{m_1}{m_2} = \frac{2}{1}$  and the collision angle between the particles is  $60^\circ$  symmetrically.  
 4)  $\frac{m_1}{m_2} = \frac{2}{1}$  and the particles fly apart symmetrically at an angle  $90^\circ$

**COLLISIONS SYNOPSIS-2**

**Head on inelastic collision:**

Consider two bodies of masses  $m_1$  and  $m_2$  moving with initial velocities  $u_1$  and  $u_2$  ( $u_1 > u_2$ ) after collision two bodies will move with velocities  $v_1$  and  $v_2$ .

Applying law of conservation of linear momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \text{---(1)}$$

By the def of coefficient of restitution  $e = \frac{v_2 - v_1}{u_1 - u_2}$ ;  $v_2 - v_1 = e(u_1 - u_2)$  ----(2)

Substituting  $v_1$  and  $v_2$  values from eq 2 in eq 1 we get

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 + \frac{m_2}{m_1 + m_2}(1 + e)u_2; \quad v_2 = \frac{m_1(1 + e)u_1}{m_1 + m_2} + \left(\frac{m_2 - em_1}{m_1 + m_2}\right)u_2$$

If  $m_1 = m_2 = m$   $u_2 = 0$  then  $v_1 = (1 - e)\frac{u_1}{2}$ ;  $v_2 = (1 + e)\frac{u_1}{2}$   $\frac{v_1}{v_2} = \frac{1 - e}{1 + e}$

**Loss of KE of the system:**

$$\Delta K.E = K.E_i - K.E_f = \frac{1}{2} \left( \frac{m_1 m_2}{m_1 + m_2} \right) (u_1 - u_2)^2 (1 - e^2)$$

In case of perfectly in-elastic collision  $e = 0$

∴ loss in K.e of system is  $\Delta K.E = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (u_1 - u_2)^2$

If two bodies are approaching each other then

Loss in K.E of the system is  $\Delta K.E = \frac{1}{2} \left( \frac{m_1 m_2}{m_1 + m_2} \right) (u_1 + u_2)^2$

**Coefficient of restitution:**

Newton introduced a dimensionless parameter called the coefficient of restitution (e) to measure the elasticity of collision. It is defined as the ratio of the relative velocity of separation to the relative velocity of approach of the two colliding

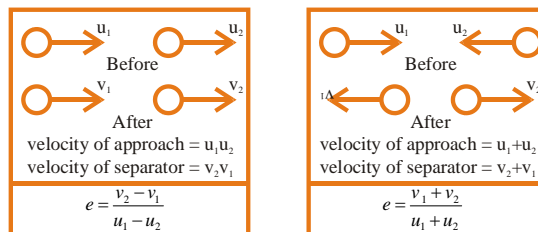
bodies  $e = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}} = \frac{(v_2 - v_1)}{(u_1 - u_2)}$

This formula is applied along the line of impact. Here the velocities mentioned in the expression should be taken along the line of impact.

For a perfectly elastic collision  $e = 1$

For an inelastic collision  $0 < e < 1$

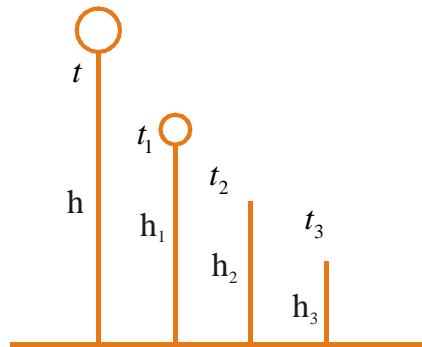
For completely inelastic collision  $e = 0$



- A body dropped freely from a height 'h' strikes the floor and rebounds to a height  $h_1$ ,  $e = \sqrt{\frac{h_1}{h}}$  and after  $n^{\text{th}}$  rebound  $h_n = e^{2n} h$
- When a freely falling ball strikes the ground with a velocity 'V' and rebounds with a velocity  $V_1$  then  $e = \frac{V_1}{V}$  and after  $n^{\text{th}}$  rebound  $V_n = e^n V$
- Total distance travelled by the ball before it stops bouncing  $S = h + 2h_1 + 2h_2 + 2h_3 + \dots = h + 2e^2 h + 2e^4 h + 2e^6 h + \dots$

$$= h + 2e^2h[1 + e^2 + e^4 + \dots] = h + 2e^2h\left[\frac{1}{1-e^2}\right] = h\left[1 + \frac{2e^2}{1-e^2}\right] = h\left[\frac{1-e^2+2e^2}{1-e^2}\right]$$

$$S = h\left[\frac{1+e^2}{1-e^2}\right]$$



➤ Total time taken by the ball to stop bouncing

$$= t + 2t_1 + 2t_2 + 2t_3 + \dots$$

$$= \sqrt{\frac{2h}{g}} + 2\sqrt{\frac{2h_1}{g}} + 2\sqrt{\frac{2h_2}{g}} + 2\sqrt{\frac{2h_3}{g}} + \dots = \sqrt{\frac{2h}{g}} + 2\sqrt{\frac{2e^2h}{g}} + 2\sqrt{\frac{2e^4h}{g}} + 2\sqrt{\frac{2e^6h}{g}} + \dots$$

$$= \sqrt{\frac{2h}{g}} + 2e\sqrt{\frac{2h}{g}} + 2e^2\sqrt{\frac{2h}{g}} + 2e^3\sqrt{\frac{2h}{g}} + \dots$$

$$= \sqrt{\frac{2h}{g}} + 2e\sqrt{\frac{2h}{g}}[1 + e + e^2 + \dots]$$

$$= \sqrt{\frac{2h}{g}} + 2e\sqrt{\frac{2h}{g}}\left[\frac{1}{1-e}\right] = \sqrt{\frac{2h}{g}}\left[1 + \frac{2e}{1-e}\right] = \sqrt{\frac{2h}{g}}\left[\frac{1-e+2e}{1-e}\right] = \sqrt{\frac{2h}{g}}\left[\frac{1+e}{1-e}\right]$$

➤ Change in momentum in 1<sup>st</sup> collision =  $mv_1 - (-mu) = (mv_1 + mu) = me u + mu = mu(1+e)$ .

Change in momentum in 2<sup>nd</sup> collision =  $m(v_2 + v_1) = m(e^2u + eu) = me u(1+e)$

Total change in momentum after nth collision.

$$\Delta p = mu(1 + e + e^2 + \dots) \left[U = \sqrt{2gh}\right] = mu\left[\frac{1+e}{1-e}\right] = m\sqrt{2gh}\left[\frac{1+e}{1-e}\right]$$



## COLLISSIONS WORKSHEET-2

- CUQ** 1. In a one dimensional elastic collision, the relative velocity of approach before collision is equal to
- 1) sum of the velocities of the body
  - 2) relative velocity of separation after collision
  - 3)  $e$  times relative velocity of separation after collision
  - 4)  $1/e$  times relative velocity of separation after collision
2. A ball is released from some height which strikes the ground. If the collision is perfectly elastic
- 1) The ball sticks to the ground
  - 2) The ball rebounds to the same height
  - 3) The ball rebounds to a greater height
  - 4) The ball moves along the ground
3. In the above case, if the collision is perfectly inelastic
- 1) The ball rebounds to the same height
  - 2) The ball rebounds to a greater height
  - 3) The ball rebounds to a smaller height
  - 4) The ball sticks to the ground
4. A ball hits the ground and rebounds after collision. In this process
- 1) Momentum of the ball is conserved
  - 2) Momentum of the earth is conserved
  - 3) Total momentum of the earth and ball is conserved
  - 4) Total momentum of the earth and ball is not conserved
5. A bullet hits and gets embedded in a solid block resting on a horizontal frictionless table, what is conserved?
- 1) Momentum and K.E
  - 2) K.E alone
  - 3) Momentum alone
  - 4) Neither momentum nor K.E
6. The coefficient of restitution for a perfectly inelastic collision is
- 1) 1
  - 2) 0
  - 3)  $\infty$
  - 4) -1
7. The co-efficient of restitution for a perfectly elastic collision is
- 1) 1
  - 2) 0
  - 3)  $\infty$
  - 4) -1
8. A bullet hits horizontally and gets embedded in a solid block resting on a frictionless surface. In this process:
- 1) momentum is conserved.
  - 2) kinetic energy is conserved.
  - 3) both momentum and K.E. are conserved.
  - 4) neither momentum nor K.E. is conserved.

9. In an inelastic collision
- 1) the initial kinetic energy is equal to the final kinetic energy
  - 2) the final kinetic energy is less than the initial kinetic energy.
  - 3) the kinetic energy remains constant
  - 4) the kinetic energy first increases then decreases.
10. The collision in which the relative velocity is zero after collision is
- 1) perfectly elastic
  - 2) perfectly inelastic
  - 3) partially elastic
  - 4) sometimes elastic and sometimes inelastic

### JEE MAIN & ADVANCED

#### LEVEL-1 Single Correct Choice Type:

1. A block of mass  $m$  moving at a speed  $v$  collides with another block of mass  $2m$  at rest. The lighter block comes to rest after the collision. Find the coefficient of restitution.
- 1)  $1/2$
  - 2)  $1/3$
  - 3)  $1/4$
  - 4)  $1/5$
2. Ball 1 collides directly with another identical ball 2 at rest. Velocity of second ball becomes two times that of 1 after collision. Find the coefficient of restitution between the two balls ?
- 1)  $1/2$
  - 2)  $1/3$
  - 3)  $1/4$
  - 4)  $1/5$
3. A  $5\text{ kg}$  mass moving at a speed of  $3\text{ ms}^{-1}$  collides head on with a body of mass  $1\text{ kg}$  at rest, if they move with a common velocity after collision in the same direction, the velocity is
- 1)  $25\text{ ms}^{-1}$
  - 2)  $250\text{ ms}^{-1}$
  - 3)  $2.5\text{ ms}^{-1}$
  - 4)  $20\text{ ms}^{-1}$
4. A bullet of mass  $50\text{ grams}$  going at a speed of  $200\text{ ms}^{-1}$  strikes a wood block of mass  $950\text{ gm}$  and gets embedded in it. The velocity of the block after the impact is
- 1)  $5\text{ ms}^{-1}$
  - 2)  $10\text{ ms}^{-1}$
  - 3)  $20\text{ ms}^{-1}$
  - 4)  $50\text{ ms}^{-1}$
5. A body of mass  $3\text{ kg}$  is moving with a velocity of  $4\text{ m/s}$  towards right, collides head on with a body of mass  $4\text{ kg}$  moving in opposite direction with a velocity of  $3\text{ m/s}$ . After collision the two bodies stick together and move with a common velocity, which is \_\_\_\_\_.
- 1)  $1\text{ m/s}$
  - 2)  $0\text{ m/s}$
  - 3)  $3\text{ m/s}$
  - 4)  $4\text{ m/s}$

6. A body of mass  $m$  moving at a constant velocity  $v$  hits another body of the same mass moving at the same velocity but in the opposite direction and sticks to it. The common velocity after collision is
- 1)  $v$                                       2)  $0$                                       3)  $2v$                                       4)  $v/2$

**Reasoning Type:**

7. A body moving towards a finite body at rest collide with it.

*Statement I :* It is possible that both the bodies move after collision

*Statement II :* It is possible that the moving body comes to rest and the stationary body starts moving

- 1) Both Statements are true, Statement II is the correct explanation of Statement I.  
 2) Both Statements are true, Statement II is not correct explanation of Statement I.  
 3) Statement I is true, Statement II is false.  
 4) Statement I is false, Statement II is true.

**Comprehension:**

Two pendulum bobs of mass  $m$  and  $2m$  collide elastically at the lowest point in their motion. If both the balls are released from height  $H$  above the lowest point,

8. Velocity of the bob of mass  $m$  just after collision is

- 1)  $\sqrt{\frac{2gH}{3}}$                                       2)  $\frac{5}{3}\sqrt{2gH}$                                       3)  $\sqrt{2gH}$                                       4) None of these

9. The bob of mass  $m$  rise after the collision is

- 1)  $\frac{25H}{9}$                                       2)  $\frac{H}{9}$                                       3)  $\frac{16H}{9}$                                       4)  $\frac{H}{4}$

10. A 2 kg mass moving on a smooth frictionless surface with a velocity of  $10ms^{-1}$  hits another 2kg mass kept at rest, in an inelastic collision. After collision, if they move together

- 1) they travel with a velocity of  $5ms^{-1}$  in the same direction  
 2) they travel with a velocity of  $10ms^{-1}$  in the same direction  
 3) they travel with a velocity of  $10ms^{-1}$  in opposite direction  
 4) they travel with a velocity of in opposite direction

**LEVEL-2 & 3** 11. A rubber ball drops from a height  $h$  and after rebounding twice from the ground, it rises to  $h/2$ . The co-efficient of restitution is

- 1)  $\frac{1}{2}$                       2)  $\left(\frac{1}{2}\right)^{1/2}$                       3)  $\left(\frac{1}{2}\right)^{1/4}$                       4)  $\left(\frac{1}{2}\right)^{1/6}$

12. A ball is dropped on to a horizontal floor. It reaches a height of 144 cm on the first bounce and 81 cm on the second bounce. The coefficient of restitution is

- 1) 0                      2) 0.75                      3) 81/144                      4) 1

13. A ball is dropped from a height  $h$  above a tile floor and rebounds to a height of  $0.64h$ . The coefficient of restitution between the ball and the floor is

- 1) 0.64                      2) 0.8                      3)  $1/0.64$                       4)  $1/0.8$

14. A truck of mass 10 metric ton runs at  $3 \text{ ms}^{-1}$  along a level track and collides with a loaded truck of mass 20 metric ton, standing at rest. If the trucks couple together, the common speed after collision is

- 1)  $1 \text{ ms}^{-1}$                       2)  $0.1 \text{ ms}^{-1}$                       3)  $0.5 \text{ ms}^{-1}$                       4)  $0.3 \text{ ms}^{-1}$

15. A body dropped freely from a height  $h$  on to a horizontal plane, bounces up and down and finally comes to rest. The coefficient of restitution is  $e$ . The ratio of velocities at the beginning and after two rebounds is

- 1)  $1 : e$                       2)  $e : 1$                       3)  $1 : e^2$                       4)  $e^2 : 1$

**Integer Answer Type:**

16. A metal ball is released from a height of 32 m on a steel plate. If the coefficient of restitution is 0.5, the height will the ball rise after second bounce is \_\_\_\_\_ m.

**LEVEL-4 & 5** 17. A ball A moving with velocity  $u$ , impinges directly on an equal ball B, moving with velocity  $v$  in the opposite direction. If  $e$  is the coefficient of restitution and ball A comes to rest after impact, find the ratio of  $u : v$ .

- 1)  $\frac{1-e}{1+e}$                       2)  $\frac{1+e}{1-e}$                       3)  $\frac{e}{1-e}$                       4)  $\frac{1+e}{e}$

18. Three balls of masses  $m_1$ ,  $m_2$  and  $m_3$  are lying in straight line. The first ball is moved with a certain velocity so that it strikes the second ball directly and itself comes to rest. The second ball collides with the third and is itself comes to rest. If 'e' be the coefficient of restitution for each ball, then choose the correct relation

- 1)  $m_1 m_3 = m_2^2$                       2)  $m_1 m_3 = 2 m_2^2$                       3)  $m_1 m_3 = 5 m_2^2$                       4)  $m_1 m_3 = 4 m_2^2$

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**COLLISSIONS WORKSHEET-1 KEY**

- CUQ:** 1) 1      2) 3      3) 4      4) 1      5) 3      6) 3      7) 2      8) 1  
 9) 3      10) 2

**JEE MAINS AND ADVANCED:**

- 1) 1      2) 2      3) 2      4) 1      5) 3      6) 2      7) a-3;b-1,5;c-4;d-2  
 8) 1,3      9) 4      10) 4      11) 3      12) 1      13) 1      14) 1      15) 1  
 16) 2      17) 4      18) 2,3

**HINTS AND SOLUTIONS**

2. 
$$V_2^1 = \left( \frac{m_2 - m_1}{m_1 + m_2} \right) V_2 + \left( \frac{2m_1}{m_1 + m_2} \right) V_1$$

3. 
$$m_1 u_1 = m_1 v_1 + m_2 v_2$$

4. 
$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

5. 
$$m_1 u_1 = m_1 v_1 + m_2 v_2 \quad e=1$$

7. 
$$P_1 + P_2 = P \quad \text{further, } K_1 + K_2 = K \quad \text{or} \quad \frac{P_1^2}{2m} + \frac{P_2^2}{4m} = \frac{P^2}{2m} \quad \text{or} \quad 2P_1^2 + P_2^2 = 2P^2$$

Solving these two equations we get,

$$P_2 = \frac{4}{3}P \quad \text{and} \quad P_1 = -\frac{P}{3}, K_1 = \frac{K}{9} \quad \text{and} \quad K_2 = \frac{8K}{9}$$

8. There is no external force on the block. Internal forces break the block in two parts. The linear momentum of the block before the break should, therefore, be equal to the linear momentum of the two parts after the break. As all the velocities

are in same direction, we get, 
$$M(20\text{m/s}) = \frac{M}{2}(30\text{m/s}) + \frac{M}{2}v$$

where v is the speed of the other part. From this equation v = 10 m/s. The change in kinetic energy is

$$\frac{1}{2}M(30\text{m/s})^2 + \frac{1}{2}M(10\text{m/s})^2 - \frac{1}{2}M(20\text{m/s})^2 = \frac{M}{2}(450 + 50 - 400) \frac{\text{m}^2}{\text{s}^2} = \left( 50 \frac{\text{m}^2}{\text{s}^2} \right) M$$

Hence, the fractional change in the kinetic energy = 
$$\frac{M \left( 50 \frac{\text{m}^2}{\text{s}^2} \right)}{\frac{1}{2}M(20\text{m/s})^2} = \frac{1}{4}$$

14. Since the two masses exchange their velocities they should have equal mass.

**COLLISIONS WORKSHEET-2\_KEY**

**CUQ:** 1) 2      2) 2      3) 4      4) 3      5) 3      6) 2      7) 1      8) 1  
 9) 2      10) 2

**JEE MAINS AND ADVANCED:**

1) 1      2) 2      3) 3      4) 2      5) 2      6) 2      7) 2  
 8) 2      9) 1      10) 1      11) 3      12) 2      13) 2      14) 1  
 15) 3      16) 2      17) 2      18) 1

**HINTS AND SOLUTIONS**

**JEE-MAINS**

1. Suppose the second block moves at a speed  $v'$  after the collision. By conservation of momentum,  $mv = 2mv'$  or,  $v' = v/2$   
 Hence, the velocity of separation =  $v/2$  and the velocity of approach =  $v$ .

By definition,  $e = \frac{\text{velocity of separation}}{\text{velocity of approach}} = 1/2$ .

2. Let us take left wards as -ve and right ward as +ve.

$$m_1 = m \quad v_1 = v \quad m_2 = m \quad v_2 = 0 \quad v_1' = v' \quad v_2' = 2v'$$

From the conservation of linear momentum  $mv + 0 = m_1v' + m_2(2v')$

$$\Rightarrow v = 3v' \text{-----(1)}$$

$$\Rightarrow v' = \frac{v}{3}; e = \frac{v_1' - v_2'}{v_2 - v_1} = \frac{v' - 2v'}{0 - v} = \frac{-v'}{-v} = \frac{v'}{v} \Rightarrow \frac{v}{3v} = \frac{1}{3}$$

$\therefore$  Coefficient of restitution between two balls =  $\frac{1}{3}$ .

3.  $v_c = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}$

4.  $v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}$

5.  $u_1 = 4 \text{ m/s}$        $u_2 = 3 \text{ m/s}$



$m_1 = 3 \text{ kg}$

$m_2 = 4 \text{ kg}$

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$\Rightarrow 3 \times 4 + 4 \times (-3) = (3 + 4)v \Rightarrow v = 0$$

6.  $\frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}$

8.  $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u_1 + \frac{2m_2}{m_1 + m_2} u_2$ ;  $v_1 = \left(\frac{m - m \times 2}{m + 2m}\right) (-\sqrt{2gH}) + \left(\frac{4m}{m + 2m}\right) \sqrt{2gH} = \frac{5}{3} \sqrt{2gH}$

9.  $h_1 = \frac{v_1^2}{2g} = \frac{\left[\frac{5}{3} \sqrt{2gH}\right]^2}{2g}$ ,  $h_1 = \frac{25}{9} H$

11.  $h_n = e^{2n} \cdot h$

12.  $e = \sqrt{\frac{h_2}{h_1}}$

13.  $e = \sqrt{\frac{h_2}{h_1}}$

14.  $v = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}$

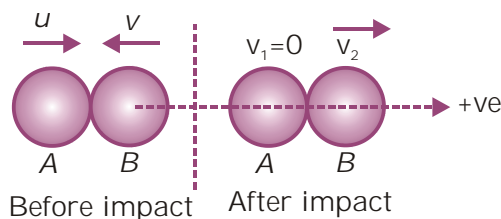
15.  $v_n = e^n \cdot v$

16.  $h_n = h e^{2n} = 32 \left(\frac{1}{2}\right)^4 = \frac{32}{16} = 2m$ , (here  $n = 2$ ,  $e = \frac{1}{2}$ )

17. Let 'm' be the mass of each ball A and B. From, principle of conservation of linear momentum,  $mu - mv = mv'$   $\Rightarrow u - v = v'$  - - - - - (1)

(where  $v_2$  is the velocity of ball B after impact).

From Newton's experimental law,



(vel. of separation) = e (vel. of approach)  $\Rightarrow v_2 = e(u + v)$  - - - - - (2)

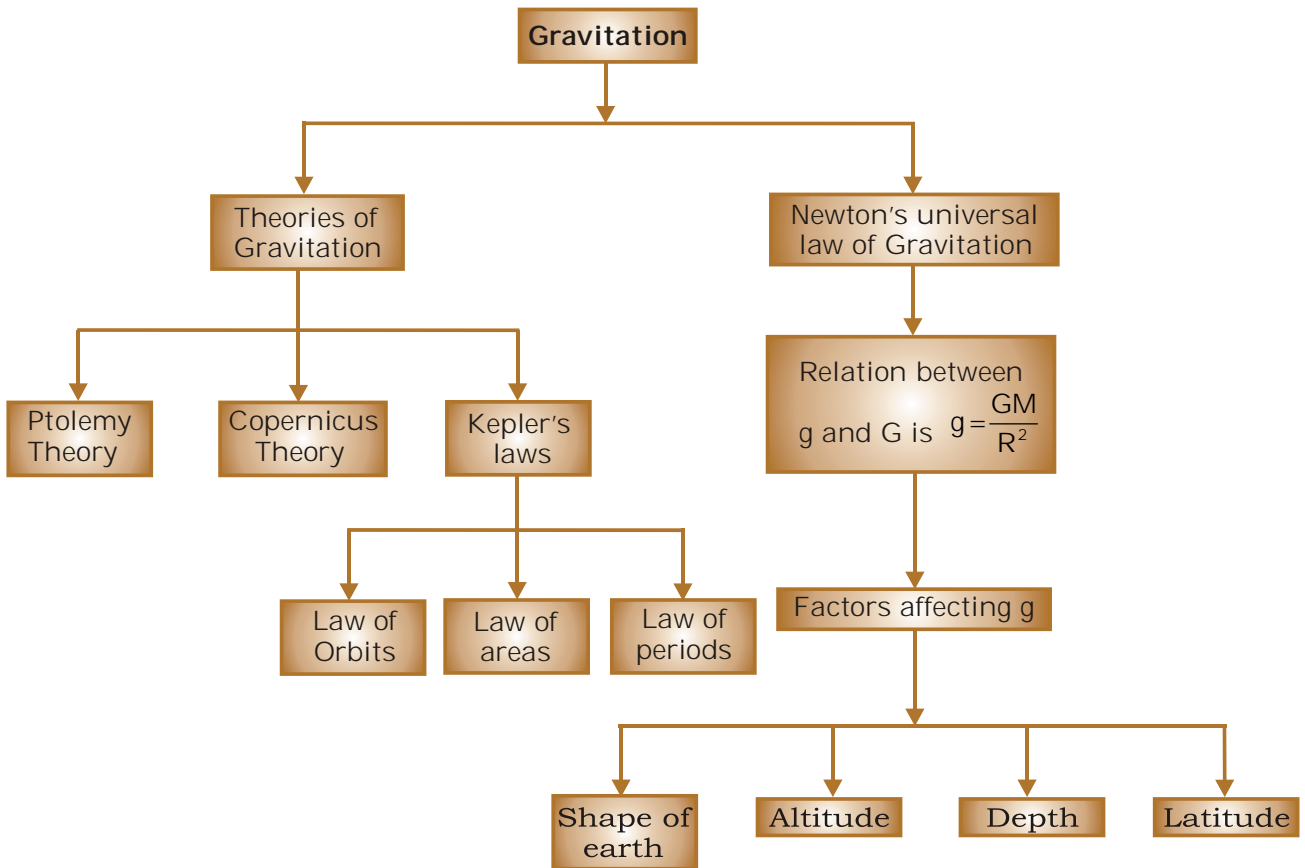
From eqns. (1) and (2) we have  $e(u + v) = (u - v)$  or  $\frac{u + v}{u - v} = \frac{1}{e}$

From component and dividend or  $\frac{u}{v} = \frac{1 + e}{1 - e}$

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# GRAVITATION

## MEMO GRAPH





**SUBRAMANYA CHANDRA SEKHAR**

Subramanya Chandrasekhar was born in Lahore. While studying the constitution of the stars he has proved that the maximum mass that a 'white dwarf' can have is 1.4 times the solar mass. This mass is known as Chandrasekhar limit. If a star crosses this limit, it has to face catastrophic collapse. Chandrasekhar got Nobel prize for physics in 1982. He was so passionate about the study of stars, when he died it was remarked that the stars were orphaned by his demise.

**GRAVITATION****SYNOPSIS-1*****Gravitation and gravity:***

The earth attracts (or pulls) all the objects towards its centre. The force with which the earth pulls the objects towards it is called the gravitational force of earth or gravity (of earth). It is due to the gravitational force of earth that all the objects fall towards the earth when released from a height.

The gravitational force of earth (or gravity of earth) is responsible for holding the atmosphere above the earth; for the rain falling to the earth and for the flow of water in the rivers. It is also the gravitational force of earth and for the flow of water in the rivers. It is also the gravitational force of earth (or gravity of earth) which keeps us firmly on the ground.

***Gravitation:***

Every body in this universe attracts every other body with a force known as 'force of gravitation'. Gravitation is the force of attraction between any two bodies in the universe. The attraction between the sun and the earth, the attraction between a table and a chair lying in a room, the attraction between the earth and a satellite revolving around it, etc.; are all examples of gravitation.

***Gravity:***

*Gravity is a special case of gravitation.* Gravity is the attraction between the earth and any object lying on or near its surface. A body thrown up falls back on the surface of the earth due to earth's force of gravity.

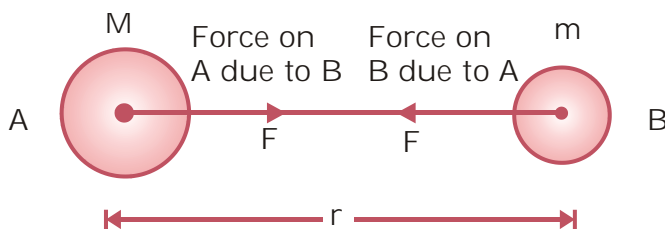
***Universal law of gravitation or Newton's law of gravitation:***

The law states that everybody in this universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres.

**Note:-** The force acts along the line joining centres of the two bodies.

**Relation between gravitational force between two bodies and the distance between them.**

Consider two bodies A and B having masses  $M$  and  $m$  respectively. Let the distance between these bodies be  $r$ .



If  $F$  is the force with which the two bodies attract each other, then according to the

law of gravitation  $F \propto Mm$  .....(1) and  $F \propto \frac{1}{r^2}$  .....(2)

Combining eqns. (1) and (2), we get  $F \propto \frac{Mm}{r^2}$  or  $F = G \frac{Mm}{r^2}$  .....(3)

where  $G$  is constant and is known as universal gravitational constant.

Eqn (3) gives the magnitude of gravitational force between two interacting bodies of masses  $M$  and  $m$  separated by distance ' $r$ '.

**Watch out!**

The equation (3) implies that force exerted by earth on an apple is same as the force exerted by apple on the earth

**Definition of universal gravitational constant (G):**

We know,  $F = \frac{Mm}{r^2}$  or  $G = \frac{Fr^2}{Mm}$

If  $M = 1$  unit,  $m = 1$  unit and  $r = 1$  unit, then  $G = F$

Thus, universal gravitational constant ( $G$ ) is defined as the force of attraction between two bodies of unit masses separated by a unit distance.

**Units of universal gravitational constant (G):**

We know,  $F = \frac{GMm}{r^2}$  or  $G = \frac{Fr^2}{Mm}$   $\therefore G = \frac{\text{unit of force} \times (\text{unit of distance})^2}{\text{unit of mass} \times \text{unit of mass}}$

Since S.I. unit of force is newton (N).

S.I. unit of distance is metre (m), S.I. unit of mass is kilogram (kg).

$\therefore$  S.I. unit of  $G = \frac{Nm^2}{kg^2}$  or  $Nm^2 kg^{-2}$

**Numerical value of gravitational constant G:**

Henry Cavendish first determined the value of G experimentally in the year 1778, by using a sensitive balance.

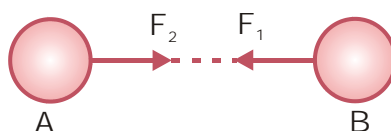
The numerical value of G is experimentally found to be  $6.67 \times 10^{-11} \text{Nm}^2\text{Kg}^{-2}$ .

**Newton's law of gravitation is known as universal law of gravitation:**

This is because the law of gravitation holds good for any pair of bodies in the universe, whether the bodies are big or small, or whether they are celestial or terrestrial.

**Characteristics of Gravitational force:**

1. Gravitational force between two bodies or objects does not need any contact between them. It means, gravitational force is **action at a distance**.
2. Gravitational force between two bodies varies inversely proportional to the square of the distance between them, Hence, gravitational force is an **inverse square force**.
3. The gravitational forces between two bodies or objects form an action reaction pair. If object A attracts object B with a force  $F_1$  and the object B attracts object A with a force  $F_2$ , then  $F_1 = -F_2$

**Free fall:**

The falling of a body (or object) from a height towards the earth under the gravitational force of earth (with no other forces acting on it) is called free fall.

**KEY POINTS**

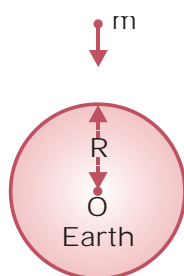
- 1) The acceleration of an object falling freely towards the earth does not depend on the mass of the object. Hence it is same for bodies of any mass
- 2) A freely falling body has acceleration equal to acceleration due to gravity (g).
- 3) The acceleration produced in the freely falling bodies is the same for all the bodies and it does not depend on the mass of the falling body.

**Acceleration due to gravity:**

The acceleration with which a body falls towards the earth due to earth's gravitational pull is known as acceleration due to gravity. It is denoted by 'g'.

**Expression for the acceleration due to gravity (Relation between G and g)**

Consider a body of mass m near the surface of the earth



The force acting on the body is the gravitational force of the earth. The magnitude of the gravitational force acting on the body due to the earth is given by

$$F = \frac{GMm}{R^2} \dots\dots(1)$$

where, M = mass of the earth, R = radius of the earth

[Here, height of the body from the surface of the earth is neglected as compared to the radius of the earth because R = 6400 km is very large.]

This gravitational force (F) produces acceleration equal to 'g' in the body of mass m. So according to Newton's second law of motion,

$$F = mg \dots\dots\dots(2)$$

Equating equations(1) and (2)  $mg = \frac{GMm}{R^2}$  or  $g = \frac{GM}{R^2} \dots\dots\dots(3)$

which is the expression for the acceleration due to gravity.

### Do you know ?

1. The force between two material bodies is not affected by the intervening medium  
G is very small-hence gravitational forces are very small, unless one(or both) of the masses is huge

## GRAVITATION WORKSHEET -1

### CUQ

1. Gravitational force is
  - 1) Mass and charge dependent
  - 2) Mass and charge independent
  - 3) Mass dependent and charge independent
  - 4) Mass independent and charge dependent
2. Gravitational force between two bodies exists
  - 1) When they are in contact only
  - 2) When they are not incontact only
  - 3) Any of the above two cases
  - 4) None of these
3. The force with which the earth pulls the objects towards it is called
  - 1) Gravitational force
  - 2) Frictional force
  - 3) Normal force
  - 4) Tension force
4. Every body in the universe attracts every other body with a force known as
  - 1) Force of friction
  - 2) Force of repulsion
  - 3) Force of gravitation
  - 4) both(1) & (2)
5. Universal law of Gravitation was proposed by\_\_\_\_\_.
  - 1) Pascal
  - 2) Newton
  - 3) Archimede's
  - 4) C.V.Raman
6. The gravitational force acting between two bodies is always\_\_\_\_
  - 1) repulsive
  - 2) attractive
  - 3) zero
  - 4) both (1) and (2)
7. The gravitational constant (G) is a\_\_\_\_\_
  - 1) scalar
  - 2) vector
  - 3) both (1) and (2)
  - 4) neither (1) nor (2)



8. Gravity is a \_\_\_\_\_ quantity  
1) vector                      2) scalar                      3) both (1) and (2)    4) neither (1) nor (2)
9. The acceleration due to gravity does not depend upon  
1) mass of the body                      2) shape or size of the body  
3) both (1) and (2)                      4) neither (1) nor (2)
10. Newton's laws of motion failed to explain  
1) The existence of gravity between bodies  
2) The non dependence of the intervening medium between the bodies on the gravitational field  
3) The reason for the variation of gravitational force with square of distance between the bodies  
4) All the above
11. Two bodies at a certain separation experience some gravitational force. If they are brought into contact the gravitational force between them.  
1) Increases                      2) Decreases                      3) Remains the same    4) Becomes zero
12. Universal law of gravitation is derived from  
1. Kepler's law                      2. Aryabhattiyam  
3. Einsteins equation                      4. Newton's law of motion
13. Newton's laws of motion do not apply when  
1. The bodies are moving with smaller velocities  
2. The bodies are at rest  
3. The bodies are moving with velocities comparable to the velocity of light  
4. The bodies move with uniform velocities
14. Newton's laws of motion are applicable for  
1. The gravitational fields which are very strong  
2. The gravitational fields which are not strong  
3. The gravitational fields which are weak as well as strong  
4. All bodies which move with the speed of light
15. Gravitational force can be called as  
1. Force without any field                      2. Force at a distance  
3. Contact force                      4. Fictitious force
16. How nature appears when viewed from different inertial frames is explained by  
1. Planck's theory                      2. Newton's theory  
3. Einstein's special theory of relativity    4. All the above
17. Newton's law of gravitation is universal because  
1. It is always attractive  
2. It acts on all the masses at all distances and not affected by the medium  
3. It acts on all bodies and particles  
4. No reason

18. Which of the following interaction is the weakest ?  
 1. Gravitational      2. Electrostatic      3. Nuclear      4. Electro magnetic
19. The unit of the quantity  $g/G$  in SI will be  
 1.  $\text{kg m}^{-2}$       2.  $\text{kg m}^{-2}$       3.  $\text{m}^2\text{kg}^{-1}$       4.  $\text{kg}^2\text{m}^{-1}$
20. The gravitational force between two bodies does not depend upon  
 1. Their separation      2. Product of their masses  
 3. Both      4. None

### JEE-MAINS

#### Single Correct Choice Type:

1. The value of  $G$  depends upon  
 1. The masses of bodies      2. The medium between the bodies  
 3. The temperature of bodies      4. None of these
2. Force of gravitation can be between  
 1) Moon and the earth      2) Sun and the earth  
 3) Moon and the sun      4) All of these
3.  $F = \frac{Gm_1m_2}{r^2}$  is valid  
 1) Between bodies with any shape      2) Between particles  
 3) Between any bodies with uniform density      4) Between any bodies with same shape
4.  $F_g$ ,  $F_e$  and  $F_n$  represent the gravitational, electro-magnetic and nuclear forces respectively, then arrange the increasing order of their strengths  
 1)  $F_n, F_e, F_g$       2)  $F_g, F_e, F_n$   
 3)  $F_e, F_g, F_n$       4)  $F_g, F_n, F_e$
5. Find the false statement  
 1) Gravitational force acts along the line joining the two interacting particles  
 2) Gravitational force is independent of medium  
 3) Gravitational force forms an action-reaction pair  
 4) Gravitational force does not obey the principle of superposition.
6. Law of gravitation is not applicable if  
 A) Velocity of moving objects are comparable to velocity of light  
 B) Gravitational field between objects whose masses are greater than the mass of sun.  
 1) A is true, B is false      2) A is false, B is true  
 3) Both A & B are true      4) Both A&B are false

7. Among the following find the wrong statement
- 1) Law of gravitation is framed using Newton's third law of motion
  - 2) Law of gravitation cannot explain why gravity exists
  - 3) Law of gravitation does not explain the presence of force even when the particles are not in physical contact
  - 4) When the range is long, gravitational force becomes repulsive.
8. Out of the following interactions the weakest is
- 1) gravitational
  - 2) electromagnetic
  - 3) nuclear
  - 4) electrostatic
9. Attractive Force exists between two protons inside the Nucleus this is due to
- 1) Gravitational Forces
  - 2) Electro magnetic Forces
  - 3) Weak Nuclear Forces
  - 4) Strong Nuclear Forces
10. Repulsive force exists between two protons outside the nucleus this is due to
- 1) Gravitational Forces
  - 2) Electro magnetic Forces
  - 3) Weak Nuclear Forces
  - 4) Strong Nuclear Forces
11. Which of the following is the evidence to show that there must be a force acting on earth and directed towards Sun?
- 1) Apparent motion of sun around the earth
  - 2) Phenomenon of day and night
  - 3) Revolution of earth round the Sun
  - 4) Deviation of the falling body towards earth
12. If suddenly the gravitational force of attraction between earth and satellite revolving around it becomes zero, then the satellite will (AIEEE 2002)
- 1) Continue to move in its orbit with same velocity
  - 2) Move tangential to the original orbit with the same velocity
  - 3) Become stationary in its orbit
  - 4) Move towards the earth
13. Two identical trains A and B move with equal speeds on parallel tracks along the equator. A moves from east to west and B moves from west to east. Which train will exert greater force on the track?
- 1) A
  - 2) B
  - 3) they will exert equal force
  - 4) The mass and the speed of each train must be known to reach a conclusion.

14. Assuming the earth to be a sphere of uniform density the acceleration due to gravity
- 1) at a point outside the earth is inversely proportional to the square of its distance from the centre
  - 2) at a point outside the earth is inversely proportional to its distance from the centre
  - 3) at a point inside is zero
  - 4) at a point inside is inversely proportional to its distance from the centre.
15. The tidal waves in the sea are primarily due to
- 1) the gravitational effect of the moon on the earth
  - 2) the gravitational effect of the sun on the earth
  - 3) the gravitational effect of the venus on the earth
  - 4) the atmospheric effect of the earth itself
16. If  $F_1$  is the force of attraction for the bodies at separation  $r_1$  and  $F_2$  is the force for the separation  $r_2$ , then
- 1)  $F_1 r_1^2 = F_2 r_2$
  - 2)  $F_1 r_1 = F_2 r_2^2$
  - 3)  $F_1 r_1^2 = F_2 r_2^2$
  - 4)  $F_1 r_1 = F_2 r_2$
17. The force of attraction between two bodies of masses 1gram each separated by a distance of 1 metre is
1.  $6.67 \times 10^{-11} \text{N}$
  2.  $6.67 \times 10^{-8} \text{N}$
  3.  $6.67 \times 10^{-17} \text{N}$
  4.  $6.67 \times 10^{17} \text{N}$
18. Two brass balls of masses 2 kg and 0.5 kg experience a force of attraction of 2N. when the distance between their centres is doubled. the force of attraction is
1.  $1/2 \text{ N}$
  2.  $1/4 \text{ N}$
  3.  $1 \text{ N}$
  4.  $2 \text{ N}$
19. Two bodies of masses  $m_1$  and  $m_2$  are separated by certain distance If  $\vec{F}_{12}$  is the force on  $m_1$  due to  $m_2$  and  $\vec{F}_{21}$  is the force on  $m_2$  due to  $m_1$ , then
1.  $F_{12} = F_{21}$
  2.  $\vec{F}_{12} = \vec{F}_{21}$
  3.  $\vec{F}_{12} = -\vec{F}_{21}$
  4. None
20. You are given a total mass M. How do you divide it into two parts so that the gravitational force between them at a distance is maximum ?
1.  $\frac{m}{4}, \frac{3m}{4}$
  2.  $\frac{m}{3}, \frac{2m}{3}$
  3.  $\frac{m}{5}, \frac{4m}{5}$
  4.  $\frac{m}{2}, \frac{m}{2}$



**SYNOPSIS-2**

**Factors on which the acceleration due to gravity depends:**

Acceleration due to gravity is

- i) directly proportional to the mass of the earth and
- ii) inversely proportional to the square of the radius of the earth.

**The value of acceleration due to gravity (g) on the earth:**

We know,  $g = \frac{GM}{R^2}$

Now,  $G = 6.673 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$ ,

$M = 5.98 \times 10^{24} \text{ kg}$  (Mass of earth)

$R = 6.4 \times 10^6 \text{ m}$  (Radius of earth). Substituting these values in equation (1), we get

$$g = \frac{6.673 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2} \times 5.98 \times 10^{24} \text{ kg}}{(6.4 \times 10^6 \text{ m})^2} = 9.8 \text{ Nkg}^{-1} = 9.8 \text{ kgms}^{-2}\text{kg}^{-1} = 9.8 \text{ ms}^{-2}$$

**Value of 'g' on the surface of the moon:**

We know,  $g_{\text{moon}} = \frac{GM_m}{R_m^2}$  .....(1)

$M_m$  (mass of the moon) =  $7.4 \times 10^{22} \text{ kg}$ ,

$R_m$  (radius of the moon) =  $1.75 \times 10^6 \text{ m}$

$G = 6.673 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$

Then, from eqn. (1)  $g_{\text{moon}} = \frac{6.673 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2} \times 7.4 \times 10^{22} \text{ kg}}{(1.75 \times 10^6 \text{ m})^2} = 1.6 \text{ ms}^{-2}$

Now,  $\frac{g_{\text{moon}}}{g_{\text{earth}}} = \frac{1.7\text{ms}^{-2}}{9.8\text{ms}^{-2}} = \frac{1}{6}$  or  $g_{\text{moon}} = \frac{1}{6} g_{\text{earth}}$

Thus, acceleration due to gravity on the surface of moon is  $\frac{1}{6}$  the times the acceleration due to gravity on the surface of the earth.

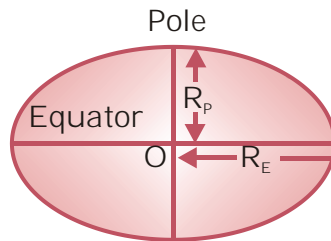
**Variation in the value of 'g':**

1. **Variation in the value of 'g' with the shape of the earth;**

The acceleration due to gravity 'g' on the surface of the earth is given by

$G = \frac{GM}{R^2}$  \_\_\_\_\_(1)

This expression for 'g' is calculated by considering the earth as a spherical body. In fact, the earth is not spherical in shape but it is egg shaped as shown in figure.



Therefore, the radius of the earth (R) is not constant throughout. Hence, the value of 'g' is different at different points on the earth.

The equatorial radius ( $R_E$ ) of the earth is about 21 km longer than its polar radius ( $R_p$ ).

Now from equation (1) value of 'g' at equator is given by  $g_e = \frac{GM}{R_p^2}$  \_\_\_\_\_(2)

Value of 'g' at pole is given by  $g_p = \frac{GM}{R_p^2}$  \_\_\_\_\_(3)

Dividing equation (3) by equation (2), we get  $\frac{g_p}{g_e} = \left(\frac{R_E}{R_p}\right)^2$

Since  $R_E > R_p \therefore g_p > g_e$

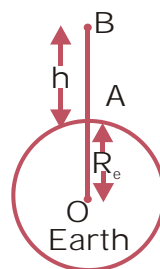
Thus, value of 'g' is more at equator than at poles.

2. **Variation in the value of 'g' with the altitude (or height) above the surface of the earth.**

We know, acceleration due to gravity on the surface of the earth is given by

$$g = \frac{GM}{R^2} \dots\dots(1)$$

Now, let a body be at a height h above the surface of the earth.



The distance of the body from the centre of the earth = (R + h).

Therefore, acceleration due to gravity at height 'h' is given by

$$g_h = \frac{GM}{(R + h)^2} \dots\dots(2)$$

Dividing (2) by (1) we get  $\frac{gh}{g} = \frac{GM}{(R+h)^2} \times \frac{R^2}{GM} = \frac{R^2}{(R+h)^2}$

$$\text{or } \frac{gh}{g} = \left(\frac{R}{R+h}\right)^2 \quad \dots\dots(3)$$

Since  $(R + h) > R \quad \therefore \frac{gh}{g} < 1$  or  $gh < g$

This shows that the value of 'g' decreases as we go higher and higher.

Thus, value of 'g' decreases with the height from the surface of the earth.

3. **Variation in the value of 'g' with depth below the surface of the earth.**

The value of 'g' decreases with depth below the surface of the earth.

The value of 'g' at depth d below the surface of the earth is given by

$$g_d = \left(1 - \frac{d}{R}\right)g \quad g_d = g\left(\frac{R-d}{R}\right)$$

This shows that the value of 'g' decreases as we go deep into the crest of the earth.

**Note:-**At the centre of the earth, depth,  $d = R$

$\therefore g$  (at centre of the earth) = 0

Thus, value of 'g' at the centre of the earth is zero.

4. **Effect of latitude (Effect of rotation of the earth about its own axis).** Due to the rotational motion of the earth about its own axis, the value of g at a place increases with the increase in latitude of the place. Hence due to rotation of the earth, the weight of a body is maximum at the poles and minimum at the equator. In fact rotation has no effect on the value of g at the poles.

**Gravity meters:**sensitive instrument used to measure small changes in the value of g at a given location are called gravity meters.

**KEY POINTS:**

1. g is independent of mass, shape and size of 'falling body', i.e., a given reference body produces same acceleration in a light and heavy falling body.
2. g is not a universal constant and depends on place, position and planet.

## GRAVITATION WORKSHEET - 2

- CUQ** 1. The value of  $g$  at poles\_\_\_\_\_.
- 1) maximum                      2) minimum                      3) constant                      4) zero
2. The value of ' $g$ ' at the equator
- 1) maximum                      2) minimum                      3) constant                      4) zero
3. The acceleration due to the gravity at the centre of the earth is
- 1) infinite                      2) zero                      3)  $9.8 \text{ m/s}^2$                       4) all of these
4. The value of acceleration due to gravity decreases with \_\_\_\_\_in height from the surface of earth
- 1) decreases                      2) increases                      3) remain same                      4) both (1) and (2)
5. The value of acceleration due to gravity decreases with \_\_\_\_\_in depth inside the earth.
- 1) decreases                      2) increases                      3) remain same                      4) both (1) and (2)
6. The value of acceleration due to gravity above the surface of the earth is inversely proportional to
- 1) square of the distance of that point from the centre of earth.  
2) distance of that point from the centre of earth  
3) square root of the distance of that point from the centre of earth.  
4) cube of the distance of that point from the centre of earth
7. The value of acceleration due to gravity below the surface of the earth is directly proportional to\_\_\_\_\_.
- 1) square of the distance of that point from the centre of earth.  
2) distance of that point from the centre of earth  
3) square root of the distance of that point from the centre of earth.  
4) cube of the distance of that point from the centre of earth
8. The instrument used to measure the value of ' $g$ ' at a given location is called
- 1) Thermometer                      2) Barometer                      3) Gravity meter                      4) Manometer
9. If the speed of rotation of earth about its axis increases, then the weight of the body at the equator will
- 1) increase                      2) decrease  
3) remain unchanged                      4) sometimes decrease and sometimes increase
10. The ratio of acceleration due to gravity at a depth ' $h$ ' below the surface of earth and at a height ' $h$ ' above the surface for  $h \ll R$
- 1) constant only when  $h \ll R$                       2) increases linearly with  $h$   
3) increases parabolically with  $h$                       4) decreases



11. If the gravitational force of earth suddenly disappears, then which of the following is correct?
- 1) weight of the body is zero                      2) mass of the body is zero  
3) both mass and weight become zero      4) neither the weight nor the mass is zero
12. Average density of the earth
- 1) does not depend on 'g'                      2) is a complex function of 'g'  
3) is directly proportional to 'g'              4) is inversely proportional to 'g'
13. A person will get more quantity of matter in Kg-Wt at
- 1) poles              2) at latitude of  $60^\circ$               3) equator                      4) satellite
14. Two identical trains A and B move with equal speeds on parallel tracks along the equator. A moves from east to west and B moves from west to east. Which train will exert greater force on the track?
- 1) A    2) B  
3) they will exert equal force  
4) The mass and the speed of each train must be known to reach a conclusion.
15. If earth were to rotate faster than its present speed, the weight of an object
- 1) increase at the equator but remain unchanged at poles  
2) decrease at the equator but remain unchanged at the poles  
3) remain unchanged at the equator but decrease at the poles  
4) remain unchanged at the equator but increase at the poles
16. A body of mass 5 kg is taken into space. Its mass becomes.
- 1) 5 kg                      2) 10 kg                      3) 2 kg                      4) 30 kg
17. Earth is flattend at poles and bulging at equators this is due to
- 1) revolution of earth around the sun is an elliptical orbit  
2) angular of velocity of spinning about its axis is more at equator  
3) centrifugal force is more at equator than poles  
4) more centrifugal force at poles than equator
18. Consider earth to be a homogeneous sphere. Scientist A goes deep down in a mine and scientist B goes high up in a baloon. The gravitational field measured by
- 1) A goes on decreasing and that of B goes on increasing  
2) B goes on decreasing and that of A goes on increasing  
3) Each decreases at the same rate  
4) Each decreases at different rates.

## JEE-MAINS

## SINGLE CORRECT CHOICE TYPE:

- Find the value of acceleration due to gravity at a height of 12,800 km from the surface of the earth. Earth radius = 6,400 km  
 1)  $1.7 \text{ m/s}^2$                       2)  $1.09 \text{ m/s}^2$                       3)  $2.09 \text{ m/s}^2$                       4)  $4.09 \text{ m/s}^2$
- Density of earth in terms of 'g' is acceleration due to gravity, M is mass of the earth, R is radius of earth  
 1)  $\frac{3g}{4\pi Rg}$                       2)  $\frac{3G}{4\pi Rg}$                       3)  $\frac{4G}{3\pi Rg}$                       4)  $\frac{4G}{3Rg}$
- If g on the surface of the earth is  $9.8 \text{ m/s}^2$ , its value at a height of 6400 km is (Radius of the earth = 6400km).  
 1)  $4.9\text{ms}^{-2}$                       2)  $9.8\text{ms}^{-2}$                       3)  $2.45\text{ms}^{-2}$                       4)  $19.6\text{ms}^{-2}$
- If g on the surface of the earth is  $9.8\text{ms}^{-2}$ , its value at a depth of 3200km (Radius of the earth = 6400km) is  
 1)  $9.8\text{ms}^{-2}$                       2) zero                      3)  $4.9\text{ms}^{-2}$                       4)  $2.45\text{ms}^{-2}$
- The mass of a body on the surface of the earth is 70 kg. What will be its (i) mass and (ii) weight at an altitude of 100 km? Radius of the earth is 6371 km.  
 1) 70 kg, 664.46 N                      2) 70 kg, 66.446 N  
 3) 70 kg, 6.6446 N                      4) 70 kg, 6644.6 N
- The mass of the body on the surface of the earth 80kg. What is the weight at an altitude of 10km?(Radius of earth is 6371 km)  
 1) 664N                      2) 784N                      3) 1000N                      4) 7.84N
- Density of earth is  $5.488 \times 10^3 \text{ kgm}^{-3}$ . Assume earth to be a homogeneous sphere. Find the value g on the surface of the earth. Use the known values of R and G  
 1)  $8.9 \text{ ms}^{-2}$                       2)  $9.8 \text{ ms}^{-2}$                       3)  $8.9 \text{ ms}^{-1}$                       4)  $9.8 \text{ ms}^{-1}$

8. The radius and density of two artificial satellites are  $R_1$ ,  $R_2$  and respectively. The ratio of acceleration due to gravities on them will be
- 1)  $\frac{R_2\rho_2}{R_1\rho_1}$                       2)  $\frac{R_1\rho_2}{R_2\rho_1}$                       3)  $\frac{R_1\rho_1}{R_2\rho_2}$                       4)  $\frac{R_2\rho_1}{R_1\rho_2}$
9. A body of 200 kg wt is lying in the surface of the earth. Find its weight at a place 'R' above the surface of the earth (Radius of the earth is R)
- 1) 25 kg wt                      2) 100 kg wt                      3) 50 kg wt                      4) remains same
10. Two planets are of the same material but their radii are in the ratio 2 : 1. Then ratio of acceleration due to gravity on those two planets is
- 1) 2 : 1                      2) 1 : 2                      3) 4 : 1                      4) 1 : 4
11. If 'w' is the weight of a body on the surface of the earth, its weight at a height equal to radius of the earth would be
- 1) w/2                      2) 2w                      3) w/4                      4) 4w

**GRAVITATION WORKSHEET-I KEY**

- CUQ:** 1) 3      2) 3      3) 1      4) 3      5) 2      6) 2      7) 1  
 8) 1      9) 3      10) 4      11) 1      12) 4      13) 3      14) 2  
 15) 2      16) 3      17) 2      18) 1      19) 1      20) 4

**JEE MAINS AND ADVANCED:**

- 1) 4      2) 4      3) 2      4) 2      5) 4      6) 3      7) 4      8) 1      9) 4  
 10) 2      11) 3      12) 2      13) 1      14) 1      15) 1      16) 3      17) 3  
 18) 1      19) 3      20) 4

**GRAVITATION WORKSHEET-2 KEY**

- CUQ:** 1) 1      2) 2      3) 2      4) 2      5) 2      6) 1      7) 2      8) 3  
 9) 2      10) 2      11) 1      12) 3      13) 3      14) 1      15) 2      16) 1  
 17) 3      18) 4

**JEE MAINS**

- 1) 2      2) 1      3) 3      4) 3      5) 1      6) 2      7) 2      8) 3  
 9) 3      10) 1      11) 3

**HINTS AND SOLUTIONS**

3.  $g \propto \frac{1}{R^2}$

4.  $g' = g \left( 1 - \frac{d}{R} \right)$

5. Mass  $m = 70$  kg    Weight on the surface of the earth  $= mg = 70 \times 9.8 = 686$  N  
 The mass of the body at the altitude of 100 km is also the same as that on the surface of the earth i.e., 70 kg

Weight of the body at a height  $h$  is  $mg'$ ;     $h = 100 \times 10^3$  m  $= 10^5$  m

$$\text{Weight} = mg' = mg \left( 1 - \frac{2h}{R} \right) = 70 \times 9.8 \left( 1 - \frac{2 \times 10^5}{6371 \times 10^3} \right) = 664.46 \text{ N}$$

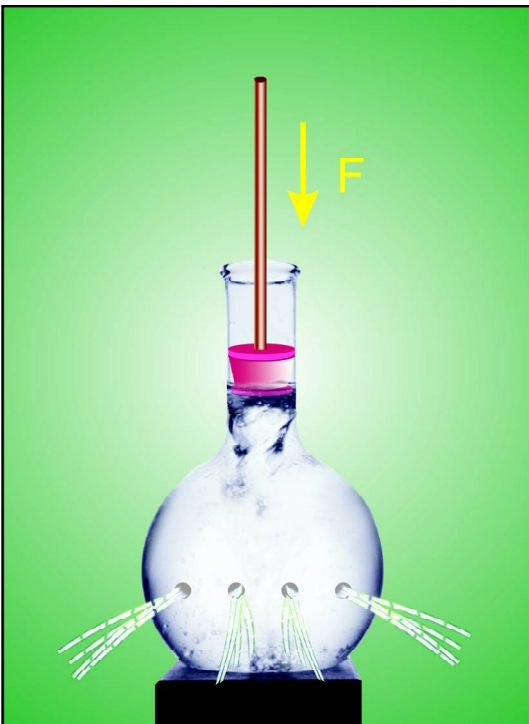
7.  $g = \frac{4\pi R G D}{3} = \frac{4 \times 3.14 \times 6371 \times 10^3 \times 6.67 \times 10^{-11} \times 5.488 \times 10^3}{3} = 9.8 \text{ ms}^{-2}$   
 8.  $g \propto R \rho$

\*\*\*\*\*



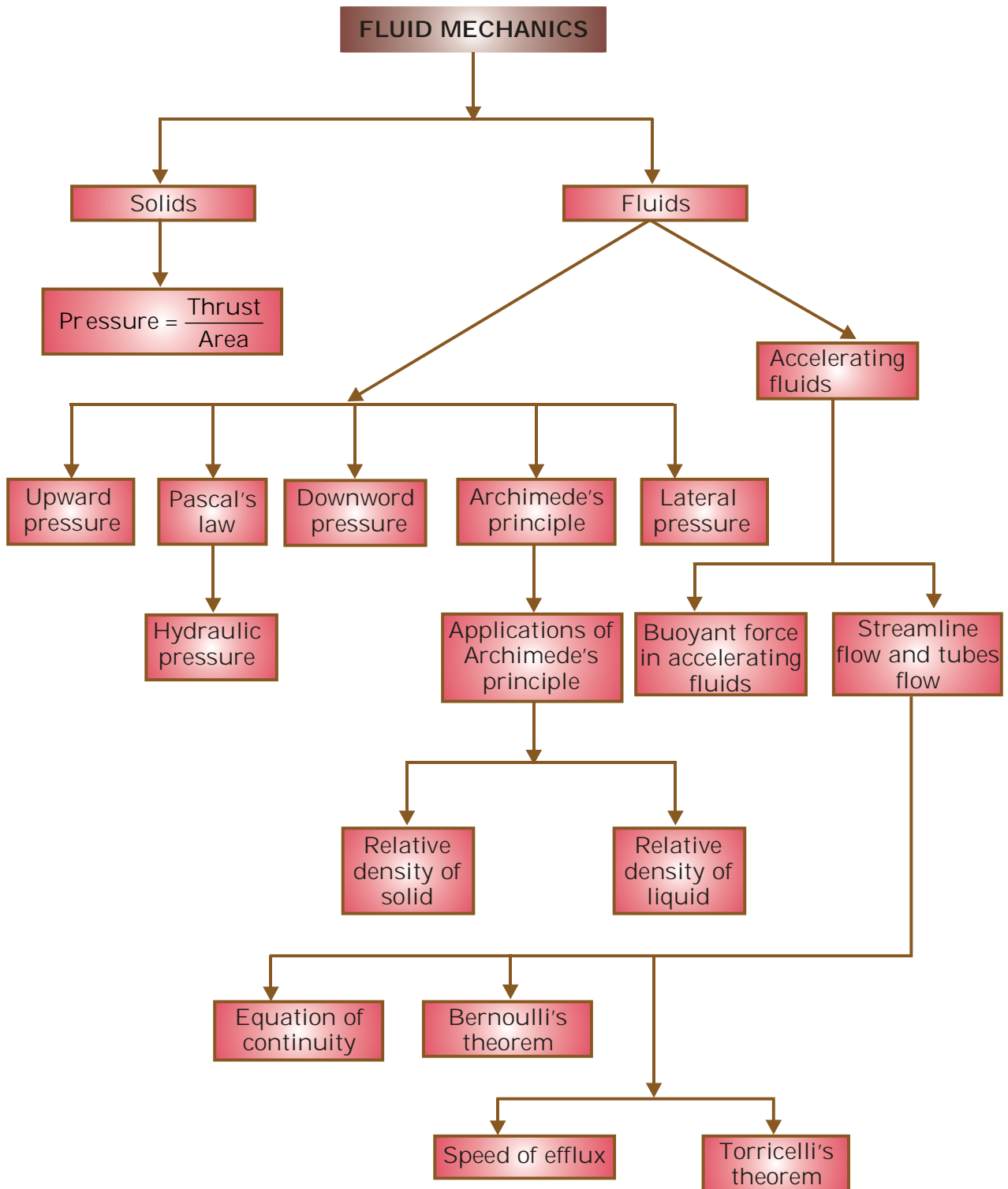
# FLUID MECHANICS

## CONTENTS



- Introduction
- Density and pressure
- Density of liquid
- Pressure in a fluid
- Pascal's law
- Pressure difference in accelerating fluids
- Archimedes' principle
- Law of floatation

# MEMO GRAPH



## KNOW YOUR SCIENTIST



Bernoulli  
(1700 - 1782)

## Bernoulli (1700 - 1782)

Bernoulli, a Swiss physicist and mathematician, made important discoveries in fluid dynamics and published 'Hydrodynamica' in 1738. In this famous work he incorporated theoretical and practical study of equilibrium, pressure, and speed in fluids. Along with Leonard Euler Bernoulli won the French Academy prize for mathematics ten times. He also studied medicine and worked as a professor of anatomy and Botany for a short period at Basle, Switzerland.

## FLUID MECHANICS

## FLUID MECHANICS SYNOPSIS-1

## 1. INTRODUCTION

The substances which flow are called fluids. Fluids include both liquids and gases. The science of fluids at rest is called fluid statics while that of moving fluids is hydro-dynamics. Fluid statics includes hydrostatic pressure, floatation, Pascal's law and Archimedes' principle while hydrodynamics includes continuity equation Bernoulli's principle and Torricelli's theorem. This all the subject of this chapter.

## 2. DENSITY AND PRESSURE

While dealing with fluids we are more interested in properties that vary from point to point in the extended substance rather than properties of a small piece of the substance. This is why we talk about density and pressure rather than mass and force in case of fluids.

## 3. DENSITY OF LIQUID

i) **Density ( $\rho$ )** of any substance is defined as the mass per unit volume or

$$\rho = \frac{\text{mass}}{\text{volume}} \quad \text{or} \quad \rho = \frac{m}{v}$$

ii) **Relative Density (R.D)**

In case of a liquid, sometimes another term relative density (R.D.) is defined. It is the ratio of density of the substance to the density of water at 4°C.

$$\text{Hence,} \quad \text{R.D} = \frac{\text{Density of substance}}{\text{Density of water at } 4^{\circ}\text{C}}$$

R.D. is a pure ratio. So, it has no units. It is also sometimes referred as specific gravity. Density of water at 4°C in CGS is 1 gm/cm<sup>3</sup>. Therefore, numerically the R.D. and density of substance (in CGS) are equal, In SI units the density of water at 4°C is 1000 kg/m<sup>3</sup>. Hence; Density of substance in S.I.units = R.D × 1000.

Ex: If R.D of kerosene is 0.8 then density in C.G.S is  $0.8 \text{ gm/cm}^3$  and density in S.I is  $0.8 \times 1000 = 800 \text{ kg/m}^3$ .

### iii) Density of a mixture of two or more liquids

Here, we have two cases.

**Case 1:** Suppose two liquids of densities  $\rho_1$  and  $\rho_2$  having masses,  $m_1$  and  $m_2$  are mixed together. Then the density of the mixture will be

$$\rho = \frac{\text{Total mass}}{\text{Total volume}} = \frac{(m_1 + m_2)}{(V_1 + V_2)} = \frac{(m_1 + m_2)}{\left(\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}\right)} \quad \text{If } m_1 = m_2, \text{ then } \rho = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2}$$

**Case 2:** If two liquids of densities  $\rho_1$  and  $\rho_2$  having volumes  $V_1$  and  $V_2$  are mixed, then the density of the mixture is,

$$\rho = \frac{\text{Total mass}}{\text{Total volume}} = \frac{(m_1 + m_2)}{(V_1 + V_2)} = \frac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2} \quad \text{If } V_1 = V_2, \text{ then } \rho = \frac{\rho_1 + \rho_2}{2}$$

#### Note:

- Density of substance means the ratio of mass of substance to the volume occupied by the substance while density of a body means the ratio of mass of a body to the volume of the body. So for a solid body

Density of body = Density of substance

while for a hollow body, density of body is lesser than that of substance [ as  $V_{\text{body}} > V_{\text{sub}}$  ].

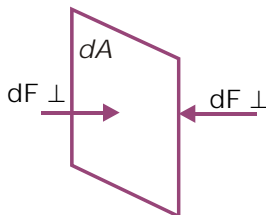
- When immiscible liquids of different densities are poured in a container, the liquid of highest density will be at the bottom while that of lowest density at the top and interfaces will be plane.

### 3. PRESSURE IN A FLUID

When a fluid (either liquid or gas) is at rest, it exerts a force perpendicular to any surface in contact with it, such as a container wall or a body immersed in the fluid.

While the fluid as a whole is at rest, the molecules that makes up the fluid are in motion, the force exerted by the fluid is due to molecules colliding with their surroundings.

If we think of an imaginary surface within the fluid, the fluid on the two sides of the surface exerts equal and opposite forces on the surface, otherwise the surface would accelerate and the fluid would not remain at rest.





Consider a small surface of area  $dA$  centered on a point on the fluid, the normal force exerted by the fluid on each side is  $dF_{\perp}$ . The pressure  $P$  is defined at that

point as the normal force per unit area, i.e.,  $P = \frac{dF_{\perp}}{dA}$

If the pressure is the same at all points of a finite plane surface with area  $A$ ,

then  $P = \frac{F_{\perp}}{A}$

Where  $F_{\perp}$  is the normal force on one side of the surface. The SI unit of pressure is pascal, where 1 pascal = 1 Pa = 1.0 N/m<sup>2</sup>

One unit used principally in meteorology is the Bar which is equal to 10<sup>5</sup> Pa.

1 Bar = 10<sup>5</sup> Pa

4. **Atmospheric Pressure ( $P_0$ )**

It is pressure of the earth's atmosphere. This changes with weather and elevation. Normal atmospheric pressure at sea level (an average value) is  $1.013 \times 10^5$  Pa. Thus, 1 atm =  $1.013 \times 10^5$  Pa = 1.013 Bar

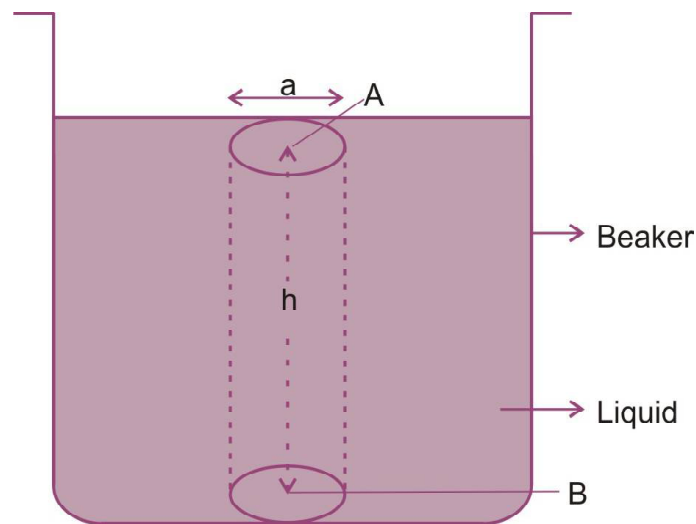
**Note:** Fluid pressure acts perpendicular to any surface in the fluid no matter how that surface is oriented. Hence, pressure has no intrinsic direction of its own, it's a scalar. By contrast, force is a vector with a definite direction.

5. **Absolute Pressure and Gauge Pressure**

The excess pressure above atmospheric pressure is usually called gauge pressure and the total pressure is called absolute pressure. Thus,

Gauge pressure = absolute pressure – atmospheric pressure

6. **Mathematical Expression for Pressure in Fluids:**



Consider a liquid contained in a beaker, such that ' $\rho$ ' is the density of liquid.

Consider a point B at the base of liquid and the liquid column of area cross-section ' $a$ ' around it, such that ' $h$ ' is the height of the liquid column.

∴ Volume of imaginary column of liquid = area of cross-section  $\times$  length =  $a.h$ .

∴ Mass of liquid column = volume  $\times$  density =  $v \times \rho$

∴ Weight of liquid column = mass  $\times$   $g$  =  $mg = a.h.\rho.g$

∴ Thrust exerted by liquid column on the base of the beaker =  $a.h.\rho.g$

∴ Pressure due to liquid column =  $P = \frac{\text{Force}}{\text{Area}} = \frac{F}{a} = \frac{a.h.\rho.g}{a}$

∴  $P = h.\rho.g$

### 7. Factors on which the pressure at a point in fluids depends

1. Pressure in a fluid is directly proportional to its height (or depth).
2. Pressure in a fluid is directly proportional to its density.
3. Pressure in a fluid is directly proportional to the acceleration due to gravity
4. Pressure in a fluid is independent of the area of cross-section.
5. If  $P_A$  is pressure due to atmosphere then total pressure at the point B is  $P_A + h\rho g$

### 8. Laws of Liquid Pressure

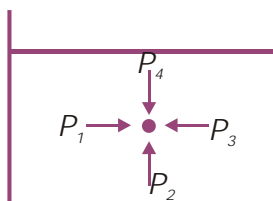
Following are the laws of Liquid Pressure:

1. Pressure at a point inside the liquid increases with the depth from the free surface of the liquid.
2. Pressure at a point inside the liquid at a given depth increases with the increase in the density of the liquid.
3. Pressure is same in all directions, about a given point within the liquid.
4. Pressure is same at all points in the horizontal plane at a given depth in a stationary liquid.
5. A liquid seeks its own level.

#### Note:

i) At same point in a fluid pressure is same in all directions.

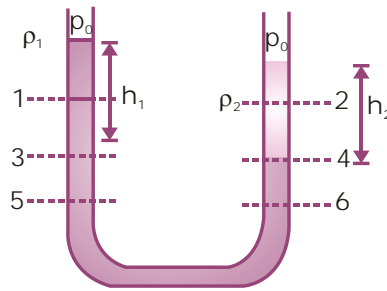
In the figure,  $P_1 = P_2 = P_3 = P_4$



ii) Forces acting on a fluid in equilibrium have to be perpendicular to its surface.

iii) In the **same liquid** pressure will be same at all points at the same level.

For example, in the figure.



$$P_1 \neq P_2, P_3 = P_4 \text{ and } P_5 = P_6; \text{ Further, } P_3 = P_4$$

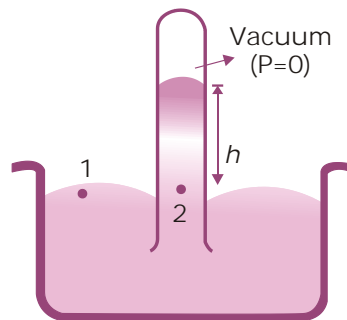
$$\therefore P_0 + \rho_1 g h_1 = P_0 + \rho_2 g h_2 \quad \text{or} \quad \rho_1 h_1 = \rho_2 h_2 \quad \text{or} \quad h \propto \frac{1}{\rho}$$

iv) In case of a given mass of an ideal gas at constant temperature

$$V \propto \frac{1}{P} \text{ or } pV = \text{const. This law is called Boyle's law.}$$

9. **BAROMETER:**

It is a device used to measure atmospheric pressure.



In principle, any liquid can be used to fill the barometer, but mercury is the substance of choice because its great density makes possible an instrument of reasonable size.

$$P_1 = P_2 \quad \text{Here, } P_1 = \text{atmospheric pressure } (P_0) \text{ and } P_2 = 0 + \rho g h = \rho g h$$

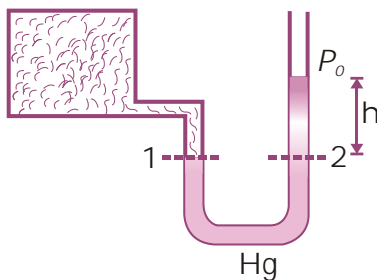
$$\text{Here, } \rho = \text{density of mercury} \quad \therefore P_0 = \rho g h$$

Thus, the mercury barometer reads the atmospheric pressure ( $P_0$ ) directly from the height of the mercury column. For example if the height of mercury in a barometer is 760 mm, then atmospheric pressure will be,

$$P_0 = \rho g h = (13.6 \times 10^3)(9.8)(0.760) = 1.01 \times 10^5 \text{ N/m}^2$$

9. **MANOMETER:**

It is a device used to measure the pressure of a gas inside a container. The U-shaped tube often contains mercury.  $P_1 = P_2$



Here,  $P_1$  = pressure of the gas in the container ( $P$ ) and  $P_2$  = atmospheric pressure ( $P_0$ ) +  $\rho gh$   $\therefore P = P_0 + \rho gh$

This can also be written as  $P - P_0 = \text{gauge pressure} = \rho gh$

Here,  $\rho$  is the density of the liquid used in U-tube.

Thus by measuring  $h$  we can find absolute (or gauge) pressure in the vessel.

**SCIENTIFIC FACT**

**How does a pressure cooker works?**

The principle of a pressure cooker is cooking under increased pressure. It is well known that food gets cooked fast at high temperatures. Generally we cook food in water kept in open vessels. In these vessels, when the water is heated to 100 degrees centigrade it begins to boil, becomes steam and escapes. Thus there is no possibility of heating the water beyond 100 degrees in open vessels. Hence it takes a lot of time to cook the food.

From physics, we know the boiling point increases with inverse in pressure. Hence in pressure cookers, the steam is not allowed to escape but enclosed with in the vessel. As more water is converted into gaseous steam, the pressure increases which in a feed back mechanism increases the boiling point to well beyond 100 degrees enabling fast cooking. Normally the temperature reaches about 120 degrees inside the pressure cooker.

The fundamental equation in physics that relates pressure( $P$ ), volume ( $V$ ) and temperature( $T$ ) is given by Boyle's law.

$PV = k T$  where  $k$  is a constant

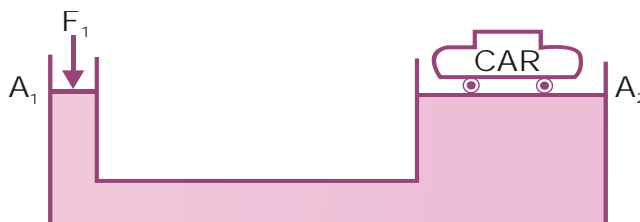
According to the equation, if  $V$  is kept constant as in a pressure cooker and  $P$  or  $T$  is increased, the other parameter increases. In the cooler, both of them increase to enable fast cooking.

In order that the pressure does not reach very high values so as to cause an explosion, a weight and safety valve are provided to let out the excess steam. Also the body of the cooker is made of an alloy which can withstand high pressures

10. **Pascal's Law**

It states that "pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel".

A well known application of Pascal's law is the hydraulic lift used to support or lift heavy objects. It is schematically illustrated in figure.





A piston with small cross section area  $A_1$  exerts a force  $F_1$  on the surface of a liquid such as oil. The applied pressure  $P = \frac{F_1}{A_1}$  is transmitted through the connecting pipe to a larger piston of area  $A_2$ . The applied pressure is the same in both cylinders, so

$$P = \frac{F_1}{A_1} = \frac{F_2}{A_2} \quad \text{or} \quad F_2 = \frac{A_2}{A_1} \cdot F_1$$

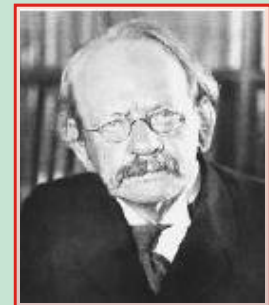
Now, since  $A_2 > A_1$ , therefore,  $F_2 > F_1$ . Thus, hydraulic lift is a force multiplying device with a multiplication factor equal to the ratio of the areas of the two pistons. Dentist's chairs, car lifts and jacks, many elevators and hydraulic brakes all use this principle.

## NOBEL LAUREATE IN PHYSICS - 1906

SIR JOSEPH JOHN THOMSON (1856 - 1940)

Cambridge University, Cambridge, Great Britain

"in recognition of the great merits of his theoretical and experimental investigations on the conduction of electricity by gases"

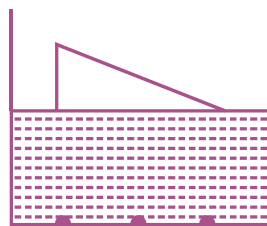


## FLUID MECHANICS WORKSHEET-1

**CUQ** 1. 100 kg of iron and cotton are weighed by using a spring balance on the surface of the earth. If  $R_1$  and  $R_2$  are the reading shown by the balance, then

- 1)  $R_1 < R_2$                       2)  $R_1 = R_2$                       3)  $R_1 > R_2$                       4)  $R_1 = R_2 = 0$

2. An object of uniform density is allowed to float in water kept in a beaker. The object has triangular cross-section as shown in the figure. If the water pressure measured at the three point A, B and C below the object are  $P_A$ ,  $P_B$  and  $P_C$  respectively then:



- 1)  $P_A > P_B > P_C$                       2)  $P_A > P_B < P_C$                       3)  $P_A = P_B = P_C$                       4)  $P_A = P_C < P_B$

3. The pressure inside a liquid of density  $d$  at a depth 'h' below its surface is
- 1)  $\frac{h}{dg}$                       2)  $hdg$                       3)  $\frac{d}{hg}$                       4)  $\frac{hg}{d}$
4. Pressure at any point inside a liquid is
- 1) directly proportional to density of the liquid  
 2) inversely proportional to density of the liquid  
 3) directly proportional to square root of density of the liquid  
 4) inversely proportional to square of density of liquid
5. Liquid pressure at a point in a liquid does not depend on
- 1) density of liquid                      2) shape of the vessel in which the liquid is kept  
 3) depth of the point from the surface    4) acceleration due to gravity
6. As the depth of a liquid increases, the pressure of liquid
- 1) decreases                      2) increases                      3) remains same    4) cannot say
7. Pressure at a certain depth in river water is  $P_1$ , and at the same depth in sea water is  $P_2$ . Then [ here density of sea water is greater than that of river water)
- 1)  $P_1 = P_2$                       2)  $P_1 > P_2$                       3)  $P_1 < P_2$     4)  $P_1 - P_2 =$  atmospheric pressure
8. Which device is used to measure atmospheric pressure?
- 1) Barometer                      2) Hydraulic press  
 3) Manometer                      4) Thermometer
9. 1 Bar = \_\_\_\_\_ pa
- 1)  $10^6$  pa                      2)  $10^3$ pa                      3)  $10^5$ pa                      4)  $10^7$ pa
10. Units of Relative Density
- 1)  $m/s^2$                       2) pa                      3)  $kg/m^3$                       4) no units

### JEE MAIN AND ADVANCED

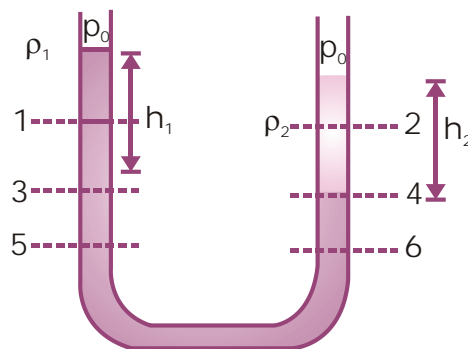
#### LEVEL-1 Single Correct Choice Type:

1. If density of kerosene in C.G.S is  $0.8 \text{ gm/cm}^3$  then density in S.I is
- 1)  $800 \text{ kg/m}^3$                       2)  $1000 \text{ kg/m}^3$                       3)  $600 \text{ kg/m}^3$                       4)  $1200 \text{ kg/m}^3$
2. Mass of stone is 900 gm dropped in a liquid and density of the stone is  $3 \frac{\text{g}}{\text{cm}^3}$ . Then volume of the stone is
- 1)  $200 \text{ cm}^3$                       2)  $300 \text{ cm}^3$                       3)  $400 \text{ cm}^3$                       4)  $100 \text{ cm}^3$
3. Water of density  $4 \text{ kg/m}^3$  and ice of density  $2 \text{ kg/m}^3$  are mixed together. If their masses are equal then the density of mixture is
- 1)  $\frac{8}{3} \text{ kg m}^{-3}$                       2)  $\frac{7}{3} \text{ kg m}^{-3}$                       3)  $\frac{5}{3} \text{ kg m}^{-3}$                       4)  $\frac{2}{3} \text{ kg m}^{-3}$
4. In the above problem, if their volumes are equal then the density of mixture is
- 1)  $5 \text{ kg m}^{-3}$                       2)  $3 \text{ kg m}^{-3}$                       3)  $1 \text{ kg m}^{-3}$                       4)  $7 \text{ kg m}^{-3}$

5. If two densities of two liquids are  $2 \frac{\text{kg}}{\text{m}^3}$  and  $3 \frac{\text{kg}}{\text{m}^3}$ , their volumes are 5 lit and 6 lit. Find the mixed density of two liquids [hint:- 1 lit = 1kg]
- 1)  $\frac{11}{28} \text{ kg m}^{-3}$       2)  $\frac{13}{28} \text{ kg m}^{-3}$       3)  $\frac{28}{11} \text{ kg m}^{-3}$       4)  $\frac{12}{28} \text{ kg m}^{-3}$
6. If two masses of liquids 1 kg and 3 kg and their densities are  $2 \frac{\text{kg}}{\text{m}^3}$  each. Then density of mixture of two liquids is
- 1)  $4 \text{ kg m}^{-3}$       2)  $3 \text{ kg m}^{-3}$       3)  $1 \text{ kg m}^{-3}$       4)  $2 \text{ kg m}^{-3}$
7. If a force is 50 dyne acting on area of block is  $10\text{cm}^2$ . Then the pressure is on block is
- 1)  $5 \frac{\text{dyne}}{\text{cm}^2}$       2)  $4 \frac{\text{dyne}}{\text{cm}^2}$       3)  $6 \frac{\text{dyne}}{\text{cm}^2}$       4)  $7 \frac{\text{dyne}}{\text{cm}^2}$
8. The pressure at the bottom of a lake, due to water is  $4.9 \times 10^6 \text{ N/m}^2$ . What is the depth of the lake?
- 1) 500m      2) 400m      3) 300m      4) 200m

**Comprehension Type:**

In the same liquid pressure will be same at points at the same level, which is shown in figure



9. From the above figure the pressure of point 3 is equal to the pressure at point \_\_\_\_
- 1) 3      2) 4      3) 5      4) 6
10. In the above question the pressure at point 5 is same as that of the pressure at point \_\_\_\_
- 1) 3      2) 4      3) 5      4) 6
11. Choose the correct one
- 1)  $\rho_1 h_1 = \rho_2 h_2$       2)  $\frac{\rho_1}{h_1} = \frac{\rho_2}{h_2}$       3)  $\rho_1 \rho_2 = h_1 h_2$       4)  $\rho_1 + h_1 = \rho_2 + h_2$

**Matrix Match Type:**

12. Density of blood is  $1025 \text{ kg/m}^3$ ,  $g = 10 \text{ m/s}^2$  and density of mercury is  $13.6 \text{ g/cm}^3$

**Column-I**

**Column-II**

- |  |   |
|--|---|
| a) 1 atm pressure is equal to          | 1) $1.01 \times 10^5 \text{ N/m}^2$         |
| b) 1 Bar is equal to column            | 2) pressure due to 76 cm of mercury         |
| c) Pressure due to 10.3 m water column | 3) pressure due to 760 mm of mercury column |
| d) Pressure due to 9.76 m blood column | 4) $1.01 \times 10^6 \text{ dyne/cm}^2$     |
|  | 5) pressure due to 0.76 m of mercury column |

**LEVEL-2 & 3** 13. Piston of cross -section area  $100 \text{ cm}^2$  is used in a hydraulic press to exert a force of  $10^7$  dynes on the water. The cross-sectional area of the piston which support an object having a mass  $2000 \text{ kg}$  is ( $g=10 \text{ m/s}^2$ )

- |                       |                        |                                 |                                    |
|-----------------------|------------------------|---------------------------------|------------------------------------|
| 1) $100 \text{ cm}^2$ | 2) $10^9 \text{ cm}^2$ | 3) $2 \times 10^4 \text{ cm}^2$ | 4) $2 \times 10^{10} \text{ cm}^2$ |
|-----------------------|------------------------|---------------------------------|------------------------------------|
14. Two pistons of a hydraulic press have diameters of 30 cm & 2.5cm . What is the force exerted by a large piston,when 50kg weight is placed on the smaller piston?
- |              |              |              |              |
|--------------|--------------|--------------|--------------|
| 1) 7200kg wt | 2) 7800kg wt | 3) 7000kg wt | 4) 8000kg wt |
|--------------|--------------|--------------|--------------|
15. The force does water exert on the base of a house tank of base area  $1.5 \text{ m}^2$  when it is filled with water up to a height of 1 m if ( $g = 10 \text{ m/s}^2$ )
- |              |              |              |              |
|--------------|--------------|--------------|--------------|
| 1) 1200 kgwt | 2) 1500 kgwt | 3) 1700 kgwt | 4) 2000 kgwt |
|--------------|--------------|--------------|--------------|

**Comprehension Type:**

Two liquids of density  $3 \text{ kg/m}^3$  and  $4 \text{ kg/m}^3$  are mixed together then,

16. If their masses are equal then the density of mixture is \_\_\_\_\_  $\text{kg/m}^3$
- |                   |                   |                  |                  |
|-------------------|-------------------|------------------|------------------|
| 1) $\frac{24}{7}$ | 2) $\frac{7}{24}$ | 3) $\frac{7}{2}$ | 4) $\frac{2}{7}$ |
|-------------------|-------------------|------------------|------------------|
17. If their volumes are equal then the density of mixture is \_\_\_\_  $\text{kg/m}^3$
- |                   |                   |                  |                  |
|-------------------|-------------------|------------------|------------------|
| 1) $\frac{24}{7}$ | 2) $\frac{7}{24}$ | 3) $\frac{7}{2}$ | 4) $\frac{2}{7}$ |
|-------------------|-------------------|------------------|------------------|
18. If the ratio of masses is 1 : 2 than the value of the density of mixture is \_\_\_\_  $\text{kg/m}^3$
- |                   |                   |                  |                  |
|-------------------|-------------------|------------------|------------------|
| 1) $\frac{5}{18}$ | 2) $\frac{18}{5}$ | 3) $\frac{9}{5}$ | 4) $\frac{5}{9}$ |
|-------------------|-------------------|------------------|------------------|



**Integer Answer Type:**

19. The reading of a spring balance when a block is suspended from it in air is 60 N. This reading is changed to 40 N when the block is submerged in water. The specific gravity of block is \_\_\_\_\_

**Multi Correct Choice Type:**

20. Choose the correct statements from the following?
- 1) The pressure exerted by a liquid column at a point depends upon the height of the liquid column above it.
  - 2) The pressure exerted by a liquid column at a point depends upon the density of the liquid.
  - 3) The pressure exerted by a liquid column at a point does not depend upon the shape of the vessel containing the liquid.
  - 4) The pressure exerted by a liquid column at a point depends upon the shape of the vessel containing the liquid.

**LEVEL-4 & 5**

21. Two syringes of different cross sections (without needles) filled with water are connected with a tightly fitted rubber tube filled with water. Diameters of the smaller piston and larger piston are 1.0cm and 3.0 cm respectively. If the smaller piston is pushed in through 6.0 cm, how much does the larger piston move out
- 1) 0.18cm                      2) 0.32cm                      3) 0.67cm                      4) 0.96cm
22. In a car lift compressed air exerts a force  $F_1$  on a small piston having a radius of 5cm. This pressure is transmitted to a second piston of radius 15cm. If the mass of the car to be lifted is 1350 kg. What is  $F_1$ ?
- 1)  $14.7 \times 10^3$  N                      2)  $1.47 \times 10^3$  N                      3)  $2.47 \times 10^3$  N                      4)  $24.7 \times 10^3$  N
23. In the above problem what is the pressure necessary to accomplish this task ( $g=9.8\text{m/s}^2$ )
- 1)  $1.87 \times 10^5$  N/m<sup>2</sup>                      2)  $18.7 \times 10^5$  N/m<sup>2</sup>                      3)  $2.87 \times 10^5$  N/m<sup>2</sup>                      4)  $28.7 \times 10^5$  N/m<sup>2</sup>
24. Toricelli's barometer used mercury. Pascal duplicated it using French wine of density  $984\text{kg/m}^3$ . The height of the wine column for normal atmospheric pressure is
- 1) 9.5 m                      2) 10 m                      3) 10.5 m                      4) 11m

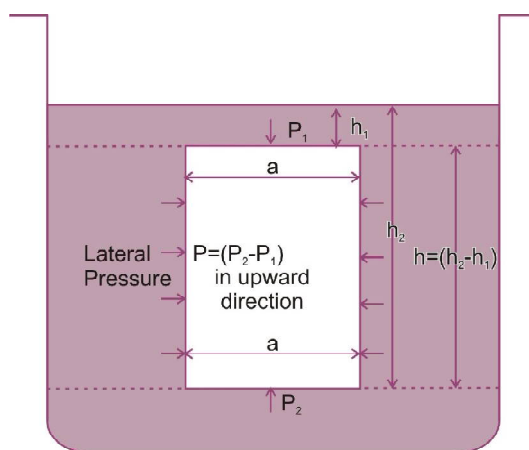
## FLUID MECHANICS SYNOPSIS-2

### I. UPTHRUST or BUOYANT FORCE:

When a body is partly or wholly immersed in a fluid (liquid or gas), the fluid exerts an upward force on the body

The upward force experienced by a body when partly or wholly immersed in a fluid is called upthrust or buoyant force, whereas the phenomenon responsible for this is called buoyancy.

### II. WHAT CAUSES UPTHRUST?



We learnt that

- Liquids exert same pressure in all directions at a given depth.
- Magnitude of pressure exerted by a liquid increases with depth and is given by the expression " $h \cdot \rho \cdot g$ ".

Where 'h' is the depth of liquid from the free surface, ' $\rho$ ', the density and 'g', the acceleration due to gravity.

When a body (say a metal block) is completely immersed in a liquid, the following kinds of pressure act on it.

- Lateral Pressure acting on the sides of the block. This pressure mutually cancels each other.
- Pressure  $P_1$  acting downward, on the upper surface of the block which is lesser in magnitude, because of lesser depth of liquid displaced.
- Pressure  $P_2$ , acting upward, on the lower surface of the block which is greater in magnitude, because of greater depth of liquid displaced.

As  $P_2$  is greater than  $P_1$ , therefore, there is a net pressure acting upward on the lower surface of the block.

As, Force = Pressure x Area of cross-section, therefore, there is a net upward force acting on the block. It is this force which is called upthrust or

buoyant force.

### Mathematical Expression for upthrust:

Consider a metal block of height  $h$  and area of cross-section ' $a$ ', completely immersed in a liquid of density ' $\rho$ ' such that the depth of liquid from the upper surface is  $h_1$ , and from the lower surface is  $h_2$ , as shown in the figure.

$\therefore$  Downward pressure  $P_1$  on the upper surface

$$P_1 = h_1 \rho g$$

Upward pressure  $P_2$  on the lower surface

$$P_2 = h_2 \rho g$$

As  $h_2 > h_1$ , therefore net pressure  $P$  in upward direction

$$P = P_2 - P_1 = h_2 \rho g - h_1 \rho g = (h_2 - h_1) \rho \cdot g = h \cdot \rho \cdot g$$

$$\{\because h_2 - h_1 = h\}$$

Furthermore, as

Force = Pressure  $\times$  Area

$$\therefore \text{Upthrust} = h \rho g \times a \text{ (or)}$$

$$\text{Upthrust} = V \rho g \{ah = \text{volume of liquid displaced}\} \text{ (or)}$$

$$\text{Upthrust} = mg \{V \times \rho = \text{mass of liquid displaced}\}$$

Upthrust =  $W$  { $W = mg$ }

Thus, upthrust is defined as the weight of the fluid displaced by the immersed part of the body.

### I. ARCHIMEDE'S PRINCIPLE

If heavy object is immersed in water, it seems to weigh less than when it is in air. This is because the water exerts an upward force called buoyant force. It is equal to the weight of the fluid displaced by the body.

"A body wholly or partially submerged in a fluid is buoyed up by a force equal to the weight of the displaced fluid". This result is known as Archimede's principle.

Thus, the magnitude of buoyant force ( $F$ ) is given by,

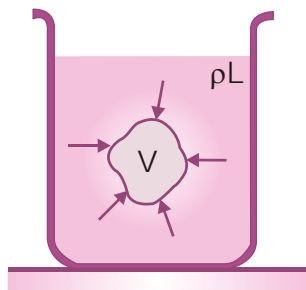
$$F = V_i \rho_L g \text{ Here, } V_i = \text{immersed volume of solid}$$

$\rho_L$  = density of liquid and  $g$  = acceleration due to gravity

Proof:

Consider an arbitrarily shaped body of volume  $V$  placed in a container filled with a fluid of density  $\rho_L$ . The body is shown completely immersed, but complete immersion is not essential to the proof. To begin with, imagine the situation before the body was immersed. The region now occupied by the body was filled with fluid, whose weight was  $V \rho_L g$ . Because the fluid as a whole was in hydrostatic equilibrium, the net upwards force (due to difference in pressure at different

depths) on the fluid in that region was equal to the weight of the fluid occupying that region.



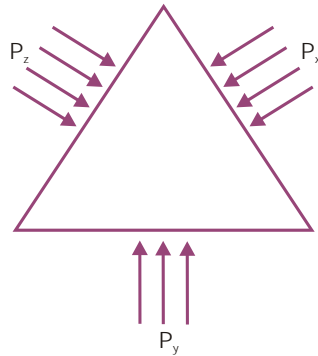
Now, consider what happens when the body has displaced the fluid. The pressure at every point on the surface of the body is unchanged from the value at the same location when the body was not present. This is because the pressure at any point depends only on the depth of that point below the fluid surface. Hence, the net force exerted by the surrounding fluid on the body is exactly the same as that exerted on the region before the body was present. But we know the latter to be  $V\rho_L g$ , the weight of the displaced fluid. Hence, this must also be the buoyant force exerted on the body. Archimede's principle is thus, proved.

## FLUID MECHANICS WORKSHEET-2

- CUQ** 1. The force of buoyancy is equal to
- 1) Weight of the body
  - 2) Weight of the liquid displaced by the body
  - 3) Apparent weight of the body
  - 4) None of these
2. When a body is fully immersed in a liquid, the loss of weight of the body is equal to
- 1) Apparent weight of the body
  - 2) force of buoyancy
  - 3) Half the force of buoyancy
  - 4) Twice the force of buoyancy
3. A boat full of scrap iron is floating on water in a lake. If all the iron is dropped into the water, the level of water will
- 1) go up
  - 2) fall down
  - 3) remain the same
  - 4) Can not be decided
4. Consider the following two statements and identify the correct choice.
- A) When a body floats in a liquid, it displaces the liquid whose weight is equal to its own weight.
- B) When a body sinks in a liquid, it displaces the liquid whose volume is equal to its own volume.
- 1) A is true but B is false.
  - 2) A is false but B is true.



- 3) Both A and B are true. 4) Both A and B are false.
5. A triangular element of the liquid is shown in the fig.,  $P_x$ ,  $P_y$  and  $P_z$  represent the pressures on the element of the liquid. Then:



- 1)  $P_x = P_y \neq P_z$     2)  $P_x = P_y = P_z$     3)  $P_x \neq P_y \neq P_z$     4)  $P_x^2 + P_y^2 + P_z^2 = \text{constant}$
6. The force of buoyancy on an immersed body is
- 1) Due to weight of the body
  - 2) Due to the pressure difference between upper surface and lower surface of the body
  - 3) Due to atmospheric pressure
  - 4) both (1) and (2)
7. A solid is completely immersed in a liquid the force exerted by the liquid on the solid will
- 1) Increase if it is pushed deeper inside the liquid
  - 2) change of its orientation is changed
  - 3) Decrease if it is taken partially out of the liquid
  - 4) None of the above
8. Buoyant force depends upon
- 1) Density of solid body immersed in a liquid
  - 2) Density of liquid in which the body is immersed
  - 3) Depends upon both density of solid and liquid.
  - 4) Independent on density of liquid
9. Buoyant force is also called as
- 1) upthrust
  - 2) upward force
  - 3) both (1) & (2)
  - 4) None of these

## JEE-MAIN AND ADVANCED

### LEVEL-1 SINGLE CORRECT CHOICE TYPE:

1. A body weighs 250g in air and 235g when completely immersed in water. Then upthrust is
- 1) 15 gf
  - 2) 20 gf
  - 3) 10 gf
  - 4) 30 gf

2. A body weighs 230g in air and 190g when completely immersed in water. Then apparent loss in weight is  
 1) 40 gf                      2) 30 gf                      3) 20 gf                      4) 50 gf
3. An iron rod of volume  $100 \text{ cm}^3$  and density  $7.8 \times 10^3 \text{ kg/m}^3$  is completely immersed in water. Calculate the upthrust acting on it.  
 (Take  $g = 10 \text{ m/s}^2$ , density of water  $10^3 \text{ kg/m}^3$ )  
 1)  $10^2 \text{ N}$                       2) 10 N                      3)  $10^{-4} \text{ m}^3$                       4) 1 N

**Comprehension type (Q.No. 4 & 5)**

An object of area of cross section  $10 \text{ cm}^2$  and length 20cm is totally immersed in water ( $g=10 \text{ m/s}^2$ ) density of object =  $1200 \text{ kg/m}^3$  density of water =  $1000 \text{ kg/m}^3$ .

4. Find the upthrust acting on it  
 1) 2 N                      2) 2.4 N                      3) 6 N                      4) 10 N
5. In above question; weight of object is  
 1) 2N                      2) 2.4N                      3) 6 N                      4) 10 N
6. A beaker containing water is placed on the platform of a spring balance. The balance reads 1.5kg. A stone of mass 0.5 kg and density  $10^4 \text{ kg/m}^3$  is immersed in water without touching the walls of beaker. What will be the balance reading now?  
 1) 2kg                      2) 2.5kg                      3) 1 kg                      4) 3 kg

**Integer type:**

7. A vessel contains oil (density  $0.4 \text{ g/cm}^3$ ) over mercury (density  $13.6 \text{ g/cm}^3$ ). A homogeneous sphere floats with half its volume immersed in mercury and the other half in oil. The density of material of the sphere in  $\text{g/cm}^3$  is \_\_\_\_\_.

**(Q.No. 8-10) Comprehension type:**

A cube of metal whose density is  $8.2 \text{ g/cm}^3$  and side 7cm is tied to a thread and completely immersed in a liquid of density  $1.2 \text{ g/cm}^3$ .

8. Calculate weight of the cube.  
 1) 2812.6 gf                      2) 28.126 gf                      3) 281.26 gf                      4) 28126 gf
9. Calculate upthrust of the cube  
 1) 411.6 gf                      2) 620 gf                      3) 120 gf                      4) 680 gf
10. Calculate tension in the string.  
 1) 2401 gf                      2) 648.126 gf                      3) 161 gf                      4) 500 gf

**LEVEL-2 & 3**

11. A tall measuring jar contains ethyl alcohol of density  $0.8 \text{ gm/cm}^3$ . An iron ball is dropped in to it and the level rises by  $20 \text{ cm}^3$ . The buoyant force acting on the ball is  
 1) 0.2N                      2) 0.25 N                      3) 0.16N                      4) 1.6N
12. A sphere of volume 'V' is immersed in two immiscible liquids (mercury and water) taken in a vessel. If half of the volume of the sphere is in mercury and the other half of its volume is in the water. Find the relative density of the sphere.  
 ( $\rho_{\text{Hg}} = 13.6 \text{ g/cm}^3$  and  $\rho_{\text{water}} = 1 \text{ g/cm}^3$ )  
 1)  $7.3 \text{ g/cm}^3$                       2)  $12 \text{ g/cm}^3$                       3)  $15 \text{ g/cm}^3$                       4)  $20 \text{ g/cm}^3$

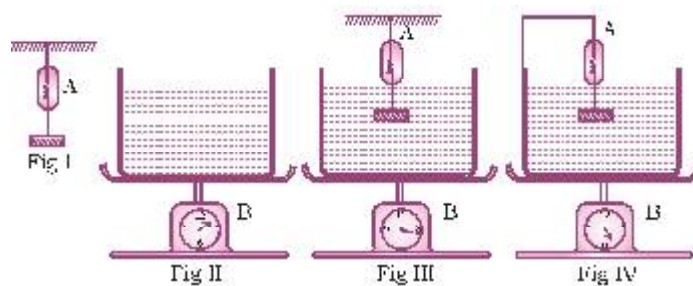
13. A boat having length 2m and width 1m is floating in a lake. When a man stands on the boat, it is depressed by 3 cm. The mass of the man is  
 1) 50kg                      2) 55kg                      3) 60 kg                      4) 70 kg

**Statement Type:**

14. *Statement I :* The apparent weight of a block of wood floating in water is zero.  
*Statement II :* The value of g in water is zero.  
 1) Both Statement-I, and Statement-II are true.  
 2) Both Statement-I, and Statement-II are false.  
 3) Statement I is true, Statement II is false.  
 4) Statement I is false, Statement II is true.
15. A block of ice of area 'a' and thickness 0.5m is floating in fresh water. In order to just support a man of 100kg. Find the area 'a' (The specific gravity of ice is 0.917 and density of water is 1000 kg/m<sup>3</sup>)  
 1) 5m<sup>2</sup>                      2) 2.41m<sup>2</sup>                      3) 4 m<sup>2</sup>                      4) 1.5 m<sup>2</sup>

**Matrix Match Type**

16. Match the column-I with column-II



The spring balance 'A' reads 'a' units in figure I and Balance 'B' reads 'b' units in figure II.

**Column - I**

- a) The reading in balance 'A' in figure-III
- b) The reading in balance 'B' in figure-III
- c) The reading in balance 'B' in figure-IV
- d) The exact decrease in weight of the body is

**Column - II**

- 1) a + b
- 2) upthrust
- 3) < a
- 4) a - Thrust force
- 5) b + Thrust force

**MULTI CORRECT CHOICE TYPE:**

17. A cylinder of area of cross section 4 cm<sup>2</sup> and length 60 cm is totally immersed in water (g = 10 m/s<sup>2</sup>) (density of solid = 1500 kg/m<sup>3</sup>, density of water = 1000 kg/m<sup>3</sup>) choose correct statements.  
 1) weight of solid is 36 N  
 2) upthrust acting on the cylinder is 2.4 N  
 3) upthrust acting on the cylinder is 24 N  
 4) weight of the solid is 3.6 N

**LEVEL-4 & 5**

18. An ornament weighing 36 g in air, weighs only 34 g in water. Assuming that some copper is mixed with gold to prepare the ornament. Find the amount of copper in it (specific gravity of gold is 19.3 and that of copper is 8.9)  
 1) 1.4 g                      2) 2.2 g                      3) 3 g                      4) 4.1 g
19. A necklace weighs 50 g in air, but it weighs 46 g in water. Assume that copper is mixed with gold to prepare the necklace. Find now much copper is present in it. (Specific gravity of gold is 20 and that of copper is 10)  
 1) 40g                      2) 30g                      3) 20g                      4) 15g
20. An iron casting has a number of cavities in it. It weighs 6000N in air and 4000N in water. Determine the total volume of all the cavities in the casting. The density of iron (without cavities) is  $8.0 \text{ g/cm}^3$ ,  $\rho_{\text{water}} = 1\text{g/cm}^3$   
 1)  $0.412 \text{ m}^3$                       2)  $0.612 \text{ m}^3$                       3)  $0.125 \text{ m}^3$                       4)  $0.250 \text{ m}^3$

\*\*\*\*\*

**FLUID MECHANICS\_WORKSHEET\_1 KEY**

- CUQ** 1) 2                      2) 3                      3) 2                      4) 1                      5) 2                      6) 2                      7) 3                      8) 1  
 9) 3                      10) 4

**JEE MAIN AND ADVANCED:**

- 1) 1                      2) 2                      3) 1                      4) 2                      5) 3                      6) 4                      7) 1                      8) 1  
 9) 2                      10) 4                      11) 1                      12) a-1,2,3,4,5; b-1,2,3,4,5; c-1,2,3,4,5; d-1,2,3,4,5  
 13) 3                      14) 1                      15) 2                      16) 1                      17) 3                      18) 2                      19) 3                      20) 1,2,3  
 21) 3                      22) 2                      23) 1                      24) 3

**HINTS AND SOLUTIONS**

1. Density in SI =  $0.8 \times 1000 = 800 \text{ kg/m}^3$   
 2. Density = mass/Volume; Volume = mass/density

3. 
$$\rho = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2}$$

4. 
$$\frac{\rho_1 + \rho_2}{2}$$

5. 
$$\frac{\frac{m_1 + m_2}{\rho_1} + \frac{m_2}{\rho_2}}$$

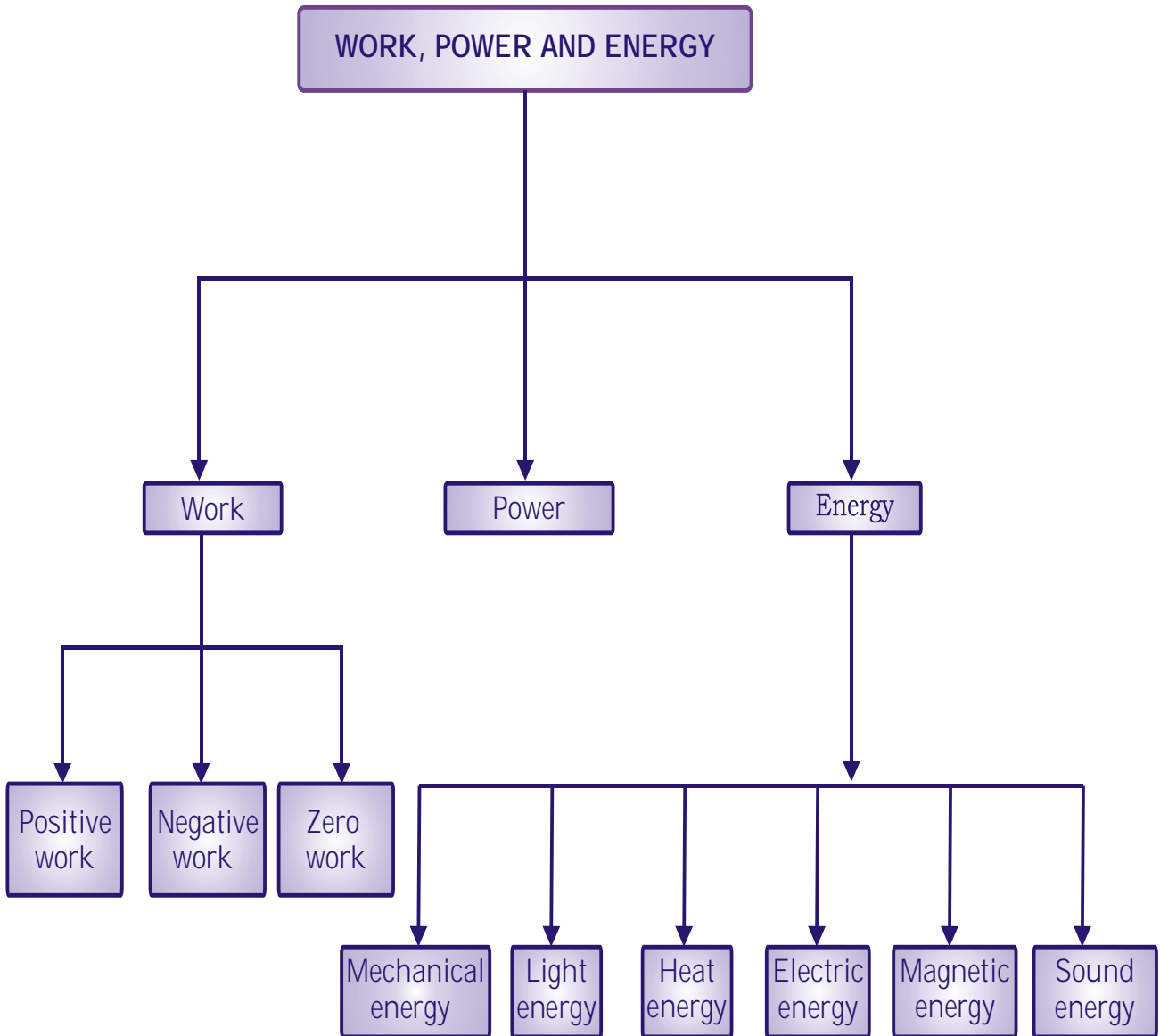




## HINTS &amp; SOLUTIONS

1. weight in air - weight in water  
= (250g - 235g) = 15 gf
2. weight in air - weight in water  
= (230g - 190g) = 40 gf
3.  $F = V \times d \times g$   
=  $10^{-4} \times 10^3 \times 10 = 1 \text{ N}$
4.  $F = v_i \rho_i g$
5.  $F = v \rho_s g$
7.  $Vdg = \frac{V}{2} d_{Hg} \times g + \frac{V}{2} d_{oil} \times g$
8. Volume of the cube =  $7 \times 7 \times 7 = 343 \text{ cm}^3$   
mass of the cube = volume  $\times$  density =  $343 \times 8.2 = 2812.6 \text{ g}$   
weight = 2812.6 gf
9. upthrust =  $V \times d \times g = 344 \times 1.2 \times g = 411.6 \text{ gf}$
10. Tension in the string =  $2812.6 - 411.6 = 2401 \text{ gf}$
11.  $Mg = Vdg$
12.  $Vdg = \frac{V}{2} \times 13.6 \times g + \frac{V}{2} \times 1 \times g$
13.  $Mg = Vdg$
15.  $m_1 = 100 \text{ kg}$ ;  $m_2 = 0.917 \times 1000 = 917$ ;  $\rho = 1000 \text{ kg/m}^3$ ;  $h = 0.5 \text{ m}$ ;  
 $100g + 917vg = v\rho g = v \times 1000 \times g$   
 $v = \frac{100}{83} g \Rightarrow Ah = \frac{100}{83} \Rightarrow A = \frac{100}{83 \times 0.5} = 2.41$
17. weight =  $mg = A \times l \times \rho_s \times g = 4 \times 10^{-4} \times 60 \times 10^{-2} \times 1500 \times 10$   
=  $3600 \times 10^3 \times 10^{-6} = 3.6 \text{ N}$   
Upthrust =  $4 \times 10^{-4} \times 60 \times 10^{-2} \times 1000 \times 10 = 2.4 \text{ N}$

### MEMO GRAPH



## WORK, POWER & ENERGY

### SYNOPSIS-1

**Definition:** Work is said to be done when a force produces motion.

**Examples:** A man climbing the stairs of a house is also doing work in moving himself against the force of gravity.

**Mathematical Expression for work :**

work = Force × Distance moved in the direction of force

**The work done by a force on a body depends on two factors.**

- i) Magnitude of the force.
- ii) Distance through which the body moves (in the direction of force)

**Units of work:** The unit of work in C.G.S. system is 'erg'.

$$W = F \cdot S = 1 \text{ dyne} \times 1 \text{ cm} = 1 \text{ erg}$$

**erg :** Work done is said to be one 'erg' if a force of one dyne displaces the body through a distance of 1 cm along the direction of force.

$$1 \text{ erg} = 1 \text{ dyne} \times 1 \text{ cm} = 1 \text{ g cm s}^{-2} \times \text{cm} \quad \text{or} \quad 1 \text{ erg} = 1 \text{ g cm}^2 \text{ s}^{-2}$$

The unit of work in S.I. system is joule (J)

$$W = F \cdot S = 1 \text{ N} \times 1 \text{ m} = 1 \text{ joule}$$

**Joule :** Work is said to be one joule, if a force 1 newton displaces a body through a distance of 1 m along the direction of force.

$$1 \text{ J} = 1 \text{ N} \times 1 \text{ m} = 1 \text{ kg} \cdot \frac{\text{m}}{\text{s}^2} \cdot \text{m} \quad 1 \text{ J} = 1 \text{ kg m}^2 \text{ s}^{-2}$$

**Relation between Joule and erg :**

$$\begin{aligned} 1 \text{ J} &= 1 \text{ N} \times 1 \text{ m} = 10^5 \text{ dyn} \times 100 \text{ cm} \\ &= 10^7 \text{ dyn cm} \quad \text{or} \quad 1 \text{ J} = 10^7 \text{ erg.} \end{aligned}$$

Work done can be positive, negative or zero depending upon the direction of force and direction of motion. (displacement)

Work done by a force on a body (or an object) is said to be positive work done when the body is displaced in the direction of applied force.

**Types of work done:**

**Positive work done :** Work done by a force on a body (or an object) is said to be positive work done when the body is displaced in the direction of applied force.

**Examples :** i) The body falling freely under the action of gravity has positive work done by the gravitational force.

ii) The work done by the engine is positive.



**Negative work done :** The work done by a force on a body is said to be negative work done when the body is displaced in a direction opposite to the direction of the force.

**Examples :**

- i) When an object is lifted upward to a certain height, then the work done by the force of gravity (equal to the weight of the object) on the object is negative.
- ii) Work done by frictional force as force of friction and the displacement are opposite to each other.
- iii) Work done by a person with a suitcase on his head moving upwards in the vertical direction. Here the displacement is in the upward direction and the force of gravity acts in the downward direction.
- iv) When brakes are applied on a moving vehicle, work done by the braking force is negative.

**Zero work :**

If  $S = 0$  i.e., the body does not move from its position on the application of force, then  $W = 0$ . Thus, no work is done by the force if it fails to displace the body.

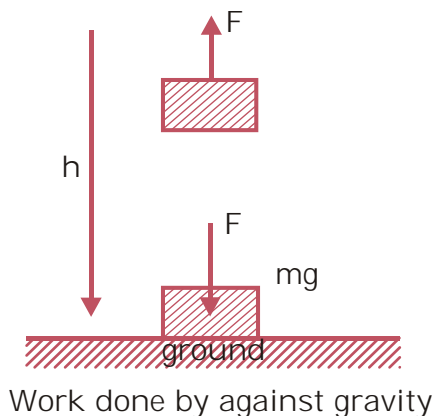
**Ex :** When a person pushes a wall but fails to move the wall, then work done by the force on the wall is zero.

- b) When a body moves in a direction perpendicular to the direction of the force no work is done by the force.

**Examples :**

- i) When a person carrying a suitcase in his hand or on his head is walking horizontally, the work done against gravity is zero.
- ii) No work is done on a body when it moves along a circular path.
- iii) Work done by the flying aeroplane is zero as the force and displacement are perpendicular to each other.

**Work done against gravity :** If a body is lifted vertically upwards, then the force required to lift the body is equal to its weight. So, whenever work is done against gravity, the amount of work done is equal to the product of weight of the body and the vertical distance through which the body is lifted.



Suppose a body of mass 'm' is lifted vertically upwards through a distance 'h'. In this case the force required to lift the body will be equal to weight of the body ( $m \times g$ ).

Work done in lifting a body = weight of body  $\times$  vertical distance

$$W = mg \times h$$

Where,  $m$  = mass of body

$g$  = acceleration due to gravity at that place

$h$  = height through which the body is lifted

**ACTIVITY :-** Lift an object up. Work is done by the force exerted by you on the object. The object moves upwards. The force you exerted is in the direction of displacement. However, there is the force of gravity acting on the object.

Which one of these forces is doing positive work ?

Which one is doing negative work ?

### WORKSHEET-1

- CUQ** 1. A bucket full of water is drawn up by a person. In this case the work done by the gravitational force is
- 1) negative because the force and displacement are in opposite directions
  - 2) positive because the force and displacement are in the same direction
  - 3) negative because the force and displacement are in the same direction
  - 4) positive because the force and displacement are in opposite directions
2. When brakes are applied on a moving vehicle, work done by the braking force is
- 1) Positive
  - 2) Negative
  - 3) Zero
  - 4) Infinity
3. A body when it moves along a circular path, the work done on the body is
- 1) Positive
  - 2) Negative
  - 3) Zero
  - 4) Infinity
4. The work done by the engine is \_\_\_\_\_
- 1) Positive
  - 2) Negative
  - 3) Zero
  - 4) Infinity
5. Work done when the body is displaced in the direction of applied force is \_\_\_\_\_
- 1) Positive
  - 2) Negative
  - 3) Zero
  - 4) Infinity
6. The work is done by the force if it fails to displace the body is
- 1) Positive
  - 2) Negative
  - 3) Zero
  - 4) Infinity
7. Work done when the body is displaced in the opposite direction of applied force is
- 1) Positive
  - 2) Negative
  - 3) Zero
  - 4) Infinity

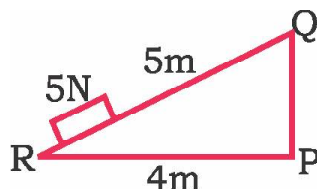
8. A ball is thrown vertically upwards from the ground. Regarding the work done by air resistance which of the following is correct?
- 1) work done is positive during the ascent and negative during the descent
  - 2) work done is positive during the ascent and positive during the descent
  - 3) work done is negative during the ascent and positive during the descent
  - 4) work done is negative during the ascent and negative during the descent

### JEE MAINS

#### Single Correct Choice Type:

1. An agent is moving a positively charged body towards another fixed positive charge. The work done by the agent is
  - 1) positive
  - 2) negative
  - 3) zero
  - 4) may be positive or negative
2. A man weighing 80 kg climbs a staircase carrying a 20 kg load. The staircase has 40 steps each of 25 cm height. If he takes 20 seconds to climb, the work done is
  - 1) 9800 J
  - 2) 490 J
  - 3)  $98 \times 10^5$  J
  - 4) 7840 J
3. Work is said to be done
  - 1) When no force is applied
  - 2) When a force produces no motion.
  - 3) When a force produces motion.
  - 4) None of these
4. Mathematical Expression for work is
  - 1) Force  $\times$  Distance moved in the direction of applied force
  - 2) 
$$\frac{\text{Force}}{\text{Dis tance moved in the direction of force}}$$
  - 3) 
$$\frac{\text{Dis tance in the diriction of force}}{\text{Force}}$$
  - 4) None of these
5. Unit of work is
  - 1) Joule
  - 2) erg
  - 3) newton
  - 4) Both (1) & (2)
6. Nature of work done by gravitational force
  - a) may be negative
  - b) may be positive
  - c) may be zero
  - d) always positive
  - 1) a,b & d are correct
  - 2) a, b & c are correct
  - 3) a,c & d are correct
  - 4) all correct
7. A rain drop of mass (1/10) gram falls vertically at constant speed under the influence of the forces of gravity and viscous drag. In falling through 100 m, the work done by gravity is
  - 1) 0.98 J
  - 2) 0.098 J
  - 3) 9.8 J
  - 4) 98 J

8. When displacement is perpendicular to the force, work done is  
 1) Positive      2) Negative      3) Zero      4) May be positive or negative
9. A weight of 5 N is moved up a frictionless inclined plane from R to Q as shown. What is the work done in joules?



- 1) 15                      2) 20                      3) 25                      4) 35
10. Work done by the gravitational force on a body of mass "m" moving on a smooth horizontal surface through a distance 's' is  
 1) mgs                      2) -mgs                      3) 0                      4) 2 mgs
11. Calculate the work done by a passenger standing on a platform holding a suitcase of 10 kgwt  
 1) 15                      2) 10                      3) 0                      4) 5

### JEE ADVANCED

#### Multi Correct Choice Type:

12. The work done by a force on a body depends on two factors  
 1) Magnitude of the force.                      2) External dimensions of the body  
 3) Distance through which the body moves      4) Colour of the body
13. Choose the correct statements :  
 1) When displacement is opposite to the force work done = - (force × displacement)  
 2) When displacement is opposite to the force work done = + (force × displacement)  
 3) Work done by frictional force is negative  
 4) Work done by frictional force is positive

#### Statement Type

14. *Statement I* : Work done is said to be one erg if a force of one dyne displaces the body through a distance of 1 cm along the direction of force.  
*Statement II* : Work is said to be one joule, if a force 1 newton displaces a body through a distance of 1 m along the direction of force.
- 1) Both Statements are true, Statement II is the correct explanation of Statement I.  
 2) Both Statements are true, Statement II is not correct explanation of Statement I.  
 3) Statement I is true, Statement II is false.  
 4) Statement I is false, Statement II is true.

#### Comprehension Type:

Work done can be positive, negative or zero depending upon the direction of force and direction of motion



15. Work done by a force on a body is said to be positive work done when
- 1) No force is applied
  - 2) The body is displaced in a direction opposite to the direction of the force.
  - 3) The body is displaced in the direction of applied force
  - 4) Both (1) & (2)
16. The work done by a force on a body is said to be negative work done when
- 1) No force is applied
  - 2) The body is displaced in a direction opposite to the direction of the force.
  - 3) The body is displaced in the direction of applied force
  - 4) The body does not move from its position on the application of force
17. The work done by a force on a body is said to be zero when
- 1) No force is applied
  - 2) The body is displaced in a direction opposite to the direction of the force.
  - 3) The body is displaced in the direction of applied force
  - 4) The body does not move from its position on the application of force

**Matrix Match Type:**

18. **Column - I**
- a) 1Joule
  - b) 1erg
  - c) Work done in lifting a body
  - d) Body falling freely under the action of gravity
- Column - II**
- 1)  $10^7$  erg.
  - 2)  $1 \text{ kg m}^2 \text{ s}^{-2}$
  - 3) Positive work done
  - 4) Weight of body  $\times$  vertical distance
  - 5)  $1 \text{ g cm}^2 \text{ s}^{-2}$

**Integer Answer Type:**

19. A boy pushes a book by applying a force 5N, the work done by this force in displacing the book through 20 cm along the direction of the push is \_\_\_\_\_J

**Multi Correct Choice Type:**

20. Choose the correct statements :
- 1) When a stone tied to a string is whirled in a circle, the work done on it by the string is zero
  - 2) When brakes are applied on a moving vehicle, work done by the braking force is negative.
  - 3) Work done by the flying aeroplane is zero as the force and displacement are perpendicular to each other.
  - 4) The body falling freely under the action of gravity has positive work done by the gravitational force.

**Matrix Match Type:**

21. **Column - I**
- a) One kilo joule
  - b) One mega joule
  - c) One giga joule
  - d) 1 erg
- Column - II**
- 1)  $10^6$  joules
  - 2)  $10^3$  joules
  - 3)  $10^9$  joules
  - 4) 1 dyne - cm
  - 5)  $10^{-7}$  joules

## SYNOPSIS-2

### Introduction to Energy

It is often said that a person A is more energetic than a person B. The meaning of this statement is that a person A can do more work than the person B. Similarly, a person after doing a lot of work gets tired and after that he is not able to do much work. It is clear that a person doing work expends something. This 'something' is known as the energy of the person. The energy spent by a person is equal to the work done by him. Human beings and animals get energy by eating food.

It may be noted that anything which is capable of doing work has energy. For example, the steam pushes up the lid placed on the boiling water container. It means, the steam has the ability or capacity to do work. The work done by the steam on the lid is equal to the energy of the steam.

### Definition of energy

If a person can do a lot of work we say that he has a lot of energy or he is very energetic. In physics also, anything which is able to do work is said to possess energy.

**Thus, energy is the ability to do work or the capacity to do work.**

### Units of energy

Unit of energy is same as that of the unit of work as work is a form of energy.

So, S.I. unit of energy is **Joule (J)**.

When we say that energy of a body is 1 joule, it means, this body has the capacity to do 1 J work.

### Commercial unit of Energy

The commercial unit (or trade unit) of energy is kilowatt-hour which is written in short form as kWh. Kilowatt-hour is usually used as a commercial unit of electrical energy.

One kilowatt-hour is the amount of electrical energy consumed when an electrical appliance having a power rating of 1 kilowatt is used for 1 hour. Since a kilowatt means 1000 watts, so we can also say that one kilowatt-hour is the amount of electrical energy consumed when an electrical appliance of 1000 watts is used for 1 hour.

1 kilowatt-hour is the amount of energy consumed at the rate of 1 kilowatt for 1 hour. That is, 1 kilowatt-hour = 1 kilowatt for 1 hour

or 1 kilowatt-hour = 1000 watts for 1 hour

**Note:** Watt or kilowatt is the unit of electrical power but kilowatt-hour is the unit of electrical energy.

Energy is a scalar quantity.

Dimensional formula of energy =  $ML^2T^{-2}$

## Kinds of Energy

In actual practice there are many kinds of energy, such as mechanical energy; heat energy; light energy; sound energy, electrical energy; nuclear energy; chemical energy, etc. Let us discuss about mechanical energy.

### Mechanical energy (M.E)

The sum of kinetic energy (K.E) and potential energy (P.E) of a body is known as mechanical energy.  $\therefore M.E = K.E + P.E$

**Kinetic energy:-** Energy possessed by a body by virtue of its motion is called kinetic energy

Ex: (1) Moving vehicle

Energy has many forms, such as thermal energy (Heat Energy), sound Energy, Light Energy.

### Expression for Kinetic energy :-

The kinetic energy of a body of mass 'm' moving with a speed v is  $\frac{1}{2} \times m \times v^2$

**Potential Energy:-** Energy possessed by a body (or) system by virtue of its position.

Ex:-(1) Water stored at a height, (2) compressed spring

### Gravitational potential energy :-

The Potential energy due to height above the earth's surface is called gravitational potential energy.

In general, if the potential energy at the ground is taken as zero, the potential energy of an object at a height h above the ground is given by

$$U = mgh$$

The energy results from the force of attraction mg between the earth and the object. From Newton's third law, both earth and the object attract each other. Hence, strictly speaking energy 'mgh' is not the potential energy of the object alone it is the potential energy of object-earth system.

**Heat Energy:-** Heat is the energy that is transferred between a system and its environment because of a temperature difference that exists between them.

Heat is an Internal energy that consists of the kinetic and potential energies associated with the random motion of the atoms, molecules and other microscopic bodies within object.

**Sound Energy:** Sound is a form of energy, that is produced by a body when it is in the state of vibration. It propagates in the form of Longitudinal waves through elastic media and causes sensation of hearing.

### Light Energy:-

- Light is a form of energy, which causes sensation of vision.
- Light travels from one place to another place in the form of Electromagnetic waves.
- E.M wave can transport energy and deliver it to a body on which it falls.

**Elastic potential energy :-**

When a spring is stretched or compressed from its natural length, it gets extra energy. It can return to its natural length by performing some work.

The extra energy stored in a stretched or compressed spring is called elastic potential energy.

A stretched rubber band also has potential energy, whereas a rubber band at its natural length lying on a table has no elastic potential energy.

**Other forms of energy :-**

Besides mechanical energy, energy can exist in several other forms.

Charged particles and electric currents can produce electrical energy and magnetic energy. Electric batteries, cooking gas, petrol etc., have chemical energy stored in them. Even matter itself is a concentrated form of energy and can be converted into other forms of energy such as kinetic energy and heat energy.

**Electrical energy :** Energy associated with electric current is called electrical energy.

The flow of electrical current causes bulbs to glow, fans to rotate and bells to ring.

**Work and energy :** Whenever work is done on an object, its energy increases.

When we push a block kept on a table, the block starts moving. We do work on the block and the block acquires kinetic energy.

**ACTIVITY :-**

- Take a rubber band.
- Hold it at one end and pull from the other. The band stretches.
- Release the band at one of the ends.
- What happens?
- The band will tend to regain its original length. Obviously the band had acquired energy in its stretched position.
- How did it acquire energy when stretched?

**WORKSHEET-2**

- CUQ** 1. The total amount of work done on a body is equal to 24J, then change in its energy.
- 1) zero                      2) 24 J                      3) 42 J                      4) Infinity
2. In the construction of electric bells, relays, electric motors and electric generators \_\_\_\_\_ energy is used
- 1) Sound                      2) Light                      3) Heat                      4) Magnetic
3. Whenever work is done on an object, its energy is
- 1) zero                      2) increases                      3) decreases                      4) infinity



4. Energy is associated with electric current is called \_\_\_\_\_ energy  
 1) Sound                      2) Magnetic                      3) Electrical                      4) Light
5. The phenomenon of transformation of the energy, from the useful form to the useless form is known as  
 1) Conservation of energy                      2) Dissipation of power  
 3) Dissipation                      4) Dissipation of energy

### JEE MAINS

#### Single Correct Choice Type:

1. The S.I unit of energy is  
 1) joule                      2) erg                      3) watt                      4) newton
2. The Dimensional formula for energy is  
 1)  $MLT^{-2}$                       2)  $ML^{-1}T^2$                       3)  $ML^2T^{-2}$                       4)  $M^{-1}L^3T^{-2}$
3. The energy possessed by a body by virtue of its position (or) configuration is called  
 1) Heat energy                      2) P.E                      3) K.E                      4) Light energy
4. The work done is always  
 1) Greater than energy spent  
 2) Less than energy spent  
 3) Some times greater than & sometimes less than energy  
 4) Equal to energy spent
5. As a body rolls down a inclined plane, it has  
 1) Only kinetic energy  
 2) Only potential energy  
 3) Both kinetic energy and potential energy  
 4) Neither kinetic energy nor potential energy

### JEE ADVANCED

#### Multi Correct Choice Type:

6. Choose the correct statements :
- 1) Unit of energy is same as that of the unit of work.  
 2) Capacity to do work is known as energy.  
 3) The commercial unit of energy is kilowatt-hour.  
 4) When we say that energy of a body is 1 joule, it means, this body has the capacity to do 1 J work.

**Reasoning Type:**

7. *Statement I* :Kinetic energy of the body is due to virtue of its motion  
*Statement II* :Potential energy of the body is due to virtue of its position
- 1) Both Statements are true, Statement II is the correct explanation of Statement I.
  - 2) Both Statements are true, Statement II is not correct explanation of Statement I.
  - 3) Statement I is true, Statement II is false.
  - 4) Statement I is false, Statement II is true.

**Comprehension Type:**

The sum of kinetic energy (K.E) and potential energy (P.E) of a body is known as mechanical energy

8. Gravitational potential energy (U) of the body is given by
- 1)  $mgh$
  - 2)  $\frac{1}{2} \times m \times v^2$
  - 3)  $\frac{1}{2} \times m \times v$
  - 4)  $mgh^2$
9. The kinetic energy of a body depends
- 1) On its mass only
  - 2) On its speed only
  - 3) On its mass as well as on its speed
  - 4) Neither on its mass nor on its speed
10. The mechanical energy is the sum of
- 1) P.E & K.E
  - 2) Thermal energy & light energy
  - 3) sound energy & nuclear energy
  - 4) K.E & heat energy

**Matrix Match Type:**11. **Column - I**

- a) Water stored at a height
- b) Moving vehicle
- c) Forms of energy
- d) Dimensional formula of energy

**Column - II**

- 1) Sound energy
- 2) Kinetic energy
- 3) Potential energy
- 4)  $ML^2T^{-2}$
- 5) Light energy

**Integer Answer Type:**

12. The kinetic energy of a ball of mass 200 g moving at a speed of 20 cm/s is \_\_\_\_\_  $\times 10^4$ erg

**Multi Correct Choice Type:**

13. Choose the correct statements :
- 1) When the body is at rest its kinetic energy is zero
  - 2) When the body is on the ground( $h=0$ ) its potential energy is zero
  - 3) Compressed spring posses potential energy
  - 4) The potential energy of an object due to its height above the earth's surface is zero

14. Choose the correct statements :

- 1) 1 kilowatt-hour is the amount of energy consumed at the rate of 1 kilowatt for 1 hour
- 2) Work done by external forces on the system is equal to the increase in the system's energy
- 3) Work done by external forces on the system is equal to the decrease in the system's energy
- 4) 1 kilowatt-hour is the amount of energy consumed at the rate of 1 kilowatt for 2 hours

### SYNOPSIS - 3

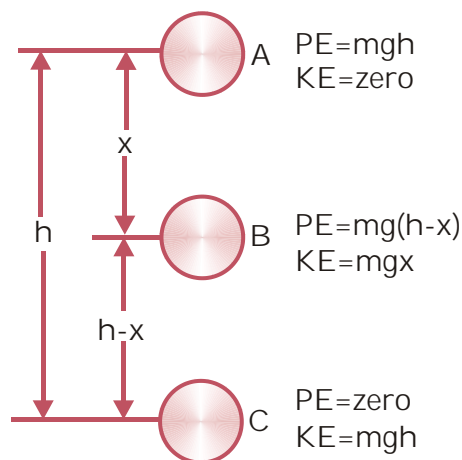
**Law of conservation of energy :** According to this law "Energy can neither be created nor be destroyed, but can be changed from one form to another form".

**Examples:**

- 1) When a body falls from a certain height, its P.E gradually changes into kinetic energy but the total sum of both the energies remains the same.
- 2) When coal is burnt in a steam engine, the chemical energy of coal disappears and an equivalent amount of heat and light energies are obtained.

**The law of conservation of energy of freely falling body :**

The total energy of a freely falling body at any instant is constant.



Consider a body of mass 'm' at a height 'h' above the ground. Suppose this position of the body is A. Suppose the body at 'A' is at rest i.e v = 0.

**At the position 'A' :**

Potential energy at 'A', P.E = mgh

$$\text{kinetic energy at 'A', K.E} = \frac{1}{2}mv^2 = \frac{1}{2}m(0)^2 = 0$$

$$\therefore \text{Total energy of the body at A} = \text{P.E} + \text{K.E} = mgh + 0 = mgh \text{ ----- (1)}$$

Let the body falls freely under the action of gravity to position 'B' through a height

x. Now, the height of the body from the ground =  $h - x$

**At position 'B'**

Potential energy of the body at 'B',  $P.E = mg(h - x) = mgh - mgx$

kinetic energy of the body at 'B',  $K.E. = \frac{1}{2} mv^2$  ----- (a)

where  $v$  is the velocity of the body at position 'B'.

But here  $a = g$  and  $s = x$ ,  $u = 0$  as body at A is at rest

using  $v^2 - u^2 = 2as \Rightarrow v^2 = 2as \Rightarrow v^2 = 2gx$  ----- (b)

from equation (a) and (b) we have  $K.E = \frac{1}{2} m \times 2gx = mgx$

Now, total energy at the point 'B' =  $P.E + K.E = mgh - mgx + mgx = mgh$  ----- (2)

Finally, let the body touches the ground at 'C', so that the distance through which it falls =  $h$

**At position 'C'**

Potential energy at 'C',  $P.E = mg(0) = 0$

Kinetic energy at 'C',  $K.E = \frac{1}{2} mv^2$  ----- (c)

where  $v$  is the velocity of the body just at position 'C'.

Here  $u = 0$  [ $\because$  body is at rest at position A,  $a = g$  and  $s = h$ ]

we know that  $v^2 - u^2 = 2as \Rightarrow v^2 - 0 = 2gh \Rightarrow v^2 = 2gh$  ----- (d)

from equation 'c' and 'd' we have  $K.E = \frac{1}{2} m (2gh) = mgh$

$\therefore$  Total energy at the point 'C' =  $P.E + K.E = 0 + mgh = mgh$  ----- (3)

From (1), (2) and (3) it is clear that the total energy of a body at any instant during free fall of a body remains constant. Hence, the law of conservation of energy is verified.

**Note :** 1) The total energy (K.E + P.E) of the freely falling body is converted into sound and heat energy, when the body strikes the ground.

2) For a freely falling body, potential energy changes into kinetic energy.

3) For a body projected vertically upwards, kinetic energy changes into potential energy.

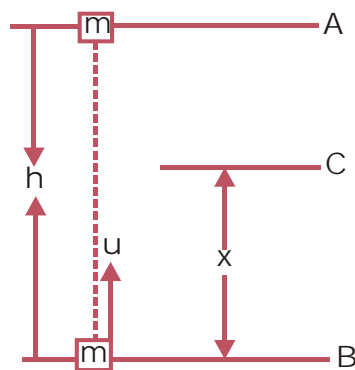
**Conservation of mechanical energy for a vertically projected body**

**Statement :** The total energy in the universe is constant and it can be neither created nor destroyed.

Consider a body of mass "m" it is projected vertically upwards with an initial



velocity "u"



**At Point B :-**

The point is on the ground so  $h = 0$

Potential energy P.E. =  $mgh = mg(0) = 0$ , Initial velocity =  $u$

Kinetic energy  $KE = \frac{1}{2} m v^2 = \frac{1}{2} m u^2$  ∴ Total energy =  $PE + KE = \frac{1}{2} m u^2$

$$E_B = \frac{1}{2} m u^2$$

**At point A :**

This point is at a height "h" from the ground. At this point velocity is zero  $v = 0$ .

$$KE = \frac{1}{2} m v^2 = \frac{1}{2} m (0)^2 = 0$$

$PE = mgh$ , Initial velocity =  $u$ , acceleration  $a = -g$ , distance travelled  $s = h$

From  $v^2 - u^2 = 2as$ .

$$\text{We will get } 0^2 - u^2 = 2(-g)h \Rightarrow -u^2 = -2gh \Rightarrow h = \frac{u^2}{2g}$$

$$\therefore PE = mgh = mg \left( \frac{u^2}{2g} \right) = \frac{1}{2} m u^2$$

$$\therefore \text{Total Energy} = PE + KE = \frac{1}{2} m u^2 + 0$$

$$E_A = \frac{1}{2} m u^2$$

**At point C :**

This point is at a height "x" from the ground and let the velocity of the body at this point be "v".  $h = x$

Potential energy  $PE = mgh = mgx$

$$KE = \frac{1}{2} mv^2$$

initial velocity =  $u$ , acceleration  $a = -g$ , displacement  $s = x$   
 from  $v^2 - u^2 = 2as$ , we will get

$$v^2 - u^2 = 2(-g)x \Rightarrow x = \frac{v^2 - u^2}{-2g} = \frac{u^2 - v^2}{2g}$$

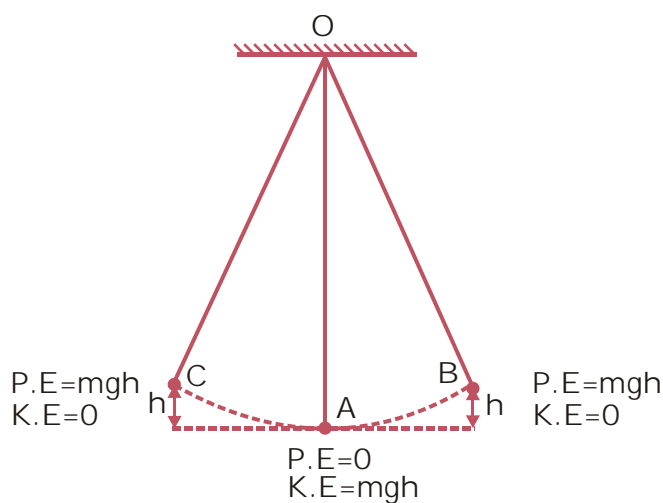
$$\therefore PE = mg \left[ \frac{u^2 - v^2}{2g} \right] = \frac{1}{2} mu^2 - \frac{1}{2} mv^2$$

$$\therefore \text{total energy} = PE + KE = \frac{1}{2} mu^2 - \frac{1}{2} mv^2 + \frac{1}{2} mv^2$$

$$E_c = \frac{1}{2} mu^2; \quad \text{At A, B and C the total energy is constant.}$$

**Law of conservation of Energy:** The sum of total energy in a system is a constant quantity. It can neither be created nor destroyed. However, energy can change from one form to another form.

**Law of conservation of energy in case of a simple pendulum :**



Motion of simple pendulum

Let us consider a simple pendulum suspended from a rigid support 'O'. Its resting position is 'A'. When it is displaced to one side and then released, it swings from one side to the other, reaching equal distance and equal height on either side. Neglecting the friction between the bob and the surrounding air (i.e., considering the pendulum as an isolated system), the motion of the pendulum can be easily explained by applying the law of conservation of energy as follows :

- (1) As the bob of the pendulum swings from A to B, the kinetic energy changes into potential energy and at B (extreme position), its total mechanical energy is the potential energy (no kinetic energy) and so it comes to rest momentarily in this position.
- (2) As it swings back from B to A, the potential energy decreases and the kinetic energy increases. At A (resting position), it has the total mechanical energy in the form of kinetic energy and the potential energy is zero.
- (3) Again, when it swings from A to C, the kinetic energy decreases and the potential energy increases. The potential energy becomes maximum at C (the other extreme position).
- (4) From C to A, the potential energy again changes into kinetic energy. At an intermediate position (between A and B or between A and C), it has both the kinetic energy and potential energy, but the sum of both the energies (i.e., the total mechanical energy) remains constant throughout the swing.

**Proof :**

Let 'm' be the mass of the bob of a simple pendulum.

Let it is displaced from A to B so that its height from the mean position is 'h'.

∴ The P.E of the bob at height 'h' =  $mgh$  ----- (1)

When the bob reaches back the mean position with velocity 'v', then K.E of the bob

at mean position (A) =  $\frac{1}{2} mv^2$  ----- (2)

Now using,  $v^2 - u^2 = 2as$ ,

we get,  $v^2 - 0^2 = 2gh$  [ $\because u = 0$  and  $s = h$ ]

Therefore the above equation can be written as  $v^2 = 2gh$  ----- [3]

using (3) in (2) we have

$$\text{K.E} = \frac{1}{2} m \times 2gh = mgh$$

Thus P.E of the bob of a simple pendulum when raised to height h is equal to the kinetic energy of the bob of pendulum at its mean position. This is the law of conservation of energy.

**Transformation of energy:** Energy changes happen all around us. The change of energy from one form to another form is called the transformation of energy.

**INTER-CONVERSION OF ENERGY:****Inter-conversion of Mechanical Energy:**

1) When an electric drill is used to bore a hole in a wooden block, smoke starts rising. It is because mechanical energy of drill changes into heat energy.

2) When two stones are struck against each other sharply, sparks of light are produced along with sound. Thus, mechanical energy changes into light and sound energies.

3) When the water stored in dams is allowed to flow from a pipe and directed against turbine, the mechanical energy of the flowing water rotates the turbine. The turbine is coupled to electric generator, where the mechanical energy of the turbine changes into electric energy.

#### Inter-conversion of Heat Energy:

1) In steam engines or diesel engines, the heat energy is converted into mechanical energy.

2) In the preparation of certain chemical compounds, the heat energy changes into chemical energy.

#### Inter-conversion of Light Energy:

1) During photosynthesis, the light energy changes into chemical energy. The plants take the carbon dioxide gas and water and these react with the help of chlorophyll and sunlight to form starch and oxygen.

2) When light energy falls on silver salts coated on photographic plate, it changes into chemical energy.

3) When light energy is absorbed by photo-voltaic cells, it changes into electric energy. Phot-voltaic cells are commonly used in electronic calculators; wrist watches photographic cameras etc.

#### Inter-conversion of Sound Energy:

When we speak in front of a microphone, the sound energy changes into electric energy.

#### Inter-conversion of Magnetic Energy:

When an electric generator works, the mechanical energy interacts with magnetic energy and changes into electric energy.

#### Inter-conversion of Electric Energy:

1) In electric motors; fans; grinders; etc. the electric energy changes into mechanical energy

2) In electric kettles and electric room heaters, the electric energy changes into heat energy.

3) In electric bulbs, tube lights, etc, the electric energy first changes into heat energy and then light energy.

4) In electromagnets, the electric energy changes into magnetic energy.

5) During the electrolysis of salt solutions the electric energy changes into chemical energy.



**Inter-conversion of Chemical Energy:**

- 1) During the process of burning, the chemical energy changes into heat energy and light energy. When wood is burnt, the chemical energy changes into heat and light energies.
- 2) In electric dry cells or car batteries, the chemical energy changes into electrical energy.
- 3) In explosive devices, such as fire crackers, bombs, etc. the chemical energy changes into heat, light and sound energies.

A few more examples of changes of energy are given below:

DEVICE	ENERGY CHANGE
Electric fan	Electrical energy into mechanical energy
Car batteries	Chemical energy into electrical energy
Cycle dynamo	Mechanical energy into electrical energy
Microphone	Sound energy into electrical energy
Loudspeaker	Electrical energy into sound energy
Solar cell	Light energy into electrical energy
Electric torch	Chemical energy into electrical energy and then into light energy
Electric motor	Electrical energy to mechanical energy

**WORKSHEET-3**

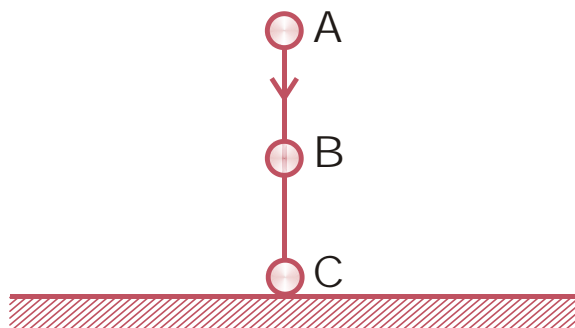
- CUQ** 1. The potential energy of a body at height 'h' is 'x' Joules, when it reaches to the ground its potential energy will become
- 1) x Joules      2) Zero      3) 2 x Joules      4)  $\frac{x}{2}$  Joules
2. If the kinetic energy of a body, when it is moving with a velocity 'v' is P Joules. If it comes to rest then its kinetic energy is
- 1) Zero      2) P Joules      3) P<sup>2</sup> Joules      4) 2P Joules
3. In case of simple pendulum change in kinetic energy is equal to
- 1) Zero      2) Change in potential energy  
3) Potential energy      4) Total energy
4. In the preparation of certain chemical compounds, the \_\_\_\_\_ energy changes into chemical energy

- 1) Light                      2) Heat                      3) Sound                      4) Electrical
5. During the electrolysis of salt solutions , the electric energy changes into \_\_\_\_\_ energy
- 1) Light                      2) Heat                      3) Sound                      4) Chemical

### JEE MAINS

#### Single Correct Choice Type:

1. The potential energy of a freely falling object decreases continuously. What happens to the loss of potential energy ?
- 1) It is continuously converted into sound energy  
 2) It is continuously converted into kinetic energy  
 3) It is continuously converted into magnetic energy                      4) none of these
2. The potential energy of a body at a height  $h$  is  $mgh$ . When it falls to the ground, its K.E becomes
- 1)  $2mgh$                       2)  $mgh/2$                       3)  $mgh$                       4)  $mgh^2$
3. A body is dropped from point A as shown in the figure. When it comes to point B, it has
- 1) only K.E.                      2) only P.E                      3) both K.E and P.E                      4) none of these



4. A ball is thrown upwards from a point 'A'. It reaches up to the highest point 'B' and returns then
- 1) K.E at 'A' = K.E at 'B'                      2) P.E at 'A' = P.E at 'B'  
 3) P.E at 'A' =  $2 \times$  K.E at 'B'                      4) P.E at 'B' = K.E at 'A'
5. A stone of mass ' $m$ ' is thrown vertically upwards with a velocity  $v$ . The K.E at the highest point is

1)  $\frac{1}{2}mv^2$

2) zero

3)  $2\left(\frac{1}{2}mv^2\right)$

4) 2 mgh

**JEE ADVANCED****Multi Correct Choice Type:**

6. When a body falls from a certain height
- 1) Its P.E gradually changes into kinetic energy
  - 2) The total sum of both the energies (K.E & P.E) remains the same
  - 3) Its kinetic energy gradually changes into P.E
  - 4) The total sum of both the energies (K.E & P.E) does not remains same

**Reasoning Type:**

7. *Statement I* : When an electric drill is used to bore a hole in a wooden block, smoke starts rising.  
*Statement II* : Mechanical energy converts into heat energy.
- 1) Both Statements are true, Statement II is the correct explanation of Statement I.
  - 2) Both Statements are true, Statement II is not correct explanation of Statement I.
  - 3) Statement I is true, Statement II is false.
  - 4) Statement I is false, Statement II is true.

**Comprehension Type:**

Energy can neither be created nor be destroyed, but can be changed from one form to another form

8. In steam engines or diesel engines
- 1) Chemical energy changes into electrical energy.
  - 2) Light energy changes into chemical energy
  - 3) Mechanical energy interacts with magnetic energy and changes into electric energy
  - 4) Heat energy is converted into mechanical energy
9. During photosynthesis
- 1) Electric energy changes into chemical energy
  - 2) Heat energy is converted into mechanical energy
  - 3) Light energy changes into chemical energy
  - 4) Chemical energy changes into electric energy.
10. When an electric generator works
- 1) Chemical energy changes into electric energy.
  - 2) Light energy changes into chemical energy

- 3) Mechanical energy interacts with magnetic energy and changes into electric energy
- 4) Heat energy is converted into mechanical energy

**Matrix Match Type:**11. **Column - I**

- a) Electron revolves in a orbit
- b) Bended bow
- c) In explosive devices Inter conversion of chemical energy changes to
- d) Freely suspended mass less spring

**Column - II**

- 1) No energy
- 2) Kinetic energy
- 3) Potential energy
- 4) Heat energy
- 5) Sound energy

**Integer Answer Type:**

12. A body is moving horizontally at a height of 10m has its P.E equal to K.E. Then velocity of that body is ( $g = 9.8 \text{ m/s}^2$ ) \_\_\_\_\_m/s

**Reasoning Type:**

13. *Statement I* : P.E of the bob of a simple pendulum when raised to height  $h$  is equal to the kinetic energy of the bob of pendulum at its mean position

*Statement II* : The sum total of energy in a system is a constant quantity

- 1) Both Statements are true, Statement II is the correct explanation of Statement I.
- 2) Both Statements are true, Statement II is not correct explanation of Statement I.
- 3) Statement I is true, Statement II is false.
- 4) Statement I is false, Statement II is true.

**Multi Correct Choice Type:**

14. Choose the correct statements :

- 1) In case of freely falling body and body projected vertically up mechanical energy remains constant
- 2) When a body is released from some height on falling through certain distance P.E lost by it is equal to K.E gained by it
- 3) When a body is projected vertically up on reaching certain height K.E lost by it is equal to P.E gained by it
- 4) When wood is burnt, the chemical energy changes into heat and light energies.



## WORK, POWER AND ENERGY

### WPE -WORKSHEET-1 -KEY

**CUQ:** 1) 1 2) 2      3) 3      4) 1      5) 1      6) 3      7) 2      8) 4

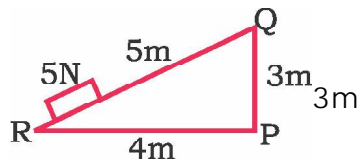
**JEE ADVANCED :**

1) 2      2) 1      3) 3      4) 1      5) 4      6) 2      7) 2      8) 3  
 9) 1      10) 3      11) 3      12) 1,3  
 13) 1,3      14) 2      15) 3      16) 2      17) 4      18) a-1,2; b-5; c-4 ; d-3  
 19) 1      20) 1,2,3,4      21) a-2;b-1;c-3;d-4,5

**HINTS AND SOLUTIONS:**

2. M man = 80kg, M Load = 20kg, Time = 20sec, g = 9.8m/s<sup>2</sup>  
 (80 + 20) × 9.8 × 40 × 0.25 = 9800J

9.



Vertical height QP = 3m;      W = F × S = 5 × 3 = 15N

19. work done = Force × displacement = 5 N × 0.2 m = 1 J

### WPE -WORKSHEET-2 -KEY

**MCQ:** 1) 2      2) 4      3) 2      4) 3      5) 4

**JEE ADVANCED :**

1) 1      2) 3      3) 2      4) 4      5) 3      6) 1,2,3,4  
 7) 2      8) 1      9) 3      10) 1      11) a-3;b-2;c-1,2,3,5;d-4      12) 4  
 13) 1,2,3,4      14) 1,2

**HINTS AND SOLUTIONS:**

12.  $\frac{1}{2} \times 200g \times (20)^2 = \frac{1}{2} \times 200 \times 400 = 100 \times 400 = 40,000 = 4 \times 10^4 \text{ erg}$

### WPE -WORKSHEET-3 -KEY

**CUQ:** 1) 2 2) 1      3) 2      4) 2      5) 4

**JEE ADVANCED :**

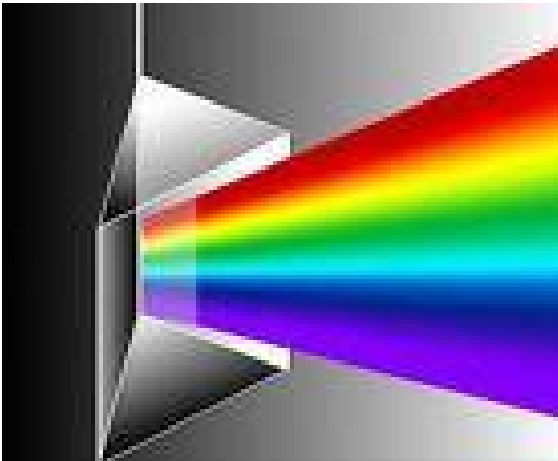
1) 2      2) 3      3) 3      4) 4      5) 2  
 6) 1,2      7) 2      8) 1      9) 3      10) 3      11) a-2,3;b-3;c-4,5;d-1  
 12) 14      13) 1      14) 1,2,3,4

**HINTS AND SOLUTIONS:**

12.  $\frac{1}{2} mV^2 = m \times g \times h$ ;  $V^2 = 2gh \Rightarrow V = \sqrt{2gh} \Rightarrow v = \sqrt{2 \times 9.8 \times 10} = 14$

\*\*\*\*\*

# RAY OPTICS



## CONTENTS

Refraction of light at Plane surface  
Refraction through Prism  
Refraction at Curved Surfaces  
Optical Instruments

RAY OPTICSSYNOPSIS-1(Refraction of light at Plane surface)

**Refraction of Light :** When a beam of light is travelling from one medium to another medium, a part of light gets reflected back into first medium at the interface of two media and the remaining part travels through second medium in another direction. The change in the direction of light take place at the interface of two media.

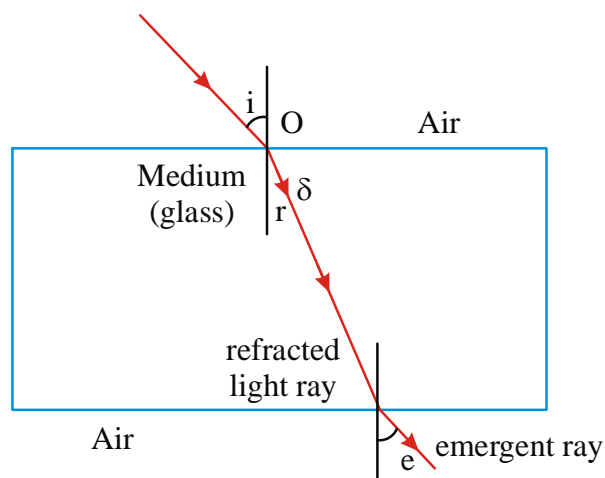
Deviation or bending of light rays from their original path while passing from one medium to another is called refraction. (or)

The phenomenon due to which light deviates from its initial path, while travelling from one optical medium to another optical medium is called refraction.

Refraction of light is due to change in speed of light passes from one medium to another medium.

In case of refraction of light frequency (colour) and phase do not change. But wavelength and velocity will change.

**Note:** When light passes from one medium to another medium, the colour of light is determined by its frequency not by its wavelength.

**Refraction of light at plane surface:**

**Incident ray:** A ray of light, traveling towards another optical medium, is called incident ray.

**Point of incidence:** The point (O), where an incident ray strikes on another optical medium, is called point of incidence.

**Normal:** A perpendicular drawn at the surface of separation of two media on the point of incidence, is called normal.

**Angle of incidence (i):** The angle which the incident ray makes with normal, is called angle of incidence.

**Refracted ray:** A ray of light which deviates from its path on entering in another optical medium is called refracted ray.

**Angle of refraction( $r$ ):** The angle which the refracted ray makes with normal, is called the angle of refraction.

**Angle of deviation due to refraction( $\delta$ ):** It is the angle between the direction of incident light ray and refracted light ray.

**Emergent ray:** A ray of light which emerges out from another optical medium as shown in the above figure is called emergent ray.

**Angle of emergence ( $e$ ):** The angle which the emergent ray makes with the normal is called the angle of emergence.

#### Laws of Refraction:

Incident ray, refracted ray and normal always lie in the same plane.

The product of refractive index and sine of angle of incidence at a point in a medium is constant,  $\mu \times \sin i = \text{constant}$   $\mu_1 \sin i_1 = \mu_2 \sin i_2$

If  $i_1 = i$  and  $i_2 = r$  then  $\mu_1 \sin i = \mu_2 \sin r$ ; This law is called Snell's law.

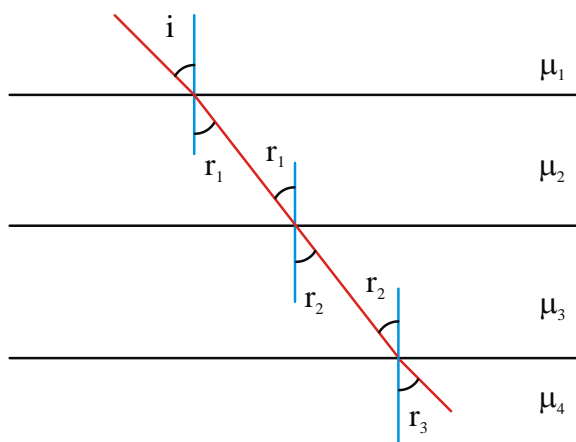
According to Snell's law,

$\frac{\sin i}{\sin r} = \text{constant} \left( = \frac{\mu_2}{\mu_1} \right)$  for any pair of medium and for light of given wavelength.

**Note:** The ratio between sine of angle of incidence to sine of angle of refraction is commonly called as refractive index of the material in which angle of refraction is situated with respect to the medium in which angle of incidence is situated.

When light ray travels from medium 1 to medium 2 then  $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} = \mu_2 = \text{refractive index of medium (2) with respect to medium (1)}$

**Note:** Let us consider a ray of light travelling in situation as shown in fig. Applying Snell's law at each interface, we get





$\mu_1 \sin i = \mu_2 \sin r_1$  ;  $\mu_2 \sin r_1 = \mu_3 \sin r_2$  ;  $\mu_3 \sin r_2 = \mu_4 \sin r_3$  ; It is clear that

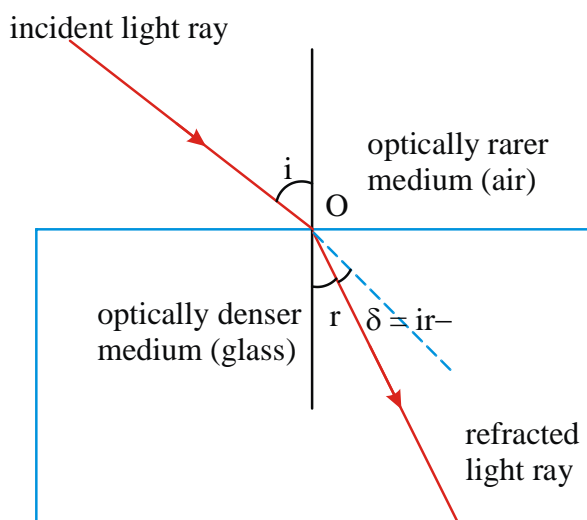
$\mu_1 \sin i = \mu_2 \sin r_1 = \mu_3 \sin r_2 = \mu_4 \sin r_3$  (or)  $\mu \sin i = \text{constant}$

**Note:** When light ray travels from medium of refractive index  $\mu_1$  to another medium of

refractive index  $\mu_2$  then,  $\mu_1 \sin i_1 = \mu_2 \sin i_2$

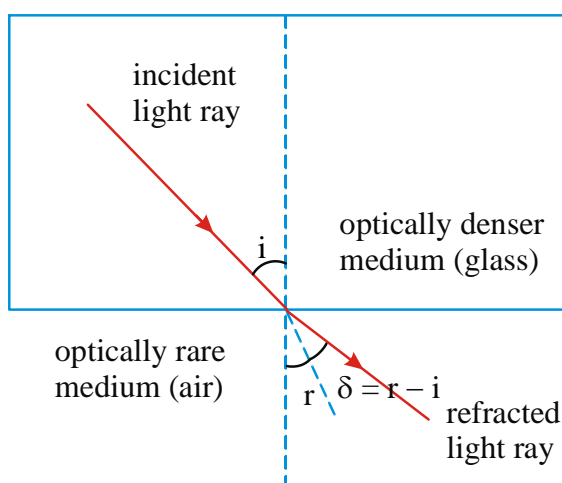
$$\frac{\sin i_1}{V_1} = \frac{\sin i_2}{V_2} = \frac{\sin i_1}{\lambda_1} = \frac{\sin i_2}{\lambda_2}$$

**When a light travels from optically rarer medium to optically denser medium obliquely:**



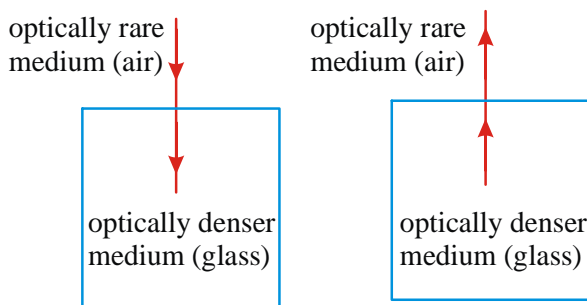
- a) it bends towards normal.
- b) angle of incidence is greater than angle of refraction.

**When a ray of light travels from optically denser medium to optically rarer medium obliquely**



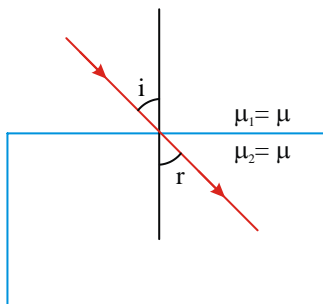
- a) it bends away from the normal at the point of incidence.
- b) angle of refraction is greater than angle of incidence.
- c) angle of deviation  $\delta = r - i$ .

**Condition for no refraction :** When an incident ray strikes normally at the point of incidence, it does not deviate from its path. i.e., it suffers no deviation.



In this case angle of incidence ( $i$ ) and angle of refraction ( $r$ ) are equal and  $\angle i = \angle r = 0$ .

If the refractive indices of two media are equal



$\mu_1 = \mu_2 = \mu$  From Snell's law,  $\mu \sin i = \mu \sin r, \sin i = \sin r \quad \angle i = \angle r$

Hence, the ray passes without any deviation at the boundary.

**Note:** Because of the above reason a transparent solid is invisible in a liquid if their refractive indices are same.

**Refractive Index :**

**Absolute refractive Index ( $\mu$ ):**

The absolute refractive index of a medium is the ratio of speed of light in free space ( $C$ ) to speed of light in a given medium ( $V$ ).

$$\mu = \frac{\text{velocity of light in free space (C)}}{\text{velocity of light in a given medium (V)}}$$

It is a scalar.

It has no units and dimensions.

For vacuum of free space, speed of light of all wavelengths is same and is equal to

C. So, for all wavelengths the refractive index of free space is  $\mu = \frac{C}{C} = 1$ .

For a given medium the speed of light is different for different wavelengths of light, greater will be the the speed and hence lesser will be refractive index.

$$\lambda_R > \lambda_V, \text{ So in medium } \mu_V > \mu_R$$

**Note:**

For a given light, denser the medium lesser will be the speed of light and so greater will be the refractive index.

Example : Glass is denser medium when compared to water, so  $\mu_{\text{glass}} > \mu_{\text{water}}$ .

The refractive index of water  $\mu_w = 4/3$  The refractive index of glass  $\mu_g = 3/2$

For a given light and given medium, the refractive index is also equal to the ratio of wavelength of light in free space to that in the medium.

$$\mu = \frac{C}{V} = \left( \frac{f\lambda_{\text{vacuum}}}{f\lambda_{\text{medium}}} \right) = \frac{\lambda_{\text{vacuum}}}{\lambda_{\text{medium}}}$$

(when light travels from vacuum to a medium, frequency does not change)

**Note:** If  $C$  is velocity of light in free space  $\lambda_0$  is wavelength of given light in free space

then velocity of light in a medium of refractive index ( $\mu$ ) is  $V_{\text{medium}} = \frac{C}{\mu}$ .

wavelength of given light in a medium of refractive index ( $\mu$ ) is  $\lambda_{\text{medium}} = \frac{\lambda_0}{\mu}$

**Relative Refractive Index:** When light passes. from one medium to the other, the refractive index of medium 2 relative to medium 1 is written as  ${}_1\mu_2$  and is given by

$${}_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} \quad \dots(1)$$

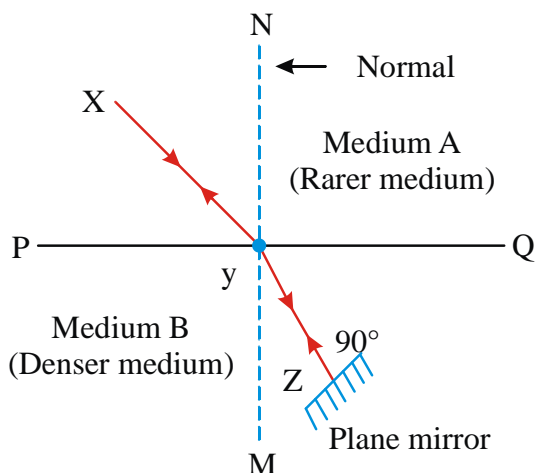
refractive index of medium 1 relative to medium 2 is

$${}_2\mu_1 \text{ and } {}_2\mu_1 = \frac{\mu_1}{\mu_2} = \frac{v_2}{v_1} = \frac{\lambda_2}{\lambda_1} \quad \dots(2)$$

From eq. (1) & (2)  ${}_1\mu_2 = \frac{1}{{}_2\mu_1}$  i.e.,  $({}_1\mu_2) \cdot ({}_2\mu_1) = 1$

**Principle of Reversibility of Light**

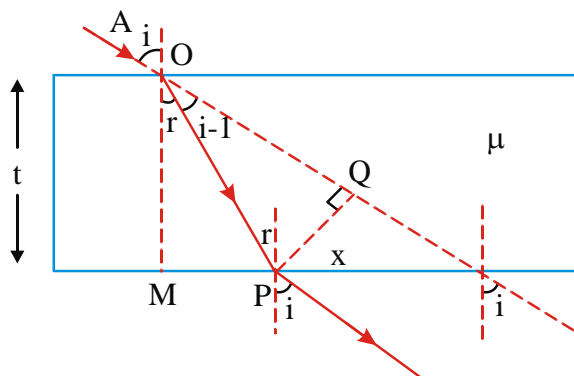
According to principle of reversibility, if a ray of light travels from X to Z along a certain path, it will follow exactly the same path, while travelling from Z to X. In other words the path of light is reversible.



Note :  ${}_a\mu_b = \frac{1}{{}_b\mu_a}$

Thus, the refractive index of medium 'b' with respect to 'a' is equal to the reciprocal of refractive index of medium 'a' with respect to medium 'b'.

**Lateral Shift :** In figure consider a ray AO incident on the slab at an angle of incidence 'i' and passing through a slab of thickness 't'. After two refractions at the boundary, the ray emerge parallel to the incident ray. The perpendicular distance between incident ray direction and emergent ray direction is called lateral shift or lateral displacement (x)



From the figure, the distance  $PQ$  is called lateral displacement (or) lateral shift

From the triangle  $PQO$ ,  $\sin(i-r) = \frac{PQ}{OP}$   $PQ(=x) = OP \sin(i-r)$   $x = OP \sin(i-r) \dots(1)$

But  $\cos r = \frac{OM}{OP}$ ,  $OP = \frac{OM}{\cos r} = \frac{t}{\cos r} \dots(2)$  from (1) and (2)  $\therefore x = t \left[ \frac{\sin(i-r)}{\cos r} \right]$



**Apparent Depth:**

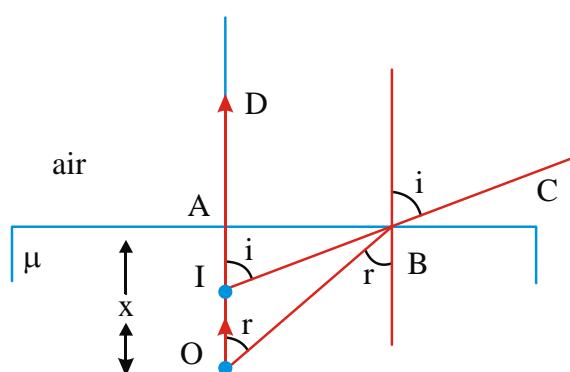
**Case(1) :** Object in denser medium and observer in rarer medium.

When object 'O' is placed at a distance 'x' from A in denser medium of refractive index  $\mu$  as shown in figure. Ray OA, which falls normally on the plane surface, passes undeviated as AD. Ray OB, which is incident (with normal) on the plane surface, bends away from the normal and passes as BC in air. Rays AD and BC meet at 'I' after extending these two rays backwards. This 'I' is the virtual image of real object 'O' to an observer in rarer medium near to transmitted ray.

$$\sin i \approx \tan i = \frac{AB}{AI} \quad \dots\dots(i)$$

$$\sin r \approx \tan r = \frac{AB}{AO} \quad \dots\dots(ii)$$

Dividing eq. (i) and (ii)



$$\frac{\sin i}{\sin r} = \frac{AO}{AI} ; \quad \text{According to Snell's law } \mu = \frac{\sin i}{\sin r}$$

$$\therefore \mu = \frac{AO}{AI} \therefore AI = \frac{AO}{\mu} = \frac{x}{\mu}$$

The distance of image AI is called apparent depth or apparent distance. The

apparent depth  $x_{app}$  is given by i.e.,  $x_{app} = \frac{x_{real}}{\mu}$

The apparent shift  $(OI) = AO - AI = x - \frac{x}{\mu}$

Hence the apparent shift  $(OI) = \left(1 - \frac{1}{\mu}\right)x$

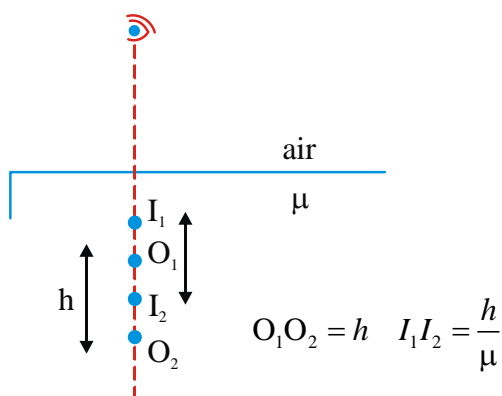
If the observer is in other than air medium of refractive index  $\mu (< \mu)$ .

Then apparent depth  $= \frac{\text{real depth}}{\mu_{relative}} = \frac{\text{real depth}}{\left(\frac{\mu}{\mu^1}\right)}$

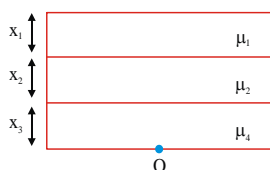
$$\therefore \text{apparent depth} = \frac{\mu^1}{\mu} (\text{real depth})$$

$$\text{apparent shift} = \left(1 - \frac{\mu^1}{\mu}\right)x$$

**Note:** If two objects  $O_1$  and  $O_2$  separated by 'h' on normal line to the boundary in a medium of refractive index  $\mu$ . These objects are observed from air near to normal line of boundary. The distance between the images  $I_1$  and  $I_2$  of  $O_1$  and  $O_2$  is  $\frac{h}{\mu}$ .



**Note:** Apparent depth of object due to composite slab is  $x_a = \frac{x_1}{\mu_1} + \frac{x_2}{\mu_2} + \frac{x_3}{\mu_3}$



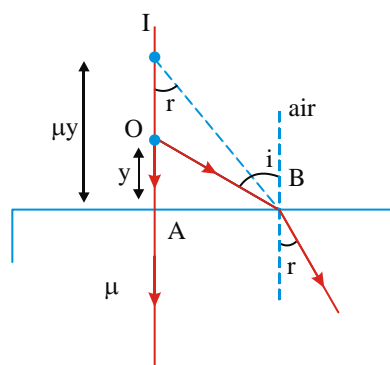
**Note:** If there are 'n' number of parallel slabs which are may be in contact or may not with different refractive indices are placed between the observer and the object, then the total apparent shift

$$s = \left(1 - \frac{1}{\mu_1}\right)x_1 + \left(1 - \frac{1}{\mu_2}\right)x_2 + \dots + \left(1 - \frac{1}{\mu_n}\right)x_n$$

Where  $x_1, x_2, \dots, x_n$  are the thickness of the slabs and  $\mu_1, \mu_2, \dots, \mu_n$  are the corresponding refractive indices.

**Object in rarer medium and observer in denser medium :** When the object in rarer medium (air) at a distance 'y' from boundary and an observer near to normal in denser medium of refractive index ' $\mu$ '. By ray diagram in figure it is observed that the image is virtual, on same side to boundary and its distance from the boundary is  $\mu$  times the object distance.

Since  $\mu > 1$  image distance is more than object distance.



$$\sin i \approx \tan i = \frac{AB}{AO}, \sin r \approx \tan r = \frac{AB}{AI} \quad \text{According to Snell's law } 1. \sin i = \mu \sin r$$

$$\frac{AB}{AO} = \mu \frac{AB}{AI}, AI = \mu \cdot AO$$

Therefore apparent height of object (AI) =  $\mu$  x real height of object (AO)

i.e.  $y_{app} = \mu \cdot y_{real}$     Apparent shift =  $AI - AO$     Apparent shift =  $(\mu - 1)y$ .

If the object is in other than air medium of refractive index  $\mu^1 (< \mu)$ . Then apparent

height =  $\mu_{rel}$  (real height) ;    i.e.,  $y_a = \left(\frac{\mu}{\mu^1}\right)y$     Apparent shift =  $\left(\frac{\mu}{\mu^1} - 1\right)y$

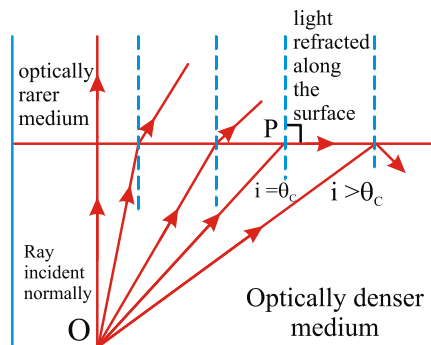
### Critical Angle and Total Internal Reflection :

Consider a point object O placed in a optically denser medium as shown in the figure. Rays of light travel from O in all possible directions.

When light is refracted at the surface into the rarer medium, it bends away from the normal.

Therefore, as the angle of incidence increases, the angle of refraction also increases till for a certain angle of incidence, the angle of refraction is  $90^\circ$  and light is refracted along the surface separating the two media. The corresponding angle of incidence is called the critical angle ( $\theta_c$ ).

When light is incidence at any point beyond P, that is when the angle of incidence is greater than the critical angle ( $i > \theta_c$ ), then no light is refracted, and the entire incident light is reflected into the same medium. This phenomenon is known as the total internal reflection.



**Expression for critical angle ( $\theta_c$ ):**

According to Snell's law, at critical angle of incidence

$$\mu_D \cdot \sin \theta_c = \mu_R \cdot \sin 90^\circ, \quad \sin \theta_c = \frac{\mu_R}{\mu_D} \quad \sin \theta_c = \frac{\mu_R}{\mu_D} = \frac{V_D}{V_R} = \frac{\lambda_D}{\lambda_R}$$

$$\text{For } \mu_R = 1, \mu_D = \frac{1}{\sin \theta_c}$$

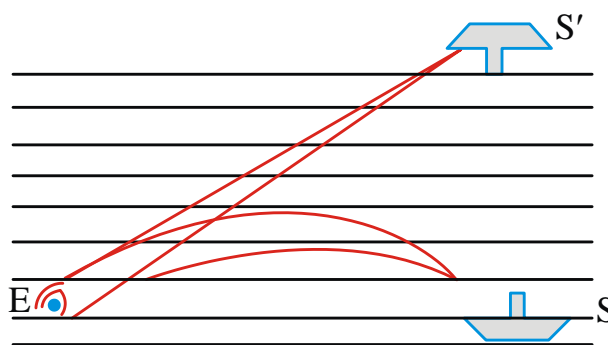
**Condition for total internal reflection :**

For total internal reflection to take place light must be propagating from denser to rarer medium. Ex: Ray from water to air, glass to water.

Total internal reflection will take place only if angle of incidence is greater than critical angle. i.e.

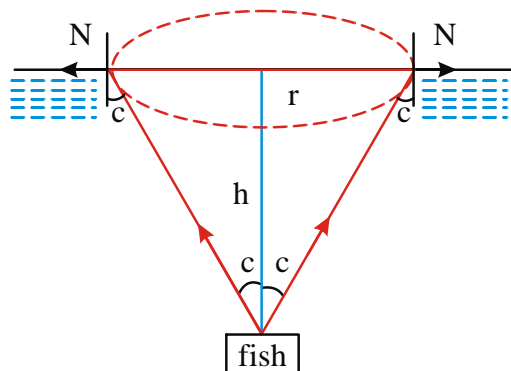
$$i > \theta_c \text{ with } \theta_c = \sin^{-1} \left( \frac{\mu_R}{\mu_D} \right)$$

- **Looming :** This effect occurs when the density of air decreases much more rapidly with increasing height than it does under normal conditions. This situation sometimes happens in cold regions particularly in the vicinity of the cold surface of sea or of a lake. Light rays starting from an object S (say a ship) are curved downward and on entering the eye the rays appear to come from S', thus giving an impression that the ship is floating in air.





**Field of vision of fish :** - A fish at a depth 'h' from the surface of water of refractive index  $\mu$  can see the outer world through an inverted cone with



Vertex angle =  $2C$

Radius of the circular base of the cone formed on surface of water is given by

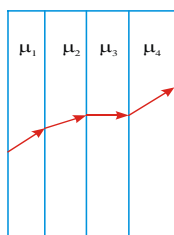
$$\left[ r = h \tan C = \frac{h}{\sqrt{\mu^2 - 1}} \right] \quad \left[ \because \sin C = \frac{1}{\mu}, \tan C = \frac{1}{\sqrt{\mu^2 - 1}} \right]$$

**RAY OPTICS WORKSHEET-1**

**CUQ**

- Light of frequency  $n$ , wave length  $\lambda$  travelling with a velocity  $v$  enters into a glass slab of R.I  $\mu$  then frequency, wave length and velocity of the wave in glass slab respectively are
  - $\frac{n}{\mu}, \lambda, \frac{v}{\mu}$
  - $n, \frac{\lambda}{\mu}, \frac{v}{\mu}$
  - $n, \lambda, \frac{v}{\mu}$
  - $\frac{n}{\mu}, \frac{\lambda}{\mu}, v$
- Absolute refractive index of a material depends upon
  - nature of material
  - nature, wavelength and size of material
  - density, temperature, wavelength of material
  - nature, temperature, wavelength of material
- If a ray of light takes  $t_1$  and  $t_2$  times in two media of absolute refractive indices  $\mu_1$  and  $\mu_2$  respectively to travel same distance, then
  - $\mu_1 t_1 = \mu_2 t_2$
  - $\mu_1 t_2 = \mu_2 t_1$
  - $t_1 \sqrt{\mu_1} = t_2 \sqrt{\mu_2}$
  - $t_1 \sqrt{\mu_2} = t_2 \sqrt{\mu_1}$
- In cold countries, the phenomenon of looming takes place, because refractive index of air
  - decreases with height
  - increases with height
  - does not change with height
  - become infinity at the surface

5. A ray of light passes through four transparent media with refractive indices  $\mu_1, \mu_2, \mu_3$  and  $\mu_4$  as shown in figure.



The surfaces of all media are parallel. If the emergent ray is parallel to the incident ray, we must have

- 1)  $\mu_1 = \mu_2$                       2)  $\mu_2 = \mu_3$                       3)  $\mu_3 = \mu_4$                       4)  $\mu_4 = \mu_1$
6. A hunter desires to shoot a fish whose image could be seen through clear water. His aim should be
- 1) Above the apparent image of fish      2) Below the apparent image of fish  
3) In the line of sight of fish              4) Parallel to the surface of water
7. A rectangular solid piece is placed in a liquid whose refractive index is the same as that of the solid
- 1) The sides of the solid will appear to be bent inward  
2) The sides of the solid will appear to be bent outward  
3) The solid will not be seen at all      4) The solid will appear as in air
8. A plane glass plate is placed over various coloured letters. The letter which appears to be raised least
- 1) violet                      2) yellow                      3) red                      4) green
9. As temperature of medium increases the critical angle
- 1) Increases      2) Decreases                      3) Remains same  
4) first increases then decreases
10. A ball coated with 'lamp black' put in a glass tank containing water appears silvery white due to
- 1) Refraction      2) Diffraction      3) Interference      4) Total internal reflection

**JEE MAIN & ADVANCED**

**LEVEL-1 Single Correct Choice Type:**

1. If the refractive index of diamond is 2.4 find the velocity of light in diamond. ( $c = 3 \times 10^8 \text{ m/s}$ )
- 1)  $1.25 \times 10^8 \text{ m/s}$     2)  $2.25 \times 10^8 \text{ m/s}$       3)  $1.5 \times 10^8 \text{ m/s}$     4)  $4.5 \times 10^8 \text{ m/s}$
2. If  ${}_i\mu_j$  represents refractive index when a light ray goes from medium i into j, then  ${}_2\mu_1 \times {}_3\mu_2 \times {}_4\mu_3$  is equal to
- 1)  ${}_3\mu_1$                       2)  ${}_3\mu_2$                       3)  $\frac{1}{{}_1\mu_4}$                       4)  ${}_4\mu_2$

**Multi Correct Choice Type:**

3. The refractive index of glass with respect to water is  $\frac{9}{8}$ . If the velocity and wavelength of light in water are  $2.25 \times 10^8 \text{ ms}^{-1}$  and  $5400 \text{ \AA}$ , then the velocity and wavelength of light in glass are
- 1) Velocity of light in glass is  $2 \times 10^8 \text{ m/s}$  2) wavelength of light in glass is  $4800 \text{ \AA}$   
 3) Velocity of light in glass is  $1 \times 10^8 \text{ m/s}$  4) wavelength of light in glass is  $6700 \text{ \AA}$
4. The angle of incidence on the surface of a diamond of refractive index 2.4, if the angle between the reflected and refracted rays is  $90^\circ$  is
- 1)  $\tan^{-1}(2.4)$  2)  $\sin^{-1}\left(\frac{1}{2.4}\right)$  3)  $\tan^{-1}\left(\frac{1}{2.4}\right)$  4)  $\cos^{-1}\left(\frac{1}{2.4}\right)$
5. A ray of light passes normally through a slab ( $\mu = 1.5$ ) of thickness 't'. If the speed of light in vacuum is 'c', then time taken by the ray to go across the slab is
- 1)  $\frac{t}{c}$  2)  $\frac{3t}{2c}$  3)  $\frac{2t}{3c}$  4)  $\frac{4t}{9c}$
6. A ray of light incident on a transparent block at an angle of incidence  $60^\circ$ . If  $\mu$  of block is  $\sqrt{3}$  then the angle of deviation of the refracted ray is
- 1)  $15^\circ$  2)  $25^\circ$  3)  $30^\circ$  4)  $45^\circ$

**Integer Answer Type:**

7. A ray of light is incident at angle of  $60^\circ$  on a  $\sqrt{3}$  cm thick plate ( $\mu = \sqrt{3}$ ). The shift in the path of the ray as it emerges out from the plate is (in cm) \_\_\_\_.
- 1) 1 2) 1.2 3) 0.5 4) 1.8
8. A bird in air is at a height 'y' from the surface of water. A fish is at a depth 'x' below the surface of water. The apparent distance of fish from the bird is (The refractive index of water is  $\mu$ )
- 1)  $x + \frac{y}{\mu}$  2)  $\mu x + y$  3)  $\frac{x}{\mu} + y$  4)  $\frac{x}{\mu} - y$
9. A fish looking up through the water see the outside world contained in a circular horizon. If the refractive index of water is  $\frac{4}{3}$  and fish is 12 cm below the surface, the radius of this circle is (in cm)
- 1)  $\frac{36}{\sqrt{5}}$  2)  $4\sqrt{5}$  3)  $\frac{36}{\sqrt{7}}$  4)  $36\sqrt{7}$

10. When a light ray is refracted from one medium into another, the wavelength changes from  $4500\text{\AA}$  to  $3000\text{\AA}$ . The critical angle for a ray from second medium to first medium is

1)  $\sin^{-1}\left(\frac{2}{13}\right)$       2)  $\cos^{-1}\left(\frac{2}{3}\right)$       3)  $\tan^{-1}\left(\frac{3}{2}\right)$       4)  $\tan^{-1}\left(\frac{2}{\sqrt{5}}\right)$

**LEVEL-2 & 3 Single Correct Choice Type:**

11. A ray of light is incident upon a parallel sided transparent slab of thickness 9 cm at an angle of incidence  $60^\circ$ . If the angle of refraction is  $30^\circ$ , the lateral displacement of the light ray is

1)  $\sqrt{3}$  cm      2)  $3\sqrt{3}$  cm      3) 3 cm      4)  $\frac{2}{\sqrt{3}}$  cm

12. A beaker contains water up to a height  $h_1$  and kerosene of height  $h_2$  above water so that the total height of ( water + kerosene ) is  $(h_1 + h_2)$ . Refractive index of water is  $\mu_1$  and that of kerosene is  $\mu_2$ . The apparent shift in the position of the bottom of the beaker when viewed from above is :

1)  $\left(1 - \frac{1}{\mu_1}\right)h_1 + \left(1 - \frac{1}{\mu_2}\right)h_2$       2)  $\left(1 + \frac{1}{\mu_1}\right)h_2 - \left(1 + \frac{1}{\mu_2}\right)h_1$   
 3)  $\left(1 - \frac{1}{\mu_1}\right)h_2 + \left(1 - \frac{1}{\mu_2}\right)h_1$       4)  $\left(1 + \frac{1}{\mu_1}\right)h_1 - \left(1 + \frac{1}{\mu_2}\right)h_2$

13. A vessel of depth 'd' filled with a liquid of refractive index  $\mu_1$  up to half its depth and the remaining space is filled with a liquid of refractive index  $\mu_2$ . The apparent depth while seeing normal to the free surface of the liquid is

1)  $d\left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right)$       2)  $d(\mu_1 + \mu_2)$       3)  $\frac{d}{2}\left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right)$       4)  $\frac{d}{2}(\mu_1 + \mu_2)$

**Comprehension Type:**

$$\mu_D = \frac{1}{\sin \theta_c}$$

14. Light ray is travelling from a denser medium into a rarer medium. The velocity of light in the denser and rarer medium is  $2 \times 10^8 \text{ m/sec}$  and  $2.5 \times 10^8 \text{ m/sec}$ . The critical angle of the two media is

1)  $\sin^{-1}\left(\frac{5}{4}\right)$       2)  $\sin^{-1}\left(\frac{4}{5}\right)$       3)  $\sin^{-1}\left(\frac{1}{2}\right)$       4)  $\sin^{-1}\left(\frac{3}{5}\right)$



15. The critical angle of light going from medium A into medium B is  $\theta$ . The speed of light in medium A is  $v$ . The speed of light in medium B is

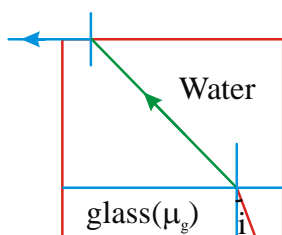
- 1)  $\frac{v}{\sin \theta}$                       2)  $v \sin \theta$                       3)  $\frac{v}{\tan \theta}$                       4)  $v \tan \theta$

16. Light takes  $t_1$  sec to travel a distance  $x$  cm in vacuum and takes  $t_2$  sec to travel  $10x$  cm in a medium. The critical angle corresponding to the media is

- 1)  $\sin^{-1}\left(\frac{10t_1}{t_2}\right)$                       2)  $\sin^{-1}\left(\frac{t_2}{10t_1}\right)$                       3)  $\sin^{-1}\left(\frac{10t_2}{t_1}\right)$                       4)  $\sin^{-1}\left(\frac{t_1}{10t_2}\right)$

**LEVEL-4 & 5 Single Correct Choice Type:**

17. A ray of light travels in the way as shown in the figure. After passing through water, the ray grazes along the water air interface. The value of  $\mu_g$  in terms of 'i' is ( $\mu_w = 4/3$ )



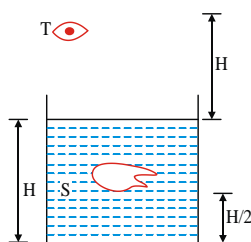
- 1)  $\frac{1}{\sin i}$                       2)  $\frac{3}{4 \sin i}$                       3)  $\frac{4}{3 \sin i}$                       4)  $\sin i$

18. A layer of oil 3 cm thick is floating on a layer of coloured water 5 cm thick. The refractive index of the coloured water is  $5/3$  and the apparent depth of the two liquids is  $36/7$ cm. Then the refractive index of the oil is

- 1)  $\frac{7}{4}$                       2)  $\frac{5}{4}$                       3)  $\frac{7}{5}$                       4)  $\frac{5}{3}$

**Comprehension Type:**

Consider the situation in the figure. The bottom of the pot is a reflecting plane mirror, S is a small fish, and T is a human eye. Refractive index of water is  $\mu$



19. At what distance from itself will the fish see the image of the eye by direct observation?

1)  $H\left(\frac{1}{2} + \mu\right)$     2)  $H\left(\frac{1}{2} - \mu\right)$     3)  $\frac{H}{2}\left(\frac{1}{2} + \mu\right)$     4)  $H\left(\frac{1-\mu}{2}\right)$

20. At what distance from itself will the fish see or observe the image of eye by observing through mirror is

1)  $H\left(\frac{3}{2} + \mu\right)$     2)  $H\left(\frac{1}{2} + \mu\right)$     3)  $H\left(\frac{3+\mu}{2}\right)$     4)  $2H\left(\frac{3}{2} + \mu\right)$

21. At what distance from it self will the eye see the image of the fish upon direct observation?

1)  $H\left(1 + \frac{1}{2\mu}\right)$     2)  $2H\left(1 + \frac{1}{2\mu}\right)$     3)  $2H\left(1 - \frac{1}{2\mu}\right)$     4)  $H\left(\frac{\mu+1}{2}\right)$

22. At what distance from itself will the eye see the image of the fish by observing from the mirror?

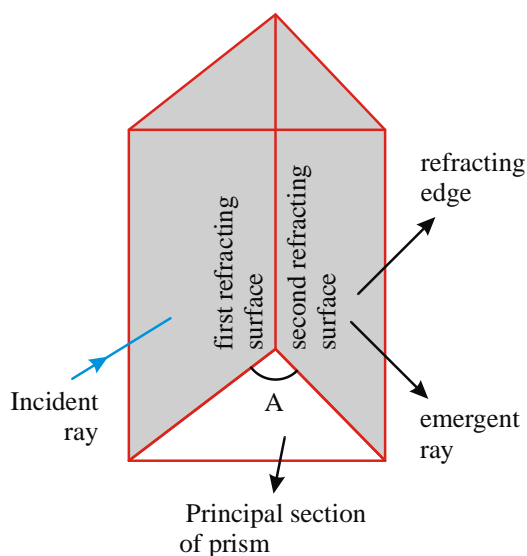
1)  $2\left(H + \frac{3}{2\mu}\right)$     2)  $\left(H + \frac{3}{2\mu}\right)$     3)  $H\left(1 + \frac{3}{2\mu}\right)$     4)  $H\left(1 - \frac{3}{2\mu}\right)$

## RAY OPTICS

### SYNOPSIS-2

#### Refraction through Prism

Prism is a transparent medium bounded by any number of surfaces in such a way that the surface on which light is incident and the surface from which the light emerges are plane and non-parallel as shown in figure.

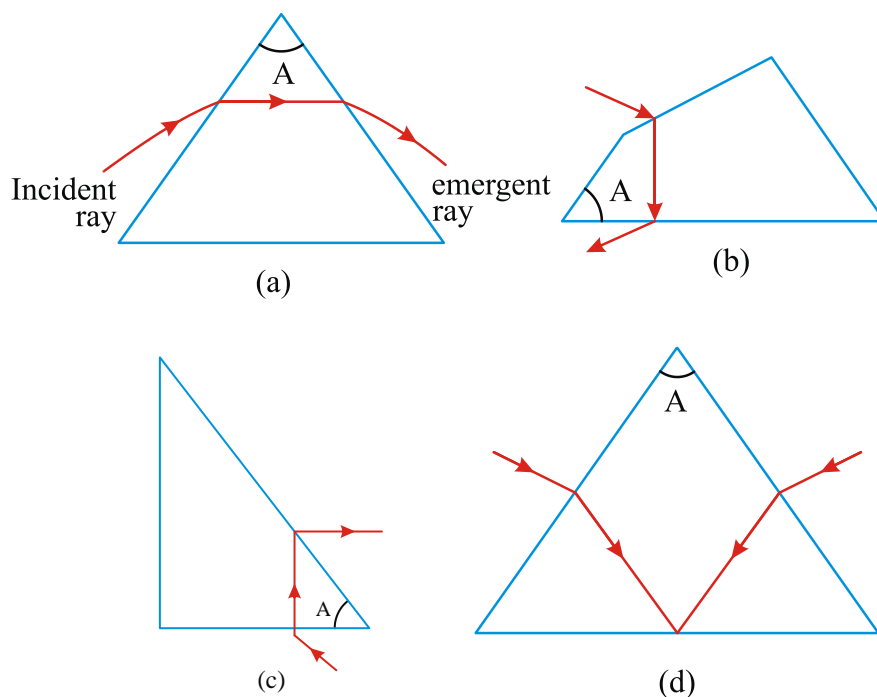


The plane surface on which light is incident and emerges are called refracting faces.

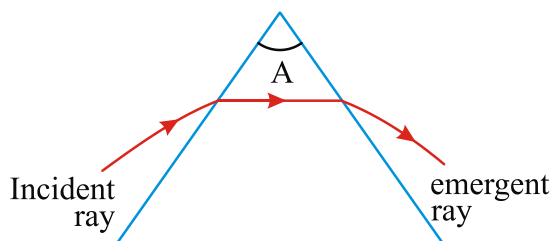
The angle between the faces on which light is incident and from which it emerges is called refracting angle or apex angle or angle of prism (A).

The two refracting surfaces meet each other in a line called refracting edge.

A section of the prism by a plane perpendicular to the refracting edge is called principal section



Angle of deviation ( $\delta$ ) means the angle between emergent and incident rays. While measuring the deviation value in anticlock wise direction is taken as positive and clock wise direction is negative.

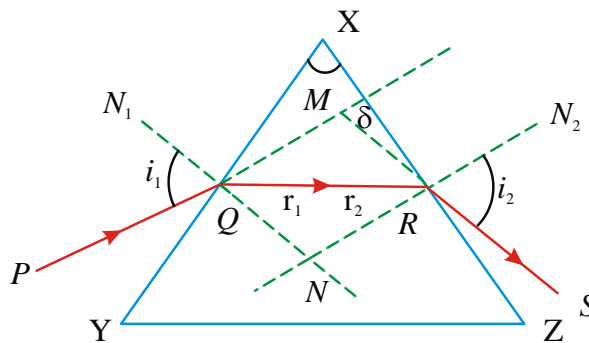


$\delta$  = deviation angle

**Note:** If refractive index of the material of the prism is equal to that of surroundings, no refraction at its surfaces will take place and light will pass through it undeviated i.e. [ $\delta = 0$ ].

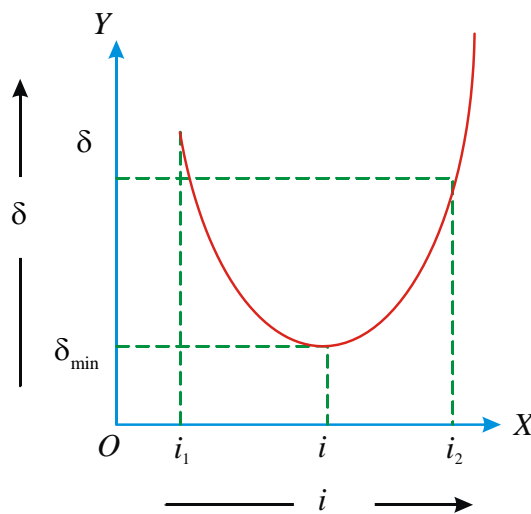
vii) Generally we use equilateral or right angled or Isosceles prism.

**Determination of Refractive index of material of the prism for minimum deviation**



**Minimum Deviation**

From the equation  $\delta = (i_1 + i_2) - A$ , the angle of deviation  $\delta$  depends upon angle of incidence ( $i_1$ ). If we determine experimentally, the angle of deviation corresponding to different angles of incidence and then plot a graph by taking angle of incidence ( $i$ ) on x-axis, angle of deviation ( $\delta$ ) on y-axis, we get the curve as shown in figure.



by snell's law  $\mu = \frac{\sin i}{\sin r} = \frac{\sin i_1}{\sin r_1} = \frac{\sin i_2}{\sin r_2}$

$\mu = \frac{\sin\left(\frac{A + \delta_{\min}}{2}\right)}{\sin\frac{A}{2}}$	$\frac{\mu_p}{\mu_m} = \frac{\sin\left(\frac{A + \delta_{\min}}{2}\right)}{\sin\frac{A}{2}}$
--	--

**Note:** Deviation produced by small angled prism for small angle, from equation above

$$\mu = \frac{i_1}{r_1} = \frac{i_2}{r_2}; i_1 = \mu r_1, i_2 = \mu r_2 \quad \text{But } \delta = (i_1 + i_2) - A$$

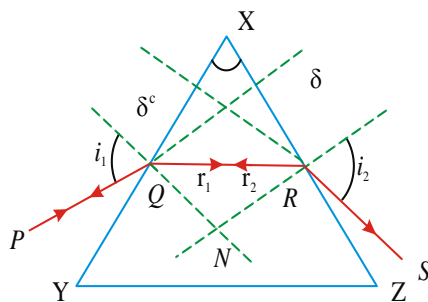


$$\delta = \mu r_1 + \mu r_2 - A; \delta = \mu(r_1 + r_2) - A \text{ But } r_1 + r_2 = A$$

For a prism immersed in a medium of refractive index  $\mu_m$

$$\delta = (\mu - 1)A \Rightarrow \delta = \left( \frac{\mu_p}{\mu_m} - 1 \right) A$$

**Note:** There are two values of angle of incidence for same angle of deviation:



When a light ray is incident at an angle  $i_1$  at the surface (xy), it emerges at an angle  $i_2$  from the surface (zx) with a deviation angle  $\delta$ . As the path of light is reversible, therefore if angle of incidence is  $i_2$ , at the face (xy), then the angle of emergence will be  $i_1$ , with the same angle of deviation ( $\delta$ )

**Note:**

- i) For a given material of prism, wave length of light and angle of incidence. When the angle of prism increases angle of deviation also increases as  $\delta \propto A$ .
- ii) With increase in wavelength, deviation decreases i.e. deviation for red is least

while maximum for violet as  $\delta \propto (\mu - 1) \left\{ \mu \propto \frac{1}{\lambda} \right\}$

- iii) When a given prism is immersed in liquid, the angle of deviation changes as  $\delta \propto (\mu_r - 1)$

**Maximum deviation:**

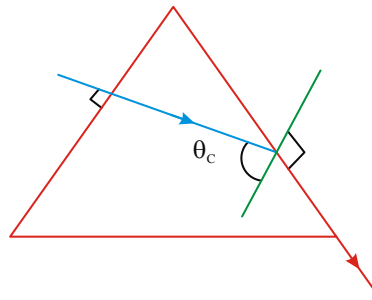
Deviation of ray will be maximum when the angle of incidence is maximum i.e  $i = 90^\circ$ .

Therefore the maximum deviation  $\delta_{\max} = 90 + i_2 - A$

**Condition of grazing emergence:** If a ray can emerge out of a prism, the value of angle of incidence  $i_1$  for which angle of emergence  $i_2 = 90^\circ$  is called condition of grazing emergence.

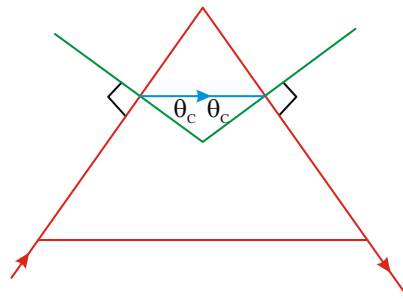
**Condition of no emergence:** A ray of light will not emerge out of a prism (what ever be the angle of incidence) if  $A > 2\theta_c$ , i.e if  $\mu > \operatorname{cosec} \left( \frac{A}{2} \right)$

**Note:** Normal incidence- grazing emergence: If the incident ray falls normally on the prism and grazes from the second surface, then



a)  $i_1 = r_1 = 0, i_2 = 90^\circ$  and  $r_2 = \theta_c = A$     b)  $A = \theta_c = \sin^{-1}\left(\frac{1}{\mu}\right)$     c) Deviation  $d = 90 - \theta_c$

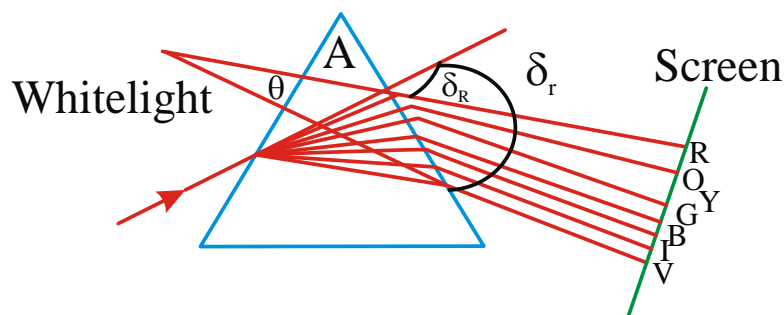
**Note:** Grazing incidence - grazing emergence: If the incident ray falls on the prism with grazing incidence and grazes from the second surface, then



(i)  $i_1 = i_2 = 90^\circ$     (ii)  $r_1 = r_2 = \theta_c$     (iii) Angle of prism  $A = 2\theta_c$   
 (iv) Deviation  $d = 180 - 2\theta_c = 180 - A$

**Dispersion by a Prism :**

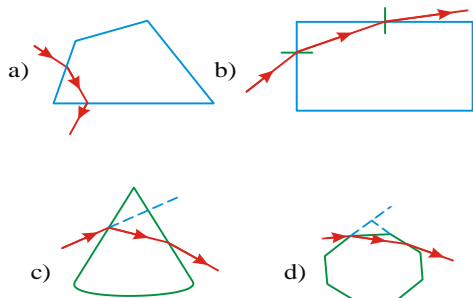
When white light passes through a prism it splits up into different component colours. This phenomenon is called dispersion and arises due to the fact that refractive index of prism is different for different wave lengths. So different wave lengths in passing through a prism are deviated through different angles and as  $\delta \propto (\mu - 1)$ , violet is deviated most while red is least deviated giving rise to display of colours known as spectrum. The spectrum consists of visible and invisible regions.



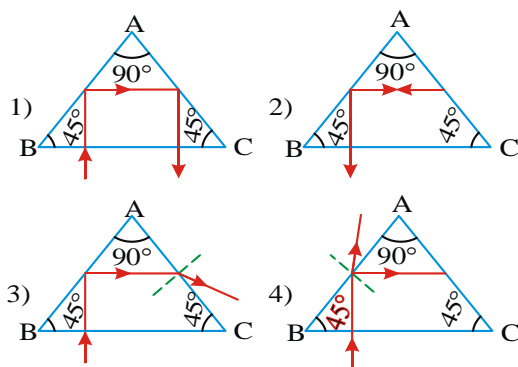
RAY OPTICS WORKSHEET-2

**CUQ**

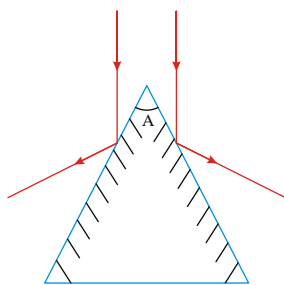
1. Recognize the prism (s) among the given figures.



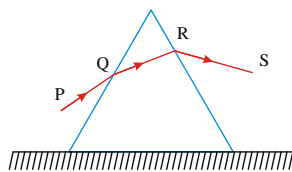
- 1) b and c      2) c, a and b      3) only b      4) a, b, c, and d
2. The refractive index of a material of a prism of angles  $45^\circ - 45^\circ - 90^\circ$  is 1.5. The path of the ray of light incident normally on the hypotenuse side is shown in



3. In the given figure, the angle between reflected ray is equal to :



- 1) A      2) 2 A      3) 3 A      4) 4 A
4. An equilateral prism is placed on a horizontal surface. A ray PQ is incident onto it. For minimum deviation



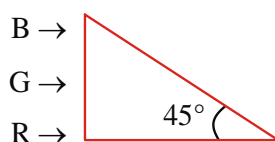
- 1) PQ is horizontal      2) QR is horizontal  
3) RS is horizontal      4) Any one will be horizontal

5. When a ray of light is refracted by a prism such that the angle of deviation is minimum, then
  - 1) the angle of emergence is equal to the angle of incidence
  - 2) the angle of emergence is greater than the angle of incidence
  - 3) the angle of emergence is smaller than the angle of incidence
  - 4) the sum of the angle of incidence and the angle of emergence is equal to  $90^\circ$
6. If a small angled prism, made of glass is immersed in a liquid of refractive index 1 and a ray of light is made incident on it, then
  - 1) its deviation will be zero
  - 2) it will suffer total reflection
  - 3) the emergent ray is bent towards the edge of the prism
  - 4) the emergent ray is bent towards the base of prism
7. Three prisms 1, 2 and 3 have  $A = 6^\circ$ , but refractive indices are 1.4, 1.5, 1.6 and their angles of deviation are  $\delta_1, \delta_2, \delta_3$  respectively. Then
  - 1)  $\delta_3 > \delta_2 > \delta_1$
  - 2)  $\delta_1 > \delta_2 > \delta_3$
  - 3)  $\delta_2 > \delta_1 > \delta_3$
  - 4)  $\delta_1 = \delta_2 = \delta_3$
8. When white light enters a prism, its gets splits into its constituent colours. This is due to;
  - 1) high density of prism material
  - 2) because  $\mu$  is different for different wavelength
  - 3) diffraction of light
  - 4) interference of light

### JEE MAIN & ADVANCED

#### LEVEL-1 Single Correct Choice Type:

1. The fig shows a mixture of blue, green, red colours incident on a right angled prism. The critical angles of the material of prism for red, green and blue colours are  $46^\circ, 44^\circ, 43^\circ$  respectively, The arrangement will separate



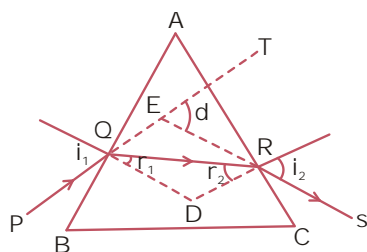
- 1) Red from Green and Blue
  - 2) Blue from Green and Red
  - 3) Green from Red and Blue
  - 4) All the colours
2. A prism has a refracting angle of  $60^\circ$ . When placed in the position of minimum deviation, it produces a deviation of  $30^\circ$ . The angle of incidence is
    - 1)  $30^\circ$
    - 2)  $45^\circ$
    - 3)  $15^\circ$
    - 4)  $60^\circ$



3. Light falls at normal incidence on one face of a glass prism of refractive index  $\sqrt{2}$ . Then the angle of emergence when the angle of the prism is  $45^\circ$ 
  - 1)  $45^\circ$
  - 2)  $60^\circ$
  - 3)  $75^\circ$
  - 4)  $90^\circ$
4. If a light ray incident normally on one of the faces of the prism of refractive index 2 and emergent ray just grazes the second face of the prism, then the angle of deviation is
  - 1)  $0^\circ$
  - 2)  $30^\circ$
  - 3)  $60^\circ$
  - 4)  $90^\circ$
5. A ray of light passes through an equilateral prism such that the angle of incidence is equal to angle of emergence and the latter is equal to  $\frac{3}{4}$ th of the angle of the prism. The angle of deviation is
  - 1)  $45^\circ$
  - 2)  $39^\circ$
  - 3)  $20^\circ$
  - 4)  $30^\circ$
6. A ray of light incident on an equilateral prism shows minimum deviation of  $30^\circ$ . The speed of light through the prism is
  - 1)  $2.121 \times 10^8 \text{ ms}^{-1}$
  - 2)  $1.50 \times 10^8 \text{ ms}^{-1}$
  - 3)  $1.25 \times 10^8 \text{ ms}^{-1}$
  - 4)  $1.75 \times 10^8 \text{ ms}^{-1}$
7. A thin prism of  $4^\circ$  angle gives a deviation of  $2.4^\circ$ . The value of refractive index of the material of the prism is
  - 1) 1.6
  - 2) 1.7
  - 3) 1.8
  - 4) 1.9

**Matrix Match Type:**

8.



In Case of the prism

Column-I

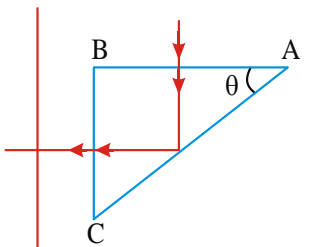
- a) for normal incidence
- b) for normal emergence
- c) for grazing incidence
- d) for grazing emergence

Column-II

- 1)  $i_1 = 90^\circ, r_1 = c, A = r_2 + c$
- 2)  $i_1 = r_1 = 0; A = r_2; D = i_2 - A = i_2 - r_2$
- 3)  $i_2 = r_2 = 0; A = r_1; D = i_1 - r_1$
- 4)  $i_2 = 90^\circ; r_2 = c; A = r_1 + c$
- 5)  $D = (i_1 + 90^\circ) - A$

**LEVEL-2 & 3 Single Correct Choice Type:**

9. A prism of  $R.I = 1.5$  is immersed in water of  $R.I = \frac{4}{3}$  as shown in the figure. For the total internal reflection the correct choice is



- 1)  $\sin \theta < \frac{8}{9}$       2)  $\sin \theta > \frac{8}{9}$       3)  $\sin \theta = \frac{8}{9}$       4)  $\sin \theta \leq \frac{8}{9}$
10. The angle of deviation by prism is  $(180^\circ - 2A)$ . Its critical angle will be
- 1)  $\sin^{-1}\left(\tan \frac{A}{2}\right)$       2)  $\sin^{-1}\left(\cot \frac{A}{2}\right)$       3)  $\cos^{-1}\left(\cot \frac{A}{2}\right)$       4)  $\cos^{-1}\left(\tan \frac{A}{2}\right)$
11. ACB is right-angled glass prism of refractive index 1.5.  $\angle A, \angle B$  and  $\angle C$  are  $60^\circ, 30^\circ$  and  $90^\circ$  respectively. A thin layer of liquid is on the surface AB. for a ray of light which is incident normally on AC to be totally reflected at AB, the refractive index of the liquid on AB should be
- 1) 1.5      2) 1.4      3) 1.3      4) 1.2
12. A beam of monochromatic light is incident on one face of the equilateral prism the angle of incidence being  $55^\circ$ . If the angle of emergence is  $46^\circ$ , then the angle of minimum deviation is
- 1)  $41^\circ$       2)  $< 41^\circ$       3)  $> 41^\circ$       4)  $\geq 41^\circ$
13. The maximum refractive index of a prism which permits passage of the light, through it when the refracting angle of the prism is  $90^\circ$ , is
- 1)  $\sqrt{3}$       2)  $\sqrt{2}$       3)  $\sqrt{\frac{3}{2}}$       4)  $\frac{3}{2}$
14. The refractive index of the material of prism is  $\sqrt{2}$  and its refracting angle is  $30^\circ$ . One of the refracting surfaces of the prism is made a mirror inwards. A beam of monochromatic light enters the prism from the other surface and the ray retraces from the mirrored surface. The angle of incidence is
- 1)  $30^\circ$       2)  $45^\circ$       3)  $60^\circ$       4)  $0^\circ$

**LEVEL-4 & 5 Single Correct Choice Type:**

15. A ray of monochromatic light is incident on one refracting face of a prism of angle  $75^\circ$ . It passes through the prism and is incident on the other face at the critical angle. If the refractive index of the material of the prism is  $\sqrt{2}$ , the angle of incidence on the first face of the prism is,
- 1)  $30^\circ$       2)  $45^\circ$       3)  $60^\circ$       4)  $0^\circ$

16. The light ray is incident at an angle of  $60^\circ$  on a prism of angle  $45^\circ$ . When the light ray falls on the other surface at  $90^\circ$ , the refractive index of the material of the prism  $\mu$  and angle of deviation 'd' are given by

- 1)  $\mu = \sqrt{2}, d = 30^\circ$     2)  $\mu = 1.5, d = 15^\circ$     3)  $\mu = \frac{\sqrt{3}}{2}, d = 30^\circ$     4)  $\mu = \sqrt{\frac{3}{2}}, d = 15^\circ$

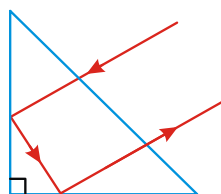
17. When a glass prism of refracting angle  $60^\circ$  is immersed in a liquid its angle of minimum deviation is  $30^\circ$ . The critical angle of glass with respect to the liquid medium is

- 1)  $30^\circ$                       2)  $45^\circ$                       3)  $60^\circ$                       4)  $50^\circ$

18. Angle of prism is 'A' and its one surface is silvered. Light rays falling at an angle of incidence  $2A$  on first surface return back through the same path after suffering reflection at second silvered surface. Refractive index of the material of the prism is

- 1)  $2\sin A$                       2)  $2\cos A$                       3)  $1/2\cos A$                       4)  $\tan A$

19. A ray of light incident normally on an isosceles right angled prism travels as shown in the figure. The refractive index of the prism must be greater than



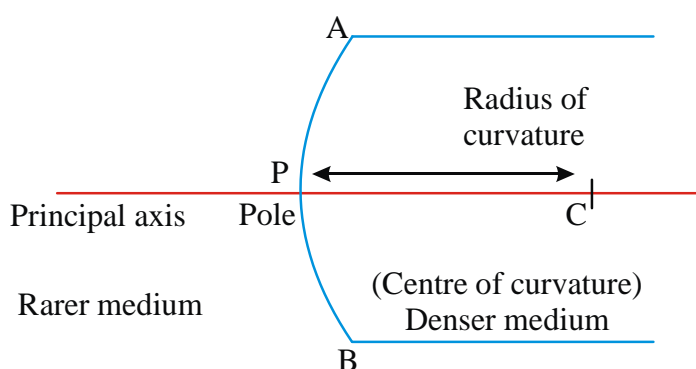
- 1)  $\sqrt{2}$                       2)  $\sqrt{3}$                       3) 1.5                      4) 2

### RAY OPTICS SYNOPSIS-3

#### (Refraction at Spherical surfaces)

A part of a sphere of refracting material is called a spherical refracting surface.

**Some important terms related to spherical refracting surface are given below:**



The point 'P' in the figure is the pole (P)

The centre of the sphere of which the refracting surface forms a part is called the centre of curvature (C) of the spherical refracting surface.

The radius of the sphere of which the refracting surface forms a part is called the radius of curvature of the spherical refracting surface (R)

The diameter of the spherical refracting surface is called its aperture. In the figure, the line joining A and B is the aperture of the spherical refracting surface. The line joining the pole and centre of curvature and extends on either side of the surface is called the principal axis.

**Sign convention:** All the distances are measured from the pole of the spherical refracting surface.

The distance measured in the direction of the incident light are taken as positive.

**Refraction at Spherical surfaces :** Consider refraction at a spherical interface between two transparent media. The normal at the point of incidence is perpendicular to the tangent plane to the spherical surface at that point and therefore, passes through its centre of curvature.

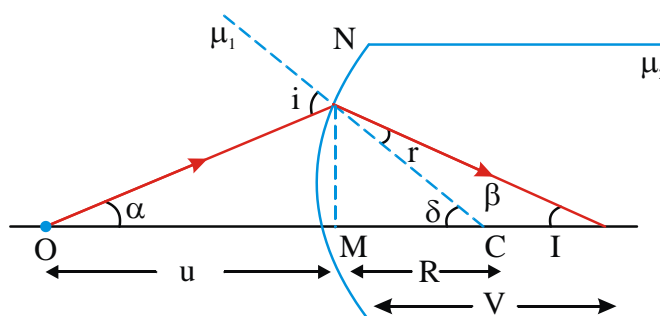


Figure shows the geometry of formation of image I of an object 'O' on the principal axis of spherical surface with centre of curvature C, and radius of curvature R. The rays are incident from a medium of refractive index  $\mu_1$  to another of refractive index  $\mu_2$ . As before, we take like in curved mirrors the aperture of the surface to be small compared to other distances involved. Hence NM will be taken to be nearly equal to the length of the perpendicular from the point N on the principal axis.

$$\tan \alpha = \frac{MN}{OM}, \tan \delta = \frac{MN}{MC}, \tan \beta = \frac{MN}{MI}$$

Now for  $\triangle NOC$ , 'i' is the exterior angle.

Therefore,  $i = \alpha + \delta$

Similarly  $r = \angle NCM - \angle NIM = \delta - \beta$

Now by Snell's law  $\mu_1 \cdot \sin i = \mu_2 \cdot \sin r$  or for small angles  $\mu_1 \cdot i = \mu_2 \cdot r$  substituting i & r,

$$\text{we get } \mu_1(\alpha + \delta) = \mu_2(\delta - \beta) \quad \mu_1\alpha + \mu_2\beta = \delta(\mu_2 - \mu_1) \quad \frac{\mu_1}{OM} + \frac{\mu_2}{MI} = \frac{\mu_2 - \mu_1}{MC}$$

Here OM, MI and MC represent magnitude of distances, applying sign conversions.

$$OM = -u, MI = +v, MC = +R$$

$$\therefore \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

This is the Gauss's relation for a single spherical refracting surface. Though above relation is derived for a convex surface and for a real object and real image, it is equally valid for all other conditions.

If we move in the direction of light,  $\mu_1$  is the refractive index of the medium which



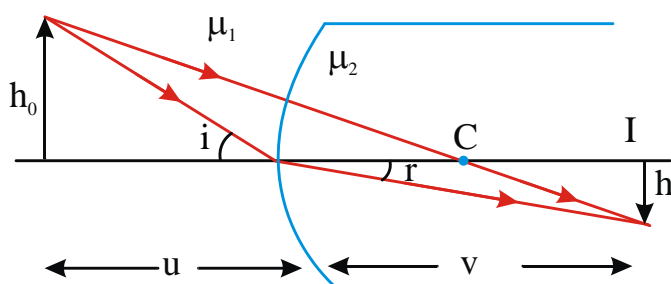
comes before the boundary and  $\mu_2$  is the refractive index of the medium which comes after the boundary.

If the object or image itself is present at a refracting surface, refraction at that surface is not considered.

It is note that with respect to real object convex refracting surfaces can form real image (for distant object) as well as virtual image (for nearer object), where as concave refracting surface forms only virtual image.

### Magnification

#### Lateral magnification or transverse magnification:



From figure, the lateral magnification is  $m_t = \frac{h_i}{h_0}$

From Snell's law:  $\mu_1 i = \mu_2 r$  (for small angles)

Therefore  $\mu_1 \frac{h_0}{u} = \mu_2 \frac{h_i}{v}$  Thus lateral magnification  $m_t = \frac{h_i}{h_0} = \frac{\mu_1}{\mu_2} \cdot \left(\frac{v}{u}\right)$

**Longitudinal magnification at refracting curved surface:** If a small object of length 'du' is placed on the axis, produces an image of length 'dv' along the axis of the refracting surface, then longitudinal magnification

$m_L = \frac{dv}{du}$ ; Since  $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$  On differentiating,  $\frac{dv}{du} = \frac{\mu_1}{\mu_2} \cdot \frac{v^2}{u^2}$

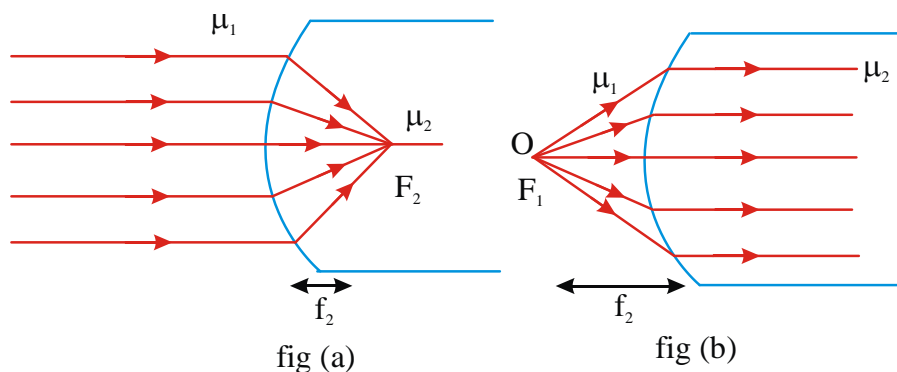
$\therefore m_L = \frac{\mu_1}{\mu_2} \frac{v^2}{u^2}$  Longitudinal magnification  $m_L = m^2 \left(\frac{\mu_2}{\mu_1}\right)$ .

Where 'm<sub>t</sub>' is transverse magnification.

**Principal FOCI :**

In the equation  $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

If the object at infinity i.e.,  $u = \infty$



$\frac{\mu_2}{v} - 0 = \frac{\mu_2 - \mu_1}{R}$  From figure, it is clear that  $v = f_2 \therefore f_2 = \frac{\mu_2 R}{\mu_2 - \mu_1}$

i.e. The position of image corresponding to the object at infinity, is called the second principal focus of the refracting surface. This is shown in fig (a)

Similarly if  $v = \infty$ , i.e., the object is so placed that the refracting rays becomes parallel to the principal axis, then  $\frac{\mu_2}{\infty} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$  From figure, it is clear

that  $u = f_1 \therefore f_1 = \frac{\mu_1 R}{\mu_2 - \mu_1}$

i.e. The position of the object, whose image is formed at infinity to known as the first principal focus of the refracting surface. This is shown in figure(b).

Hence  $\frac{f_1}{f_2} = \frac{\mu_1}{\mu_2}$

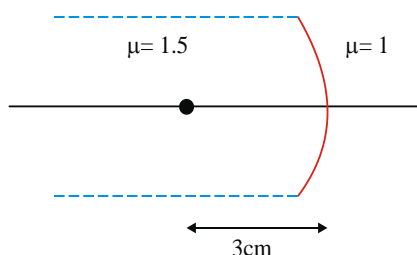
It is easy to see that first focal length  $f_1$  for spherical refracting surface is not equal to the second focal length  $f_2$ .

Further  $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\therefore \frac{\mu_2 R}{v(\mu_2 - \mu_1)} - \frac{\mu_1 R}{u(\mu_2 - \mu_1)} = 1 \therefore \frac{f_2}{v} + \frac{f_1}{u} = 1$

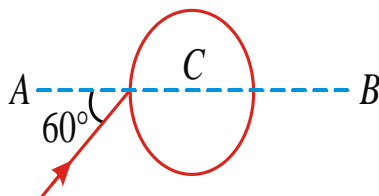
RAY OPTICS WORKSHEET-3**LEVEL-1** Single Correct Choice Type:

1. An air bubble in glass ( $\mu = 1.5$ ) is situated at a distance 3 cm from a convex surface of diameter 10 cm as shown. The distance from surface at which the image of bubble appears is



- 1) 2.5 cm                      2) 5 cm                      3) 4 cm                      4) 1.5 cm
2. Light from a point source in air falls on a spherical glass surface ( $\mu = 1.5$  and radius of curvature 20 cm). The distance of the light source from the glass surface is 100 cm. The position where image is formed is
- 1) 50 cm                      2) 100 cm                      3) 125 cm                      4) 25 cm
3. A spherical convex surface of radius of curvature  $R$  separates air ( $\mu_a = 1$ ) from glass ( $\mu_g = 1.5$ ). The centre of curvature is in the glass. A point object  $P$  placed in air is found to have a real image  $Q$  in the glass. The line  $PQ$  cuts the surface at a point  $O$  and  $PO = OQ$ . The distance  $PO$  is equal to
- 1)  $5R$                       2)  $3R$                       3)  $2R$                       4)  $1.5R$
4. A denser medium of refractive index 1.5 has a concave surface with respect to air of radius of curvature 12 cm. An object is situated in the denser medium at a distance of 9 cm from the pole locate the image due to refraction in air
- 1) A real image at 8 cm                      2) a virtual image at 8 cm  
3) A real image at 4.8 cm                      4) A virtual image at 4.8 cm
5. The human eye can be regarded as a single spherical refractive surface of curvature of cornea 7.8 mm. If a parallel beam of light comes to focus at 3.075 cm behind the refractive surface, the refractive index of the eye is
- 1) 1.34                      2) 1.72                      3) 1.5                      4) 1.61
6. A glass sphere ( $\mu = 1.5$ ) of radius 20 cm has small air bubble 4 cm below its centre. The sphere is viewed from outside and along vertical line through the bubble. The apparent depth of the bubble below the surface of sphere is (in cm)
- 1) 13.33                      2) 26.67                      3) 15                      4) 30

7. A ray of light falls on a transparent sphere with centre at C as shown in figure. The ray emerges from the sphere parallel to line AB. the refractive index of the sphere is



- 1)  $\sqrt{2}$                       2)  $\sqrt{3}$                       3)  $\frac{3}{2}$                       4)  $\frac{1}{2}$
8. A mark is made on the surface of a glass sphere of diameter 10 cm and refractive index 1.5. It is viewed through the glass from a portion directly opposite. The distance of the image of the mark from the centre of the sphere will be
- 1) 15 cm                      2) 17.5 cm                      3) 20 cm                      4) 22.5 cm
9. In a medium of refractive index 1.6 and having a convex surface with respect to air has a point object in it at a distance of 12 cm from the pole. The radius of curvature is 6 cm. Locate the image as seen from air
- 1) A real image at 30 cm                      2) A virtual image at 30 cm  
3) A real image at 4.28 cm                      4) A virtual image at 4.28 cm
10. Parallel rays are incident on a transparent sphere along its one diameter. After refraction, these rays converge at the other end of this diameter. The refractive index for the material of sphere is
- 1) 1                      2) 1.5                      3) 1.6                      4) 2

**Integer Answer Type:**

11. A mark on the surface of a glass sphere ( $\mu = 1.5$ ) is viewed from a diametrically opposite position. It appears to be at a distance 10 cm from its actual position. The radius of the sphere is \_\_\_\_\_cm.

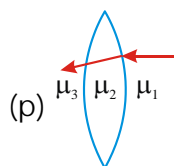
**Matrix Match Type:**

12. Two transparent media of refractive indices  $\mu_1$  &  $\mu_3$  have a solid lens shaped transparent material of refractive index  $\mu_2$  between them as shown in figures in Column - II. A ray traversing these media is also shown in the figures. In Column - I different relationships between  $\mu_1, \mu_2$  and  $\mu_3$  are given. Match them to the ray diagrams shown in Column - II.

Column - I

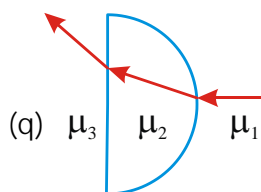
Column - II

(A)  $\mu_1 < \mu_2$

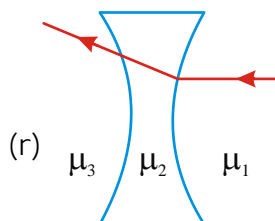




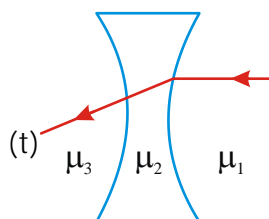
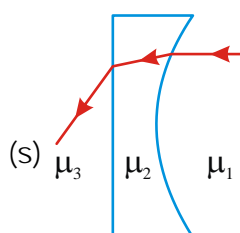
(B)  $\mu_1 > \mu_2$



(C)  $\mu_2 = \mu_3$



(D)  $\mu_2 > \mu_3$



**LEVEL-4 & 5 Single Correct Choice Type:**

13. A glass sphere ( $\mu=1.5$ ) of radius 20 cm has a small air bubble 4 cm below its centre. The sphere is viewed from outside and along a vertical line through the bubble. The apparent depth of the bubble below the surface of sphere is ( in cm)
  - 1) 13.33
  - 2) 26.67
  - 3) 15
  - 4) 30
14. A ray of light is incident on a glass sphere of refractive index  $3/2$ . What should be the angle of incidence so that the ray which enters the sphere does not come out of the sphere is
  - 1)  $\tan^{-1}(2/3)$
  - 2)  $\sin^{-1}(2/3)$
  - 3)  $90^\circ$
  - 4)  $\cos^{-1}(1/3)$
15. There is a small air bubble inside a glass sphere ( $\mu=1.5$ ) of radius 10 cm. The bubble is 4 cm below the surface and is viewed normally from the outside. The apparent depth of the bubble is
  - 1) 3 cm below the surface
  - 2) 5 cm below the surface
  - 3) 8 cm below the surface
  - 4) 10 cm below the surface

**RAY OPTICS****SYNOPSIS-4****(Thin lenses)****Refraction by Lenses**

**Lens theory** :A lens is a piece of transparent material with two refracting surfaces such that atleast one is curved and refractive index of its material is different from that of the surroundings.

If the curved surface (or surfaces) of a lens are spherical, the lens is called spherical lens and if its thickness is small the lens is called thin.

Here we shall limit ourselves to thin spherical lenses.

**Different types of spherical lens are shown in figure (a) and (b)**

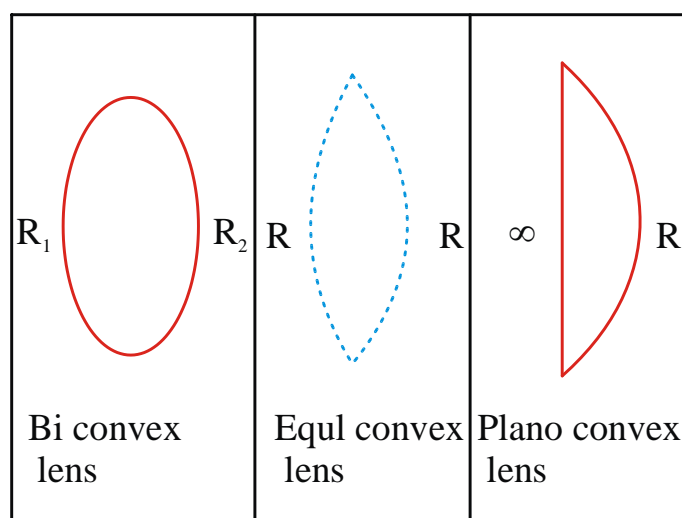


Fig. (a) Convex Lens

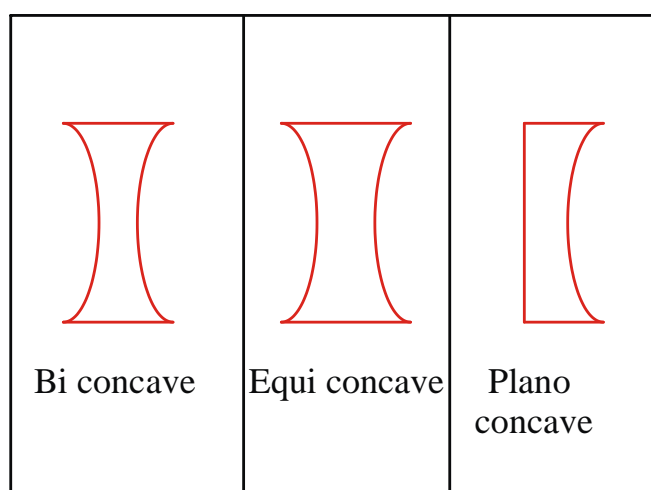


Fig. (b) Concave Lens

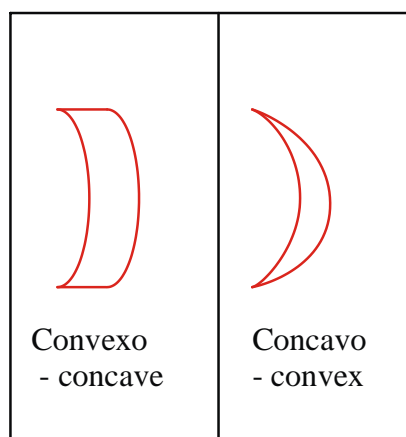


Fig. (c) Meniscus

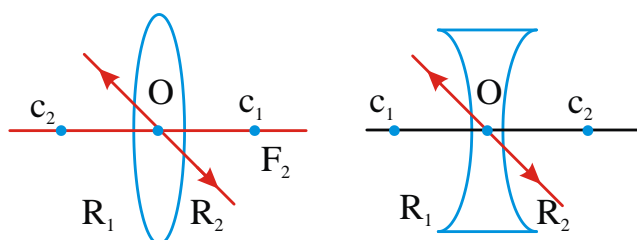
**Note:** While calling the name of the lens we called first the shape of the surface which has more radius of curvature is to be considered.

A thin lens with refractive index greater than that of surroundings behaves as a convergent or convex lens if its central portion is thicker than marginal one. i.e. it converge if parallel rays incident on it.

If the central portion of a lens (with  $\mu_L > \mu_m$ ) is thinner than marginal, it diverges parallel rays and behaves as divergent or concave lens.

**Note:** A thin lens is a lens in which the thickness of the lens is small compared to the object distance (or) the image distance or either of the two radii of curvature of the lens.

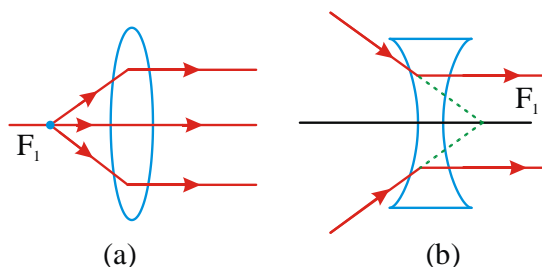
**In case of thin spherical lenses:** Optical centre (or) pole O is a point for a given lens through which any ray passes undeviated.



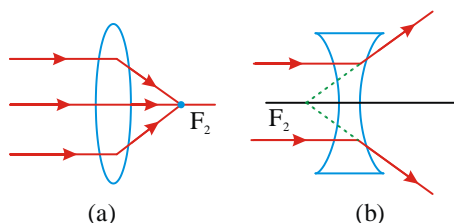
If the lens has two spherical surfaces, there are two centres of curvature  $C_1$  and  $C_2$  and correspondingly two radii of curvature  $R_1$  and  $R_2$ .

**Principal axis** ( $C_1C_2$ ) is a line passing through optical centre and centres of curvature of two refracting surfaces. It is perpendicular to the lens.

A lens has two surfaces and hence two focal points. First focal point ( $F_1$ ) is an object point (real in case of a convex lens and virtual for concave) on the principal axis for which image is formed at infinity.



Second focal point ( $F_2$ ) is an image point on the principal axis for which object lie at infinity.



The distance between optical centre of a lens and the point where the parallel beam of light converges or appears to converge. i.e., second principal focal point ( $F_2$ ), is called focal length  $f$ .

To a lens, if the media on the two sides is same, then first principal focal distance is equal to second principal focal distance. i.e.,  $|f_1| = |f_2|$ .

**Note:** We are mainly concerned with the second focus  $F_2$  because wherever we write the focal length 'f' measures second principal focal length.

**Focal plane:** It is a plane passing through the principal focus and perpendicular to the principal axis.

**Aperture:** In reference to a lens, aperture is the effective diameter of its light transmitting area. The intensity of image formed by a lens depends on square of aperture. i.e.,  $I \propto (\text{aperture})^2$

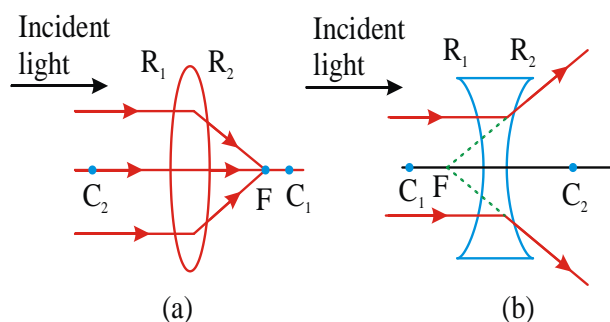
**Sign convention:** Whenever and where ever possible, rays of light are taken to travel from left to right.

Transverse distance measured above the principal axis are taken to be positive while those below it negative.



Longitudinal distances are measured from optical centre and are taken to be positive if in the direction of light propagation and negative if opposite to it.

**Note:** While using the sign convention it must be kept in mind that, to calculate an unknown quantity the known quantities are substituted with sign in a given formula.



For convex lens as shown in fig.(a)

$R_1 (= OC_1)$  is +ve;  $R_2 (= OC_2)$  is -ve

$f (= OF)$  is +ve

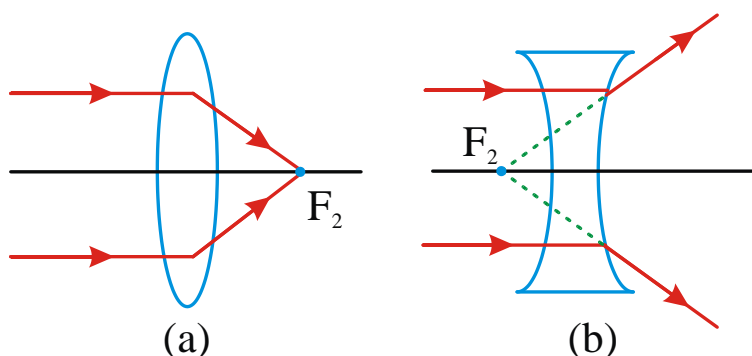
For concave lens as shown in fig. (b)

$R_1 (= OC_1)$  is -ve;  $R_2 (= OC_2)$  is +ve

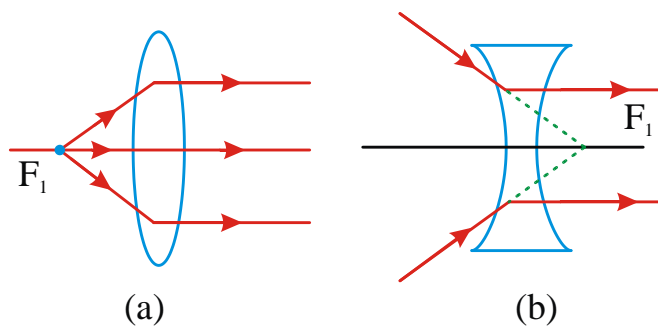
$f (= OF)$  is -ve

**Rules for image formation:** In order to locate the image and its nature by a lens graphically following rules are adopted.

A ray parallel to the principal axis after refraction passes through the principal focus for convex lens and appears to diverge from focus for concave lens.



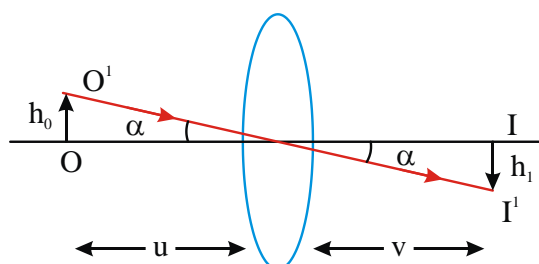
A ray passing through the first focus  $F_1$  becomes parallel to the principal axis after refraction.



**Magnification**

**Lateral magnification:** Magnification produced by a lens is defined as the ratio of the size of image to that of the object. Here the sizes being measured perpendicular to principal axis.

$$m_t = \frac{I^1}{OO^1} = \frac{h_1}{h_0} = \frac{v}{u}$$



When we apply the sign convention, for erect (and virtual) image formed by a convex or concave lens 'm' is positive, while for an inverted (and real) image, m is negative.

**Note:** Linear magnification for a lens can also be expressed as

$$m = \frac{I}{O} = \frac{v}{u} = \frac{f-v}{f} = \frac{f}{f+u}$$

**Longitudinal magnification:** Longitudinal magnification is defined as the ratio of infinitesimal axial length (dv) in the region of the image to the corresponding length (du) in the region of the object.

Longitudinal magnification  $(m_L) = \frac{dv}{du}$

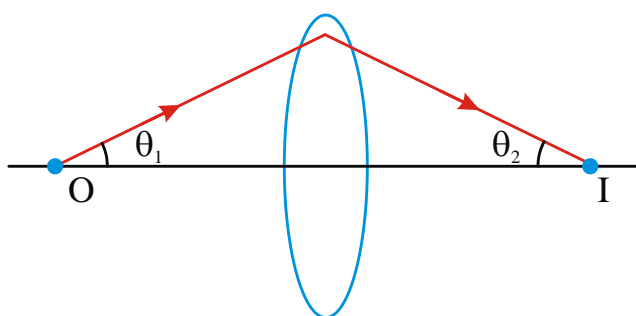
On differentiating equation  $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$-\frac{dv}{v^2} - \left(-\frac{du}{u^2}\right) = 0 \text{ or } \frac{dv}{v^2} - \frac{du}{u^2} = 0$$

$$\text{Therefore, } m_L = \frac{dv}{du} = \frac{v^2}{u^2} = \left(\frac{v}{u}\right)^2 = m^2$$

So, longitudinal magnification is proportional to the square of the lateral magnification.

**Angular magnification of lens:** The ratio of the slopes of emergent ray and corresponding incident ray with principal axis is called the angular magnification.



$$\text{Angular magnification } (\gamma) = \frac{\tan \theta_2}{\tan \theta_1}$$

**Note:** When several lenses are used co-axially, the total magnification

$$m = m_1 \times m_2 \times \dots \times m_n .$$

**From the ray diagrams it is clear that**

**Regarding convex lens:** A convex lens will form a real image for a real object when the object is placed beyond focus.

When the object comes within the focus, then a virtual image is formed for the real object.

The real image formed is always inverted while virtual image is always erect.

**Regarding concave lens:** A concave lens always form virtual image for a real object.

The image formed by a concave lens is always erect and diminished in size.

A concave lens can form a real image as well as virtual image if the object is virtual.

**Lens formula :** Lens formula is a relation connecting focal length of the lens with the object distance and image distance. The formula is  $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

**Note:** The above formula is valid for convex as well as concave lenses and it is independent of nature of the image (real or virtual)

**Note:** To solve the problems, the above equation can also be expressed as follows

$$v = \frac{uf}{u+f}, u = \frac{vf}{f-v}, f = \frac{vu}{u-v}$$

**a) Convex lens &**

**b) Concave lens**

Position of the object	Ray diagram	Image details
At Infinity		Real inverted, diminished at F
Between ∞ and 2F		Real, inverted, diminished between F and 2F
At 2F		Real, inverted, equal, at 2F
Between 2F and F		Real, inverted, enlarged, between 2F and infinity
At F		Real, inverted, enlarged at infinity
Between F and P		Virtual, erect, enlarged and on the side of the object

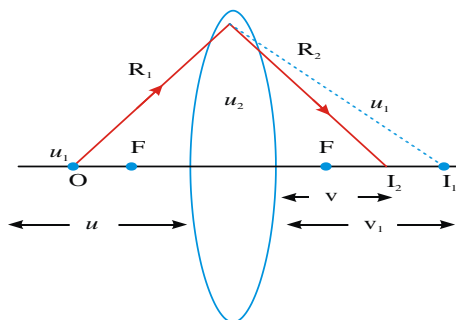
Position of the object	Ray diagram	Image details
At Infinity		Virtual, erect, diminished at F
Infront of mirror		Virtual, erect, diminished between F and P



**Lens Maker's formula and Lens formula :**

In case of image formation by a lens, the incident ray is refracted at first and second surface respectively. The image formed by the first surface acts as object for the second.

Consider an object O is placed at a distance u from a convex lens as shown in figure. Let its image is I<sub>1</sub> after refraction through first surface. So from the formula for refraction at curved surface.



$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

For first surface  $\frac{\mu_2}{v_1} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1}$  .... (1)

The image I<sub>1</sub> is acts as object to second surface, and form final image I<sub>2</sub>

For second surface  $\frac{\mu_1}{v} - \frac{\mu_2}{v_1} = \frac{\mu_1 - \mu_2}{R_2}$  .... (2)

So adding (1) and (2) equation, we have

$$\mu_1 \left[ \frac{1}{v} - \frac{1}{u} \right] = (\mu_2 - \mu_1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \text{ or } \left( \frac{1}{v} - \frac{1}{u} \right) = \left( \frac{\mu_2}{\mu_1} - 1 \right) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{v} - \frac{1}{u} = (\mu_r - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \text{ with } \mu_r = \frac{\mu_2}{\mu_1} \text{ (or) } \frac{\mu_L}{\mu_M}$$

If object is at infinity, image will be formed at the focus

i.e. for  $u = -\infty, v = f$ , so that above equation becomes  $\frac{1}{f} = (\mu_r - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$

Which is known as Lens-maker's formula and

For a lens it becomes  $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$  which is known as the "lens - formula" or "Gauss's formula" for a lens.

Though we derived it for a real image formed by a convex lens, the formula is valid for both convex as well as concave lens and for both real and virtual images.

**Note:** The lens maker's formula is applicable for thin lenses only and the value of  $R_1$  and  $R_2$  are to be put in accordance with the Cartesian sign convention.

**Power of A Lens :** The power of lens is the measure of its ability to produce convergence or divergence of a parallel beam of light.

The S.I unit of power is diopter (D) and  $1D = 1m^{-1}$

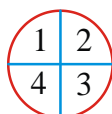
$$\text{i.e. } P = \frac{1}{f(\text{in } m)} = \frac{100}{f(\text{in } cm)} D$$

A convex lens converge the incident rays. Due to this reason, the power of a convex lens is taken as positive. On the other hand, a concave lens diverge the incident rays. Therefore its power is taken as negative.

**RAY OPTICS WORKSHEET-4**

**CUQ**

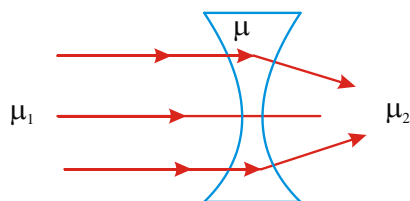
- The focal length of a lens depends on
  - 1) colour of light
  - 2) radius of curvature of the lens
  - 3) material of the lens
  - 4) all the above
- $f_B$  and  $f_R$  are focal lengths of a convex lens for blue and red light respectively and  $F_B$  and  $F_R$  are the focal lengths of the concave lens for blue and red light respectively. We must then have
  - 1)  $f_B > f_R$  and  $F_B < F_R$
  - 2)  $f_B < f_R$  and  $F_B > F_R$
  - 3)  $f_B > f_R$  and  $F_B > F_R$
  - 4)  $f_B < f_R$  and  $F_B < F_R$
- The graph drawn with object distance along abscissa and image distance along ordinate measuring the distance from the convex lens is
  - 1) Straight line
  - 2) Parabola
  - 3) Circle
  - 4) A hyperbola
- A convex lens is used to form a real image of the object shown in the following figure:



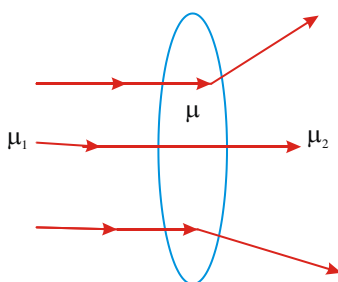
Then the real inverted image is as shown in the following figure:

- 1)
- 2)
- 3)
- 4)

5. The relation between refractive indices  $\mu, \mu_1, \mu_2$ . if the behaviour of light ray is as shown in fig.



- 1)  $\mu > \mu_2 > \mu_1$       2)  $\mu < \mu_2 < \mu_1$       3)  $\mu < \mu_2; \mu = \mu_1$       4)  $\mu_2 < \mu_1; \mu = \mu_2$
6. If parallel beam of light falls on a convex lens. The path of the rays is shown in fig. It follows that



- 1)  $\mu_1 > \mu > \mu_2$       2)  $\mu_1 < \mu < \mu_2$       3)  $\mu_1 = \mu < \mu_2$       4)  $\mu_1 = \mu > \mu_2$
7. Lens maker's formula is applicable to
- 1) Thin lenses and paraxial rays which subtend very small angles with the principal axis
  - 2) Thick lenses and paraxial rays which subtend very small angles with the principal axis
  - 3) Thin lenses and for marginal rays
  - 4) Thick lenses and for marginal rays
8. A spherical air bubble in water will act as
- 1) a convex lens
  - 2) a concave lens
  - 3) Plane glass plate
  - 4) Plano-concave lens

**JEE MAIN & ADVANCED**

**LEVEL-1 Single Correct Choice Type:**

1. A beam of light converges at a point P. Now a convex lens of focal length 20 cm placed in the path of the convergent beam 12 cm from P. The point at which the beam converges now is
- 1) 7.5cm right side of the lens
  - 2) 7.5 cm left side of the lens
  - 3) 15.2 cm right side of lens
  - 4) 15.2 cm left side of lens

2. The radius of curvature of the convex surface of a thin plano - convex lens is 15 cm and the refractive index of its material is 1.6. The power of the lens is  
1) +1 D                      2) -2 D                      3) +3 D                      4) +4 D
3. A double convex lens is made of glass which has refractive index 1.55 for violet rays and 1.50 for red rays. If the focal length of violet rays is 20 cm, the focal length of red rays is  
1) 9 cm                      2) 18 cm                      3) 20 cm                      4) 22 cm
4. The refractive index of the material of a double convex lens is 1.5 and its focal length is 5 cm. If the radii of curvatures are equal, the value of radius of curvature is (in cm)  
1) 5                              2) 6.5                              3) 8                              4) 9.5
5. A diverging meniscus lens of 1.5 refractive index has concave surfaces of radii 3 and 4 cm. The position of image if an object is placed 12cm in front of the lens is  
1) -24 cm                      2) -8 cm                      3) 8 cm                      4) 24 cm
6. A beam of light converges at a point P. Now a concave lens of focal length -16 cm is placed in the path of the convergent beam 12 cm from P. The point at which the beam converges now is  
1) 6.86cm right side of the lens                      2) 6.86cm left side of the lens  
3) 48cm right side of the lens                      4) 48cm left side of the lens
7. The radius of curvature of convex surface of plano convex lens is 10 cm and its focal length is 30 cm, then the refractive index of the material of the lens is  
1) 3                              2) 1.5                              3) 1.66                              4) 1.33
8. Focal length of a lens is 0.12 m and refractive index is 1.5. Focal length of the same lens for blue colour is 0.1m. Then refractive index of the lens for blue colour is  
1) 1.51                              2) 1.25                              3) 1.49                              4) 1.6
9. The focal length of a biconvex lens is 20 cm and its refractive index is 1.5. If the radii of curvatures of two surfaces of lens are in the ratio 1:2, then the larger radius of curvature is (in cm)  
1) 10                              2) 15                              3) 20                              4) 30
10. The radii of curvature of the two surfaces of a lens are 20 cm and 30 cm and the refractive index of the material of the lens is 1.5. If the lens is concavo convex then the focal length of the lens is  
1) 24 cm                              2) 10 cm                              3) 120 cm                              4) 24cm

**Multi Correct Choice Type:**

11. Which of the following form(s) a virtual and erect image for all positions of the real object?  
1) Convex lens                      2) Concave lens                      3) Convex mirror                      4) Concave mirror

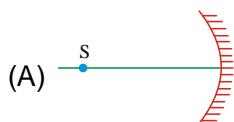


**Matrix Match Type:**

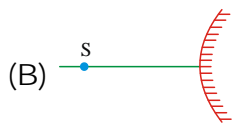
12. An optical component and an object S placed along its optic axis are given in Column - I. The distance between the object and the component can be varied. The properties of images are given in Column II. Match all the properties of images from Column - II with the appropriate components given in Column I. Indicate your answer by darkening the appropriate bubbles of the  $4 \times 4$  matrix given in the ORS.

Column - I

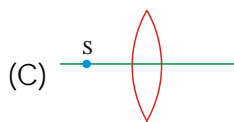
Column - II



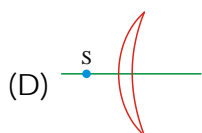
(p) Real image



(q) Virtual image



(r) Magnified image



(s) Image at infinity

**Matrix Match Type:**

13. Column - I

Column - II

(i) Converging system

(A) convex lens

(ii) Diverging system

(B) concave lens

(iii) Virtual Image is formed by

(C) concave mirror

(iv) Magnification  $< 1$  is possible with

(D) convex mirror

**LEVEL-2 & 3 Single Correct Choice Type:**

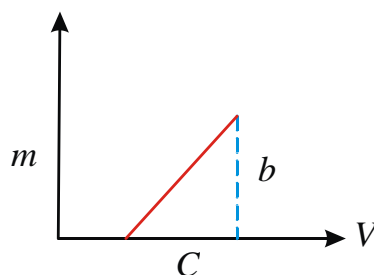
14. An object is placed first at infinity and then at 20 cm from the object side focal plane of a convex lens. The two images thus formed are 5 cm apart. The focal length of the lens is

- 1) 5 cm                      2) 10 cm                      3) 15 cm                      4) 20 cm

15. The image of a square hole in a screen illuminated by light is obtained on another screen with the help of converging lens. The distance of the hole from the lens is 40 cm. If the area of the image is nine times that of the hole, the focal length of the lens is

- 1) 30 cm                      2) 50 cm                      3) 60 cm                      4) 75 cm

16. A plano convex lens of focal length 30 cm has its plane surface silvered. An object is placed 40 cm from the lens on the convex side. The distance of the image from the lens is
- 1) 18 cm                      2) 24 cm                      3) 30 cm                      4) 40 cm
17. The graph shows the variation of magnification  $m$  produced by convex lens with image distance  $v$ . The focal length of the lens is used is :



- 1)  $\frac{b}{c}$                       2)  $\frac{b}{ca}$                       3)  $\frac{bc}{a}$                       4)  $\frac{c}{b}$
18. A convex lens of focal length  $f$  is placed somewhere in between the object and a screen. The distance between object and screen is  $x$ . If magnification produced is  $m$ , the focal length of the lens is
- 1)  $\frac{mx}{(m+1)^2}$                       2)  $\frac{mx}{(m-1)^2}$                       3)  $\frac{(m+1)^2}{m}x$                       4)  $\frac{(m-1)^2}{m}x$
19. A thin liquid convex lens is formed in glass. Refractive index of liquid is  $\frac{4}{3}$  and that of glass is  $\frac{3}{2}$ . If ' $f$ ' is the focal length of the liquid lens in air, its focal length and nature in the glass is
- 1)  $f$ , convex                      2)  $f$ , concave                      3)  $2f$ , concave                      4)  $3f$ , concave
20. A thin converging lens of refractive index 1.5 has a focal power of 5 D. When this lens is immersed in a liquid, it acts as a diverging lens of focal length 100 cm. The refractive index of the liquid is
- 1)  $\frac{11}{6}$                       2)  $\frac{9}{5}$                       3)  $\frac{5}{3}$                       4) 2

**LEVEL-4 & 5 Single Correct Choice Type:**

21. Two plano concave lenses of glass of refractive index 1.5 have radii of curvature 20 cm and 30 cm respectively. They are placed in contact with the curved surface towards each other and the space between them is filled with a liquid of refractive index  $5/2$ . The focal length of the combination is (in cm)
- 1) 6                      2) -92                      3) 108                      4) 12

22. Two converging glass lenses 'A' and 'B' have focal lengths in the ratio 2 : 1. The radius of curvature of first surface of lens 'A' is  $\frac{1}{4}$ th of the second surface where as the radius of curvature of first surface of lens 'B' is twice that of second surface. Then the ratio between the radii of the first surfaces of A and B is
- 1) 5 : 3                      2) 3 : 5                      3) 1 : 2                      4) 5 : 6
23. Image of an object at infinity is formed by a convex lens of focal length 30 cm such that the size of the image is 12 cm. If a concave lens of focal length 20 cm is placed in between the convex lens and the image, at a distance 26 cm from the convex lens, size of the new image is
- 1) 2.5 cm                      2) 2.0 cm                      3) 1.025 cm                      4) 1.05 cm

## RAY OPTICS

### SYNOPSIS-5

#### (Optical Instruments)

**Optical Instruments :** Optical instruments are used primarily to assist the eye in viewing the object. Optical instruments are classified into three groups, they are

a) visual instruments                      **Ex:** microscope, telescope  
 b) photographing and projecting instruments                      **Ex:** cameras  
 c) analysing and measuring instruments                      **Ex:** spectrometer

Optical instruments such as telescope and microscopes have one object lens and one eye lens. The lens towards the object is called objective and the lens towards eye is called eye piece. Single lens forms images with defects (aberrations). If the eye is placed near to the eye lens it will not receive marginal rays of the eye lens. This reduces the field of view and the intensity is not uniform in the field of view, the central part being brighter than the marginal part.

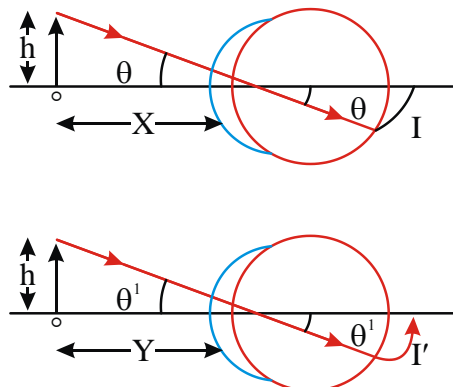
So in designing telescopes and microscopes for practical purposes, combination of lenses are used for both objective and eye lenses to minimize aberrations. A combination of lenses used as an eye lens is known as eyepiece. In any eyepiece that lens near to the objective is called field lens and the lens near to the eye is called eye lens. The field lens increase the field of view and the eye lens acts as a magnifier. We consider two eyepieces namely, Ramsden's eyepiece and Huygen's eyepiece.

**The Eye:** The light enters the eye through a curved front surface, called cornea and passes through the pupil which is the central hole in the iris. The size of pupil can change under control muscles. The cornea-lens-fluid system is equivalent to single converging lens.

The light focused by the lens on retina which is a film of nerve fibres. The retina contains rods and cones which sense the light intensity and colour respectively. The retina transmit electrical signals to the brain through optic nerve.

The shape (curvature) and focal length of the eye lens may be adjusted by the ciliary muscles. The image formed by this eye lens is real, inverted and diminished at the retina.

The size of the image on the retina is roughly proportional to the angle subtended by the object on the eye. This angle is known as the visual angle. Therefore it is known as the angular size.



When the object is distant, its visual angle  $\theta$  and hence image at retina is small and object looks smaller.

When the object is brought near to the eye its visual angle  $\theta$  and hence size of image will increase and object looks larger as shown in figure (b)

Optical instruments are used to increase this visual angle artificially in order to improve the clarity.

**Eg :** Microscope, Telescope

When the eye is focussed on a distant object ( $\theta \approx 0$ ) the ciliary muscles are relaxed so that the focal length of the eye-lens has maximum value which is equal to its distance from the retina.

When the eye is focussed on a closer object ( $\theta$  increases) the ciliary muscles of the eye are strained and focal length of eye lens decreases. The ciliary muscles adjust the focal length in such a way that the image is again formed on the retina and we see the object clearly. This process of adjusting focal length is called accommodation. If the object is brought too close to the eye the focal length cannot be adjusted to form the image on the retina. Thus there is a minimum distance for the clear vision of an object.

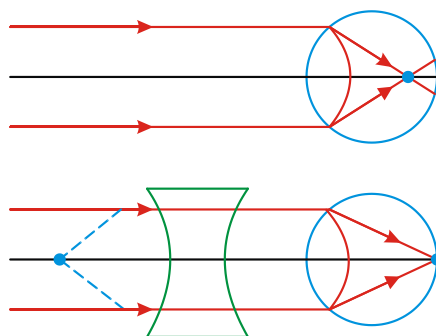
The nearest point at which an object is seen clearly by the eye is called the near point of the eye and distance of near point from the eye is called the **least distance of distinct vision**, It is equal to 25cm for normal eye and it is denoted by D.

The farthest point from an eye at which an object is distinctly seen is called **far point** and for a normal eye it is theoretically at infinity.

**Defects of Vision:** Our eyes are marvellous organs that have the capability to interpret incoming electromagnetic waves as images through a complex process. But over eye may develop some defects due to various reasons. Some common optical defects of the eye are a) myopia b) hypermetropia c) presbyopia

**Myopia:** The light from a distant object arriving at the eye lens may be converged at a point in front of the retina. This defect is called **Myopia (or) shortsightedness**. In this defect, the far point of the eye is at a distance lesser than infinity, and distant objects are not clearly visible.





This defect is rectified by using spectacles having divergent lens (concave lens) which forms the image of a distant object at the far point of defected eye. From lens formula

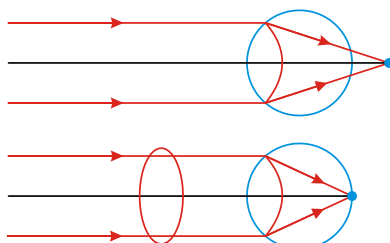
$$\frac{1}{F.P} - \frac{1}{-(distance\ of\ object)} = \frac{1}{f} = P$$

Where F.P= Far point of the defective eye. If the object is at infinity

$$Power\ of\ lens\ (p) = \frac{1}{f} = \frac{1}{F.P}$$

**Hypermetropia: (or) Long-sightedness.**

The light from an object at the eye lens may be converged at a point behind the retina. This defect is called In this type of defect, near point is at a distance greater than 25cm and near objects are not clearly visible.



This defect is rectified by using spectacles having convergent lens(i.e convex lens) which forms the image of near objects at the near point of the defected eye (which is more than 25cm)

$$\frac{1}{-N.P.} - \frac{1}{-(distance\ of\ object)} = \frac{1}{f} = P$$

N.P.= Near Point of defected eye.

If the objective is placed at D=25cm=0.25m

$$P = \frac{1}{f} = \left( \frac{1}{0.25} - \frac{1}{N.P.} \right)$$

**Presbyopia:** The power of accommodation of eyelens may change due to the decreasing effectiveness of ciliary muscles. So, far point is lesser than infinity and near point is greater than 25cm and both near and far objects are not clearly visible. This defect is called **presbyopia**. This defect is rectified by using bifocal lens.

**Astigmatism:** This defect arises due to imperfect spherical nature of lens, the focal length of eye lens in two orthogonal directions becomes different, eye cannot see objects in two orthogonal directions clearly simultaneously. This defect is remedied by cylindrical lens in a particular direction.

**W.E-1:** A person cannot see distinctly any object placed beyond 40cm from his eye. Find the power of lens which will enable him to see distant stars clearly is?.

**Sol:** The person cannot see objects clearly beyond 0.4m.

so his far point = 0.4m distance of object =  $\infty$ .

He should use lens which forms image of distant object ( $u = \infty$ ) at a distance of 40cm in front of it.

$$-\frac{1}{0.40} - \frac{1}{-\infty} = \frac{1}{f} = p; \Rightarrow P = \frac{-10}{4} = -2.5D$$

**W.E-2:** A far sighted person cannot focus distinctly on objects closer than 1m. What is the power of lens that will permit him to read from a distance of 40cm?

**Sol:** As near point is 1m and distance of objects is 0.40m

both in front of lens.

$$P = \frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{-1} - \frac{1}{-0.40} \Rightarrow P = 1.5D$$

## RAY OPTICS\_WORKSHEET-5

### **CUQ**

1. When objects at different distances are seen by the eye, which of the following remains constant
  - 1) The focal length of the eye lens
  - 2) The object distance from the eye lens
  - 3) The radii of curvature of the eye lens
  - 4) The image distance from the eye lens
2. Near and far points of healthy human eye respectively are
  - 1) 0 and 25 cm
  - 2) 0 and infinity



- 2) Because in doing so, the distance of the object is increased
- 3) Because in doing so, the focal length of the eye lens is effectively decreased
- 4) Because in doing so, the focal length of the eye lens is effectively increased

### JEE MAIN & ADVANCED

#### LEVEL-1 Single Correct Choice Type:

1. A person can see clearly upto 1m. The nature and power of the lens which will enable him to see things at a distance of 3 m is
  - 1) concave,  $-0.66D$
  - 2) convex,  $-0.66D$
  - 3) concave,  $-0.33D$
  - 4) convex,  $-0.33D$
2. The far point of a myopic eye is 10 cm from the eye. The focal length of a lens for reading at normal distance ( 25 m ) is
  - 1)  $-8.35\text{ cm}$
  - 2)  $-16.7\text{ cm}$
  - 3)  $-35.4\text{ cm}$
  - 4)  $-32.7\text{ cm}$
3. A person can see clearly objects between 15 and 100 cm from his eye. The range of his vision if he wears close fitting spectacles having a power of 0.8 diopter is
  - 1) 5 to 500 cm
  - 2) 12 to 250 cm
  - 3) 17 to 500 cm
  - 4) 17 to 250 cm
4. The near point of a hypermetropic person is 50 cm from the eye. The power of the lens required to enable the person to read clearly a book held at 25 cm from the eye is
  - 1) 2 D
  - 2) 4 D
  - 3) 8 D
  - 4) 1 D
5. A person wears glasses of power  $-2.5D$ . Is the person far-sighted or near-sighted ? The far point of the person without glasses is
  - 1) long-sighted,  $-40\text{ cm}$
  - 2) near-sighted,  $-40\text{ cm}$
  - 3) near-sighted,  $-20\text{ cm}$
  - 4) long-sighted,  $-20\text{ cm}$
6. A long sighted person has a least distance of distinct vision of 50 cm. He wants to reduce to 25 cm. He should use a
  - 1) concave lens of focal length 50 cm
  - 2) convex of focal length 25 cm
  - 3) convex lens of focal length 50 cm
  - 4) concave lens of focal length 25 cm

#### LEVEL-2 & 3 Single Correct Choice Type:

7. A person cannot see an object lying beyond 80 cm, where as a normal person can easily see the object distant 160 cm. The focal length and nature of the lens used to rectify this defect will be
  - 1) 160 cm, concave
  - 2) 160 cm, convex
  - 3) 60 cm, concave
  - 4) 60 cm, convex



8. The near point of a person is 50 cm and the far point is 1.5m. The spectacles required for reading purpose and for seeing distant objects are respectively.

1)  $+2D, -\left(\frac{2}{3}\right)D$       2)  $+\left(\frac{2}{3}\right)D, -2D$       3)  $-2D, +\left(\frac{2}{3}\right)D$       4)  $-\left(\frac{2}{3}\right)D, 2D$

9. A man cannot see clearly the objects beyond a distance of 20 cm from his eyes. To see distant objects clearly the kind of lenses and its focal length must be

1) 100 cm, convex    2) 100 cm concave    3) 20 cm convex    4) 20 cm concave

10. A short sighted person can see objects most distinctly at a distance of 16 cm. If he wears spectacles at a distance of 1 cm from the eye, then their focal length to see distinctly at a distance of 26 cm

1) 25 cm, convex    2) 25 cm, concave    3) 37.5 cm, convex    4) 37.5 cm, concave

**LEVEL-4 & 5 Single Correct Choice Type:**

11. An optician while testing the eyes finds the vision of a patient to be  $\frac{6}{12}$ . By this means that

- 1) The person can read the letters of 6 inches from a distance of 12 m  
 2) The person can read the letters of 12 inches from 6 cm  
 3) The person can read the letters from 6m which the normal eye can read from 12 m  
 4) The focal length of eye lens had become half that of the normal eye

**RAY OPTICS WORKSHEET-1 KEY**

**CUQ :** 1) 2    2) 4    3) 2    4) 1    5) 4    6) 2    7) 3    8) 3  
 9) 1    10) 4

**JEE MAIN & ADVANCED:**

1) 1    2) 3    3) 1,2    4) 1    5) 2    6) 3    7) 1    8) 3    9) 3  
 10) 4    11) 2    12) 1    13) 3    14) 2    15) 1    16) 1    17) 1    18) 3  
 19) 1    20) 1    21) 1    22) 2

1.  $\mu \propto \frac{1}{vel}$     2.  ${}_i\mu_j = \frac{\mu_j}{\mu_i}$     3.  ${}_w\mu_g = \frac{\mu_g}{\mu_w}$      $\frac{\mu_g}{\mu_w} = \frac{C_w}{C_g} = \frac{\lambda_w}{\lambda_g}$     4.  $\mu = \tan i$

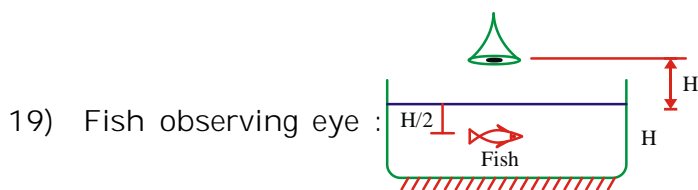
5.  $time = \frac{\mu x}{c}$      $x = \text{thickness}$     6.  $\mu = \frac{Sini}{Sinr}; d = i - r$     7.  $\mu = \frac{Sini}{Sinr}; shift = \frac{t}{cosr} \sin(i-r)$

8. Apparent distance of fish from bird =  $y + \frac{\text{real depth}}{\mu}$     9.  $r = \frac{h}{\sqrt{\mu^2 - 1}}$     10.  ${}_r\mu_d = \frac{1}{\sin \theta_c} = \frac{\lambda_r}{\lambda_d}$

11.  $x = \frac{t \sin(i-r)}{\cos r}$     12.  $S = t \left(1 - \frac{1}{\mu}\right)$     13.  $d_a = \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2}; d_1 = d_2 = \frac{d}{2}$

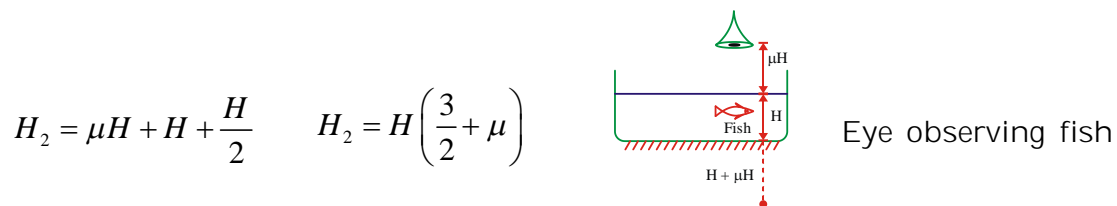
14.  $\sin C = \frac{V_d}{V_r} = \frac{\mu_r}{\mu_d}$     15.  $\frac{\mu_r}{\mu_d} = \frac{C_d}{C_r} = \sin c$     16.  $\frac{\mu_{med}}{\mu_{air}} = \frac{C_{air}}{C_{med}} = \frac{\left(\frac{d}{t}\right)_{air}}{\left(\frac{d}{t}\right)_{med}}$

17)  ${}_a\mu_g = \frac{\sin i}{\sin r}$  and  ${}_g\mu_w = \frac{\sin i}{\sin r}$     18)  $\mu = \frac{R.d}{A.d}$



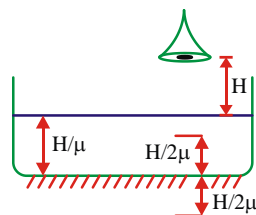
Direct observation  $H_1 = \frac{H}{2} + \mu H$      $H_1 = H \left(\frac{1}{2} + \mu\right)$

20) Fish observing image of eye by mirror, Hence, distance of the eye image from fish,



21) Direct observation  $H_1 = H + \frac{H}{2\mu} = H \left( 1 + \frac{1}{2\mu} \right)$

22) Eye observing image of the fish  $H_2 = H + \frac{H}{\mu} + \frac{H}{2\mu} = H \left( 1 + \frac{3}{2\mu} \right)$



**RAY OPTICS WORKSHEET-2 KEY**

CUQ : 1) 1      2) 1      3) 2      4) 2      5) 1      6) 4      7) 1  
8) 2

**JEE MAIN & ADVANCED**

1) 1      2) 2      3) 4      4) 3      5) 4      6) 1      7) 1  
8) a-2; b-3; c-1; d-4,5      9) 2      10) 1      11) 4      12) 2      13) 2  
14) 2      15) 2      16) 4      17) 2      18) 2      19) 1

2.  $i = \frac{A+D}{2}$

3.  $r_1 = 0 \Rightarrow r_2 = A = 45^\circ$        $\mu = \frac{\sin i_2}{\sin r_2}$

4.  $r_1 = 0$ ;       $\mu = \frac{1}{\sin r_2}$ ;  $r_2 = A$        $d = i_1 + i_2 - A$

5.  $D = 2i - A$

6.  $\mu = \frac{\sin \frac{A+D}{2}}{\sin \frac{A}{2}}$ ;       $\mu = \frac{C}{C_{\text{med}}}$

7.  $\delta = (\mu - 1)A$

9.  $i > c$  and  $i = \theta$ ;  $\Rightarrow \sin \theta > \sin c = \frac{1}{\mu_g}$

10.  $d = 180 - 2A = 2i - A$   $\mu = \frac{\sin i}{\sin r} = \frac{\sin\left(90 - \frac{A}{2}\right)}{\sin \frac{A}{2}}$ ;  $\mu = \frac{1}{\sin c}$

11.  $i_1 = r_1 = 0^\circ$ ;  $r_2 = 0^\circ$  but  $r_2 = c$   $\mu = \frac{1}{\sin c}$  and  $\mu = \frac{\mu_g}{\mu_w}$

12.  $D = i_1 + i_2 - A$ ;  $D_m < D$

13.  $A = 2C = 90^\circ$ ;  $\mu = \frac{1}{\sin c}$

14.  $r_2 = 0$  find  $r_1$  from  $r_1 + r_2 = A$ ;  $\sin i_1 = \mu \sin r_1$

15)  ${}_a\mu_g = \frac{\sin i}{\sin r}$  apply for two surfaces

16)  $\mu = \frac{\sin i}{\sin r}$  and  $d = (\mu - 1)\Delta$

17)  ${}_L\mu_g = \frac{1}{\sin c}$

18) one face is silvered  $r_2 = 0$  and  $i_2 = 0$

19) deviation is  $180^\circ$   $\delta m = \sqrt{2}$

**RAY OPTICS WORKSHEET-3 KEY**

- 1) 1      2) 2      3) 1      4) 4      5) 1      6) 2      7) 2      8) 1      9) 2  
 10) 4      11) 5      12. (A) - p,r; (B) - q,s,t; (C) - p,r,t; (D) - q,s      13) 2      14) 3

15) 1

1.  $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$   $\frac{1}{v} - \frac{(1.5)}{(-3)} = \frac{1-1.5}{(-5)}$

2.  $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ ;  $\frac{1.5}{v} - \frac{1}{(-100)} = \frac{1.5-1}{(20)}$



3.  $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ ;  $\frac{1.5}{PO} + \frac{1}{PO} = \frac{1.5-1}{R}$

4.  $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ ;  $\frac{1}{v} + \frac{1.5}{9} = \frac{1-1.5}{12}$

5.  $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ ;  $\frac{\mu_2}{30.75} + \frac{1}{\infty} = \frac{\mu_1-1}{R}$

6.  $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

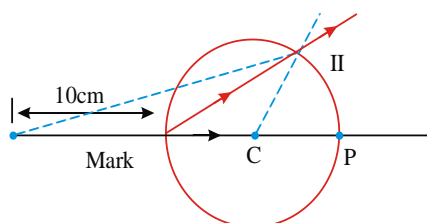
7.  $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

8.  $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ ;  $\frac{1}{v} + \frac{1.5}{10} = \frac{1-1.5}{-5}$

9.  $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ ;  $\frac{1}{v} + \frac{1.6}{12} = \frac{1-1.6}{-6}$

10.  $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ ;  $\frac{\mu_2}{2R} - \frac{1}{\infty} = \frac{\mu_2-1}{R}$

11. As the mark is viewed from the diametrically opposite position, refraction takes place at side II of the surface (the mark being on side I as shown)



Here  $\mu_1 = 1.5, \mu_2 = 1; u = -2R$

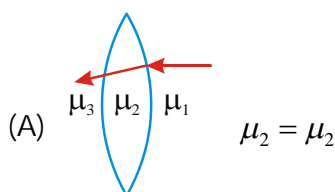
Using  $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ ;  $\frac{1}{v} - \frac{1.5}{(-2R)} = \frac{1-1.5}{-R}$

$\therefore \frac{1}{v} = \frac{0.5}{R} - \frac{1.5}{2R} = -\frac{0.5}{2R}$  or  $v = -4R$

Negative sign indicates that the image is formed to the left of refracting surface as shown in Figure. Further, it is given that the image of mark is at a distance 10 cm from the object.

Hence:  $4R = 2R + 10 \therefore R = 5\text{ cm}$

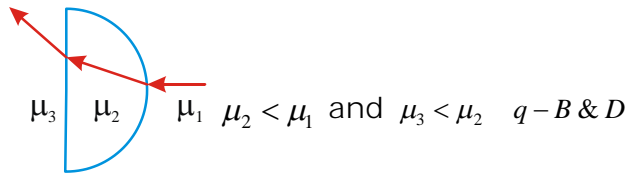
12. (A) - p,r; (B) - q,s,t; (C) - p,r,t ; (D) - q,s



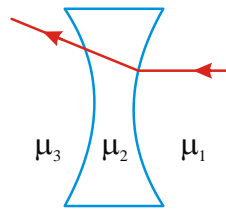
As there is no deviation. As the light bends towards normal in denser medium

$$\mu_2 > \mu_1 \quad p - A \& C$$

(B) As light bends away from normal



(C)  $\mu_2 = \mu_3$  (As no deviation)

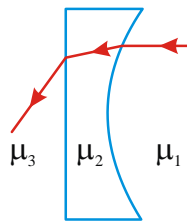


$\mu_2 > \mu_1$  (As light bends + towards normal)  $r - C \& A$

(D)  $\mu_2 < \mu_1 ; \mu_3 < \mu_2$

As light bends away from normal  $s = B, D$

(E)  $\mu_2 = \mu_3$  As no deviation of light



$\mu_2 < \mu_1$  As light bend away from normal  $t - C \& B$

$$13) \frac{\mu_2}{v} = \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$14) \mu = \frac{3}{2} \text{ on second surface} = 180 - r - 60 = 90^\circ$$

refraction is more than  $90^\circ$

$$15) \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

RAY OPTICS WORKSHEET-4 KEY

**CUQ:** 1) 4      2) 4      3) 4      4) 4      5) 3      6) 3      7) 1      8) 2

**JEE MAIN & ADVANCED**

1) 1      2) 4      3) 4      4) 1      5) 2      6) 3      7) 4      8) 4      9) 4

10) 3      11.2, 3      12. (A) – p, q, r, s (B) – q, (C) – p, q, r, s, (D) – p, q, r, s

13. (i) -AC; (ii) - BD; (iii) -ABCD; (iv) - ABCD      14) 2      15) 1      16) 2      17) 4

18) 1      19) 4      20) 3      21) 4      22) 4      23) 1

$$1. \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$2. \quad \frac{1}{f} = \frac{(\mu - 1)}{R} \quad p = \frac{100}{f}$$

$$3. \quad \frac{f_R}{f_V} = \frac{(\mu_V - 1)}{(\mu_R - 1)}$$

$$4. \quad \frac{1}{f} = (\mu - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

(Apply with sign convention)

$$5. \quad \frac{1}{f} = (\mu - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right); \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$6. \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$7. \quad \frac{1}{f} = (\mu - 1) \frac{1}{R}$$

$$8. \quad \frac{f}{f_b} = \frac{(\mu_b - 1)}{(\mu - 1)}$$

$$9. \quad \frac{1}{f} = (\mu - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$10. \quad \frac{1}{f} = (\mu - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

14.  $u_1 = \infty; v_1 = f ; u_2 = f + 20, v_2 = ? ,$

Given  $v_2 - v_1 = 5; \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

15. Areal magnification  $\frac{v^2}{u^2} = 9 \Rightarrow v = 3u; f = \frac{uv}{u+v}$

16.  $\frac{1}{F} = \frac{2}{f_L} = \frac{2}{30}; \frac{1}{F} = \frac{1}{v} - \frac{1}{u}$

17.  $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}; \frac{v}{f} = m + 1; m = \frac{v}{f} - 1 \dots (i)$

$y = mx + c \dots (ii) ; \text{Slope } \frac{1}{f} = \frac{b}{c}$

18.  $u + v = x \rightarrow (1) ; \frac{v}{u} = m \rightarrow (2); \frac{1}{f} = \frac{1}{u} + \frac{1}{v} \rightarrow (3)$

Solving (1),(2),(3),  $f = \frac{mx}{(m+1)^2}$

19.  $\frac{f^1}{f} = \frac{(\mu_{lens} - 1)}{\left(\frac{\mu_{lens} - 1}{\mu_{liq}}\right)}$

20.  $\frac{f^1}{f} = \frac{(\mu_{lens} - 1)}{\left(\frac{\mu_{lens} - 1}{\mu_{liq}}\right)}$

21)  $\frac{1}{f} = (\mu - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$

22)  $\frac{1}{f} = (\mu - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$

23)  $\frac{1}{f} = \left( \frac{1}{v} + \frac{1}{u} \right)$



RAY OPTICS WORKSHEET-5 KEY**CUQ**

- 1) 4            2) 4            3) 4            4) 2            5) 4            6) 3            7) 3  
 8) 1            9) 3

**JEE MAIN & ADVANCED**

- 1) 1            2) 2            3) 3            4) 1            5) 2            6) 3            7) 1  
 8) 1            9) 4            10) 4            11) 3

$$1. \quad u = -3m \quad v = -1m \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$2. \quad u = -25cm \quad v = -10cm \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$3. \quad p = \frac{1}{f} ; \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$4. \quad u = -25cm ; \quad v = -50cm ; \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$5. \quad f = \text{farpoint} = \frac{100}{p}$$

$$6. \quad u = -25cm ; \quad v = -50cm ; \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$7. \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$8. \quad p = \frac{100}{f}$$

$$9. \quad \frac{1}{F} = \frac{1}{v} - \frac{1}{u} ; \quad u = \infty, \quad v = -20cm$$

$$10. \quad v = -(16-1) = -15cm \quad \quad \quad u = 26-1 = 25cm \quad \text{and} \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$11) \quad \frac{1}{f.p} = \frac{1}{-(\text{distance of object})} = \frac{1}{f} = p$$

\*\*\*\*\*